Monetary Economics: Macro Aspects, 5/5 2004
Henrik Jensen

Plan for today:

Open-economy Aspects (I)

1. The Obstfeld and Rogoff two-country model
2. Solution under flexible prices

Literature: Walsh (2003, Chap. 6, pp. 269-282).

Introductory remarks

- All analyses so far, have been confined to closed-economy model
- This misses out important transmission mechanisms in most real-life economies:
  - The exchange rate channel
  - The impact of external shocks (policy- and non-policy shocks)

- Purpose today is to develop a tractable, general equilibrium model for open economies
- It builds closely on the famous paper by Obstfeld and Rogoff (1995, *Journal of Political Economy*): "Exchange Rate Dynamics Redux"
- Model initiated a new generation of micro-founded, open-economy models with incomplete nominal adjustment
  - A literature now known as the "New Open Economy Macroeconomics"

- While inherently a sticky-price model, the foundations of the model are easiest seen in a flexible price version
  - Sticky prices are introduced next time

The Obstfeld-Rogoff Two-Country Model:

The flexible price version

Basics

- World of two countries: Domestic/Home and Foreign
  - Continuum $z \in [0, n]$ of agents live in Home country
  - Continuum $z \in (n, 1]$ of agents live in Foreign country
- Each agent is monopolist supplier of distinct good, which is imperfect substitute with all other goods
  - Monopolistic competition market structure
- Utility of a representative domestic agent j:

$$U^j = \sum_{t=0}^{\infty} \beta^t \left[\log C_t^j + b \log \frac{M_t^j}{P_t} - \frac{k}{2} y_t(j)^2 \right], \quad b,\ k > 0 \quad (6.1)$$

- Definition of consumption index:

$$C^j = \left[\int_0^1 c^j(z)^q \, \mathrm{d}z \right]^{\frac{1}{q}}, \qquad 0 < q < 1. \qquad (6.2)$$

  Aggregate of **all goods** produced in the world. Note with $q < 1$, goods are imperfect substitutes

- Corresponding domestic **consumer price** index (see notes for derivation)

$$P = \left[\int_0^1 p(z)^{\frac{q}{q-1}} \, \mathrm{d}z \right]^{\frac{q-1}{q}} \qquad (6.3)$$

- Foreign agents' utility functions are similar
- Nominal budget constraint of representative domestic agent j:

$$P_t C_t^j + M_t^j + P_t T_t + P_t B_t^j \leq p_t(j) y_t(j) + R_{t-1} P_t B_{t-1}^j + M_{t-1}^j$$

  - B_t^j is internationally freely tradable bond with real (gross) return R_t
  - T_t are real government taxes
  - Balanced government budget and no government spending:

$$0 = P_t T_t + (M_t - M_{t-1})$$

  (note mistake in Walsh, p. 271: "$P_t T_t = (M_t - M_{t-1})$")

  - **Money supplies are the policy instruments** in the model

- Budget constraint in real terms:

$$C_t^j + \frac{M_t^j}{P_t} + T_t + B_t^j \leq \frac{p_t(j)}{P_t} y_t(j) + R_{t-1} B_{t-1}^j + \frac{1}{1+\pi_t} \frac{M_{t-1}^j}{P_{t-1}}$$

  π_t is domestic consumer-price inflation

- No barriers to trade in goods
  - The price of a good z is the same Home and abroad measured in common currency:

$$p(z) = Sp^*(z)$$

  - S is **nominal exchange rate** (price of foreign currency in terms of domestic currency $\Rightarrow S \uparrow$ is a domestic depreciation)
  - $p^*(z)$ is foreign currency price of good z (stars indicate foreign variables)
  - This law-of-one-price leads to PPP:

$$P_t = S_t P_t^* \tag{6.8}$$

    - * Note! Empirically problematic assumption in short run....

