

**Written exam for the M.Sc in Economics  
Institute of Economics, University of Copenhagen**

**Monetary Economics: Macro Aspects**

**Semester: Spring 2003**

**June 20, 2003**

**4-hour exam**

This set contains 4 pages. All questions must be answered.  
In the evaluation, the three questions will be weighted equally

No auxiliary material is allowed

To be answered in Danish or English

**QUESTION 1:**

Evaluate whether the following statements are true or false. Explain your answers.

- (i) In the Solow (1956) growth model with money, the so-called Tobin effect refers to the fact that a higher steady-state rate of inflation reduces physical capital accumulation and lowers the steady-state capital stock and output per capita.
- (ii) Consider a dynamic general equilibrium setting where individuals have the per-period utility function  $u(c_t, m_t, l_t)$ , where  $c_t$  is consumption,  $m_t$  denotes real money balances,  $l_t$  denotes leisure, and where output  $y_t$  is produced according to  $y_t = f(k_{t-1}, 1 - l_t)$ , where  $k_{t-1}$  is the capital stock. In this setting money is always superneutral.
- (iii) In a dynamic, general equilibrium setting, where cash is needed to purchase consumption goods (i.e., a cash-in-advance constraint is binding), and where agents' per-period utility function is given by  $u(c_t)$ , where  $c_t$  is consumption, the optimal rate of inflation is the one that implements the Friedman rule.
- (iv) In the Lucas "islands" model, anticipated aggregate money shocks have real effects as agents — due to imperfect information — cannot distinguish between local money disturbances and aggregate money disturbances.
- (v) According to the Poole (1970) model, the reason why central banks more and more rely on interest rate operating procedures is the high degree of stability on the money market relative to the goods market.

**QUESTION 2:**

**The optimal nominal interest rate rule in a simple Fuhrer-Moore model**

Consider a simplified version of the Fuhrer-Moore model:

$$y_t = a_1 y_{t-1} - a_3 (i_{t-1} - \mathbf{E}_{t-1} \pi_t) + u_t, \quad 0 < a_1 < 1, \quad a_3 > 0, \quad (1)$$

$$\pi_t = \pi_{t-1} + \gamma y_t + \eta_t, \quad \gamma > 0, \quad (2)$$

where  $y_t$  is log of output,  $i_t$  is the nominal interest rate (the monetary policy instrument),  $\pi_t$  is the inflation rate,  $u_t$  and  $\eta_t$  are independent, mean-zero, serially uncorrelated shocks.  $\mathbf{E}_i$  is the rational expectations operator conditional on information up to and including period  $i$ .

- (i) Discuss briefly equations (1) and (2)

The objective of the central bank is to conduct monetary policy so as to minimize

$$L = \frac{1}{2} \mathbf{E}_t \sum_{i=1}^{\infty} \beta^i [\lambda y_{t+i}^2 + \pi_{t+i}^2], \quad 0 < \beta < 1, \quad \lambda > 0.$$

- (ii) Find the optimal interest rate rule for  $i_t$  as a function of  $\pi_t$  and  $y_t$ .

Hints:

- a) Show first that equations (1) and (2) can be rewritten as

$$\begin{aligned} y_{t+1} &= \theta_t + u_{t+1} \\ \pi_{t+1} &= \pi_t + \gamma \theta_t + v_{t+1}, \quad v_{t+1} \equiv \gamma u_{t+1} + \eta_{t+1}, \end{aligned}$$

with  $\theta_t \equiv [a_1 y_t - a_3 (i_t - \pi_t)] / [1 - a_3 \gamma]$

- b) Then solve the amended model by dynamic programming, treating  $\theta_t$  as the policy instrument and  $\pi_t$  as the state variable  
 c) Show that the envelope theorem gives

$$V_\pi(\pi_t) = \pi_t + \gamma \theta_t + \beta \mathbf{E}_t V_\pi(\pi_{t+1}),$$

where  $V$  is the value function.

- d) Characterize the solution of the form  $\theta_t = B \pi_t$

- (iii) Comment on the coefficient on  $\pi_t$  in the optimal interest rate rule, with particular emphasis on how its value affects the stability properties of the model.

- (iv) Discuss what determines the magnitude of the coefficient on  $\pi_t$  in the optimal interest rate rule. Comment in particular on whether a relatively high value is always synonymous with a high preference for inflation stabilization.

### QUESTION 3:

#### Credibility of monetary policy and the conservative central banker

Consider the following simple model for inflation determination in a closed economy:

$$\pi_t = \mathbf{E}_{t-1}\pi_t + \kappa x_t + \varepsilon_t, \quad \kappa > 0, \quad (1)$$

where  $\pi_t$  is inflation,  $x_t$  is the output gap and  $\varepsilon_t$  is a mean-zero, serially uncorrelated shock with variance  $\sigma^2$ .  $\mathbf{E}_{t-1}$  is the rational expectations operator conditional upon all information up to and including period  $t-1$  as well as the structure of the model. The central bank is assumed to affect aggregate demand through monetary policy, and for simplicity  $x_t$  is taken to be the instrument of monetary policy. The aim of monetary policy is to maximize “social welfare” given by

$$V = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t [\lambda (x_t - k)^2 + \pi_t^2], \quad k > 0, \quad \lambda > 0, \quad 0 < \beta < 1. \quad (2)$$

- (i) Comment briefly on (1) and (2) (explain the underlying economic mechanisms and assumptions).
- (ii) Derive the optimal time-consistent outcomes for output and inflation [Hint: Maximize (2) w.r.t.  $x_t$  subject to (1), which is a sequence of one-period problems, taking as given  $\mathbf{E}_{t-1}\pi_t$ ; from the first-order condition derive  $\mathbf{E}_{t-1}\pi_t$  and the solutions]. What is the social inefficiency of this solution? Explain.
- (iii) If society delegates monetary policymaking to a “conservative” central banker with a utility function given by

$$V = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t [\lambda^c (x_t - k)^2 + \pi_t^2], \quad \lambda > \lambda^c > 0, \quad (3)$$

how will the optimal time-consistent outcomes change relative to those derived in (ii)? Will delegation as envisaged here always be beneficial?

Assume now that equation (1) is replaced by

$$\pi_t = \beta \mathbf{E}_t \pi_{t+1} + \kappa x_t + \varepsilon_t. \quad (4)$$

- (iv) Derive the optimal time-consistent outcomes for output and inflation [Hint: Maximize (2) w.r.t.  $x_t$  subject to (4), which is a sequence of one-period problems, taking as given  $E_t\pi_{t+1}$ ; use the first-order condition with (4) to derive  $\pi_t$  and thus  $x_t$ .] Discuss the social inefficiencies of this solution, and point out similarities and differences with the solution when equation (1) applies.
- (v) Discuss whether a conservative central banker will be beneficial when (4) applies, and discuss whether imposing  $k = 0$  will change the conclusions. [Hint: Remember that the shock  $\varepsilon_t$  is serially uncorrelated.]