

June 4, 2004

## On the sample question in Recap 2

**Q** Why is it stated in the question that one should maximize w.r.t.  $k_{t-1}$ ? Shouldn't one first do this when you have maximized w.r.t.  $c$ ,  $m$ , and  $n$ .

**A** First a minor detail. In the question it is stated that maximization is over  $k$ . In this context this means the current period capital, and *not*  $k_{t-1}$  which is the state variable (and thus given by history).

Nevertheless, the household's choice variables in period  $t$  are  $c_t$ ,  $n_t$ ,  $k_t$ , and  $m_t$ . The constraints are the budget constraint and the definition of  $a$ . These constraints are substituted into the value function (to eliminate  $k_t$  and  $a_{t+1}$ , respectively), and one has an unconstrained maximization problem over  $c_t$ ,  $n_t$ , and  $m_t$ . To eliminate the partial derivatives of the value function, one then differentiates it w.r.t.  $k_{t-1}$  and  $a_t$  and apply the envelope theorem to get rid of all terms involving  $\partial c_t / \partial k_{t-1}$ ,  $\partial n_t / \partial k_{t-1}$ ,  $\partial m_t / \partial k_{t-1}$ ,  $\partial c_t / \partial a_t$ ,  $\partial n_t / \partial a_t$ , and  $\partial m_t / \partial a_t$ . Look also at Walsh, pp. 81-82.