Optimal behavior

- Each agent must determine optimally:
  - Aggregate consumption dynamics (and thus asset holdings)
  - Relative demand for different goods ("composition" of aggregate demand)
  - Production effort of "own" good

Optimal production decision

- Optimal demand for good z by individual j (see notes)

$$c^j(z) = \left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C^j.$$

  - Demand for good z is increasing in total consumption
  - Demand for good z is decreasing in the real price of the good, $p(z)/P$
  - Note that $1/(1-q)$ is the elasticity of substitution between goods.
  - Hence, the limiting case of perfect competition applies when $q \to 1$ (horizontal demand curve and no monopoly power)
- **Total demand** in the two countries of a good z is:
  - n times domestic demand plus $1-n$ times foreign demand.

- A representative foreign consumer's demand function is

$$c^{*j}(z) = \left[\frac{p^*(z)}{P^*}\right]^{-\frac{1}{1-q}} C^{*j}$$

- As the nominal exchange rate is S, this is the same as

$$c^{*j}(z) = \left[\frac{Sp^*(z)}{SP^*}\right]^{-\frac{1}{1-q}} C^{*j},$$

and thus, by the law of one price,

$$c^{*j}(z) = \left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C^{*j}$$

- **Total demand**, $y^d(z)$, is therefore

$$\begin{aligned} y^d(z) &= n\left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C + (1-n)\left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C^* \\ &= \left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} [nC + (1-n)C^*] \\ &= \left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C^w, \end{aligned}$$

- $C^w \equiv nC + (1-n)C^*$ is world consumption
- Optimal production given by standard consumption-leisure trade-off:

$$\frac{MUTILIY_{leisure}}{MUTILITY_{consumption}} = MRETURN\ OF\ WORK$$

or,

$$-\frac{MLOSS_{work}}{MUTILITY_{consumption}} = MRETURN\ OF\ WORK$$

- We have from utility function:

$$-\frac{MLOSS_{work}}{MUTILITY_{consumption}} = \frac{ky_t^j}{\left(1/C_t^j\right)}$$

- What is the real return from work? It is

$$\frac{p_t(z)\,y_t(z)}{P_t}$$

so marginal return is found as

$$\frac{p_t(z)}{P_t} + \left[\partial\left(\frac{p_t(z)}{P_t}\right)/\partial y_t(z)\right] y_t(z)$$

- The real price is found by "inverting" the demand curve:

$$\frac{p_t(z)}{P_t} = \left[\frac{y_t^d(z)}{C_t^w}\right]^{q-1}$$

  - Note again, that with $q < 1$ the return for work effort is lower, as it reduces the relative price; only with $q \to 1$ is the price taken as given, and perfect competition mimicked

- The marginal product of work effort is therefore

$$q\left[\frac{y_t^d(z)}{C_t^w}\right]^{q-1}$$

- Optimal consumption-leisure choice is therefore characterized by

$$\frac{ky_t^j}{\left(1/C_t^j\right)} = q\left[\frac{y_t^j}{C_t^w}\right]^{q-1} \qquad (6.6')$$

Optimal money demand decision

- Derived from standard, general condition:

$$\frac{MUTILITY_{money}}{MUTILITY_{consumption}} = \frac{i_t}{1+i_t}$$

with $i_t = R_t(1+\pi_{t+1}) - 1$ being the nominal interest rate

- With the assumed utility function:

$$\frac{b\left[1/\left(M_t^j/P_t\right)\right]}{1/C_t^j} = \frac{i_t}{1+i_t}$$

and therefore simple money demand function

$$\frac{M_t^j}{P_t} = bC_t^j \frac{1+i_t}{i_t} \tag{6.7}$$

Optimal intertemporal consumption allocation

- Simple version of Keynes-Ramsey rule:

$$MUTILITY_{consumption}(t) = \beta R_t MUTILITY_{consumption}(t+1)$$

$$\frac{1}{C_t^j} = \beta R_t \frac{1}{C_{t+1}^j}$$

$$C_{t+1}^j = \beta R_t C_t^j \tag{6.5}$$

Equilibrium conditions and flex-price implications

- Equilibrium satisfies the optimality conditions (Home and Foreign); budget constraints, PPP, and two additional market clearing conditions:
- Goods market clearing:

$$C_t^w = n\frac{p_t(h)}{P_t}y_t(h) + (1-n)\frac{p_t^*(f)}{P_t^*}y_t^*(f) \equiv Y_t^w$$

with $p_t(i)$ and $y_t(i)$ being representative price and production in country i

- Bond-market clearing:

$$nB_t + (1-n)B_t^* = 0$$

- As well known, monopolistic competition gives no real effects of money (but is natural modelling framework when introducing sticky prices)
- This is confirmed in this model:
  - E.g., an increase in M_t leads to a proportional increase in $p_t(h)$ and P_t and a proportional increase in S_t (a nominal depreciation)
  - All real variables and foreign prices are unchanged

- Model also exhibits superneutrality
  - An increase in the growth rate of M_t will increase Home CPI inflation and home nominal interest rate, and create a higher increase in S_t
  - All real variables and foreign prices are unchanged (this follows from the separability of the utility function)
- Emphasized in steady-state solution for domestic and foreign consumption (from the budget constraints):

$$C = \frac{p(h)}{P} y(h) + (R-1)B \qquad (6.9)$$

$$C^* = \frac{p^*(f)}{P^*} y^*(f) - (R-1)\frac{n}{1-n}B \qquad (6.10)$$

Real steady-state consumption equals real steady-state income (real value of output plus/minus bond income)

A linear approximation

- As indicated, the nominal exchange rate plays a role under flexible prices to neutralize real effects of monetary shocks
- A log-linearized version of model serves to highlight this (lower-case letters are log deviations)
- Domestic optimality conditions:

$$y_t = \frac{1}{1-q}\left[p_t - p_t(h)\right] + c_t^w \qquad (6.13)$$

$$(2-q)\,y_t = (1-q)\,c_t^w - c_t \qquad (6.18)$$

$$m_t - p_t = c_t - \delta\left(r_t + \pi_{t+1}\right), \qquad \delta = \beta/\left(\overline{\Pi} - \beta\right) \qquad (6.20)$$

$$c_{t+1} = c_t + r_t \qquad (6.16)$$

- Domestic price index written in terms of foreign producer prices and the nominal exchange rate:

$$p_t = np_t(h) + (1-n)\left[s_t + p_t^*(f)\right] \qquad (6.11)$$

- These five equations plus the five Foreign analogues, and the definition of c_t^w, will determine the eleven endogenous variables:
  - Home and foreign production (y_t, y_t^*)
  - Home, foreign and world consumption (c_t, c_t^*, c_t^w)
  - Prices and the nominal exchange rate $(p_t(h), p_t, p_t^*(f), p_t^*, s_t)$
  - The real interest rate (r_t)

- Money neutrality is seen immediately from linearized model
- The two price indices, imply the PPP relationship:

$$s_t = p_t - p_t^*$$

- Used in domestic price index:

$$p_t = np_t(h) + (1-n)(p_t - p_t^* + p_t^*(f))$$

$$0 = n[p_t(h) - p_t] + (1-n)[p_t^*(f) - p_t^*]$$

or,

$$0 = n\chi_t + (1-n)\chi_t^*$$

- Demand functions are correspondingly re-written as

$$y_t = \frac{1}{1-q}\chi_t + c_t^w,$$

$$y_t^* = \frac{1}{1-q}\chi_t^* + c_t^w$$

- These three equations will along with...
  - ...the two consumption Euler equations
  - ...the two production decision equations
  - ...the definition of c_t^w
- ... determine the **real allocation**: χ_t, χ_t^*, y_t, y_t^*, c_t, c_t^*, c_t^w, r_t

- Hence, changes in m_t or m_t^* have no real effects
- Only effects on prices and the nominal exchange rate (as mentioned previously, an increase in m_t increase p_t, $p_t(h)$ and s_t proportionally)
- To see how the nominal exchange rate is determined, subtract the money market equilibrium conditions:

$$m_t - m_t^* - (p_t - p_t^*) = (c_t - c_t^*) - \delta(\pi_{t+1} - \pi_{t+1}^*)$$

(note that the inflation differential measures the nominal interest differential, as Home and Foreign face same real interest rate)

- Then use the PPP relationship in level and change form:

$$m_t - m_t^* - s_t = (c_t - c_t^*) - \delta(s_{t+1} - s_t) \qquad (6.23)$$

- First-order (expectational) difference equation in s_t :

$$s_t = \frac{\delta}{1+\delta}s_{t+1} + \frac{1}{1+\delta}[(m_t - m_t^*) - (c_t - c_t^*)]$$

- Solving (6.23) successively forward and imposing $\lim_{i\to\infty}(\delta/(1+\delta))^i s_{t+i} = 0$ yields

$$s_t = \frac{1}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{\delta}{1+\delta}\right)^i [(m_{t+i} - m_{t+i}^*) - (c_{t+i} - c_{t+i}^*)]$$

Nominal exchange rate equals present value of current and future relative money supplies and demands (the latter quantified by the consumption differential)

- Expression can be simplified as any consumption differential is **permanent** since

$$c_{t+1} = r_t + c_t$$

and

$$c^*_{t+1} = r_t + c^*_t$$

imply

$$c_{t+1} - c_t = c^*_{t+1} - c^*_t$$

and

$$c_{t+1} - c^*_{t+1} = c_t - c^*_t$$

- Hence,

$$s_t = -\left(c_t - c^*_t\right) + \frac{1}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{\delta}{1+\delta}\right)^i \left(m_{t+i} - m^*_{t+i}\right)$$

(since $\frac{1}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{\delta}{1+\delta}\right)^i = 1$)

- Hence,
  - $m_t > m^*_t$: relative supply of home currency increases, and the relative value of the currency falls (s_t increases; home currency depreciates)
  - $m_{t+i} > m^*_{t+i}$: future relative supply of home currency increases; future inflation differential increases, and reduces relative demand for home currency; relative value of the currency falls (s_t increases; home currency depreciates)
  - $c_t > c^*_t$: current or future money demand shifts towards home currency; relative value of the currency increases (s_t decreases; home currency appreciates)

- Alternative exposition featuring the relationship between nominal interest rates and the nominal exchange rates; **uncovered interest parity**
- Real interest rate is identical across countries:

$$i_t - \pi_{t+1} = i^*_t - \pi^*_{t+1}$$

and thus

$$i_t - i^*_t = \pi_{t+1} - \pi^*_{t+1}$$

- From the PPP relationship in first differences:

$$i_t - i^*_t = s_{t+1} - s_t$$

- Hence, in a world with perfect capital mobility and floating exchange rates, the nominal interest rate differential equals (expected) depreciation rate of home currency
- Only then is holding home and foreign bonds equally attractive
- Empirically, tests of the "UIP" relationship have been unsuccessful
  - Not necessarily reason for abandoning model
  - Note, as with estimations of term-structure relationships covered in Chapter 10, **how monetary policy is conducted**, will matter for regression results
    * If the nominal interest rate was instrument, and was responding to the exchange rate, a regression of the type

$$s_{t+1} - s_t = a + b\left(i_t - i^*_t\right) + \varphi_{t+1}$$

should not necessarily give $b = 1$ (as $i_t - i^*_t$ would be a positive function of s_t)

Concluding remarks

- The flex-price version of the Obstfeld-Rogoff model only gives preliminaries for the analysis of monetary policy in open economies
- The model is therefore amended with one period price rigidity. The role of the nominal exchange rate for real allocation is then evident:

$$p_t = np_t(h) + (1-n)[s_t + p_t^*(f)];$$

  with $p_t(h)$ and $p_t^*(f)$ fixed, fluctuations in the nominal exchange rates cause fluctuations in the aggregate price index

  - New transmission channel of monetary policy
  - Due to the openness of economies, monetary policy in one country will have effects on the other
  - Good or bad effects? Scope for policy coordination?
- Next time.....

Plan for next lecture

Monday, May 10, 12-14

Open-economy Aspects (II)

1. The Obstfeld and Rogoff two-country model with sticky prices
2. An example of international monetary policy coordination

Literature: Walsh (2003, Chap. 6, pp. 282-297). Read also (small) Section 6.4 on the small open economy.

As supplementary recent readings on policy coordination, I recommend Benigno (2002, *Journal of International Economics*) and Clarida et al. (2002, *Journal of Monetary Economics*)