

### Plan for today:

1. Interest rate policies (I)
2. Price level (in)determinacy
3. The term structure of interest rates
4. A model for monetary policy analysis and the impact of interest rate rule parameters

Literature: Walsh (Chapter 10, pp. 473-480; pp. 488-499; pp. 499-507)

3. Plan for next lectures

NB: Question from last lecture still not solved! Come on! The prize is still available!

## Introductory remarks

- Most central banks today uses the interest rate as the monetary policy instrument
  - Theoretical foundation: Unstable money demand, volatile financial markets . . .
  - . . . cf. Poole analysis
  - Note: Money stock may be relevant if it **provides information** about shocks to goal variables (it will then be an **intermediate target**)
    - \* (This will be the case if money market shocks are “small”; cf. slides of March 24)
- Implications to consider:
  - Theoretically, interest targeting procedures can give problems with price level determination
  - With money “out of the picture,” long term interest rates becomes more important, and thus the link between the short interest rate (the policy instrument) and the long interest rates (the rate typically affecting demand)
  - How does various interest rate policy rules perform in terms of stabilizing the economy?
- These are the subjects covered today

## Price level (in)determinacy

- Using the interest rate as monetary policy instrument may render the price level *indeterminate*
- Simple AS/IS model under an interest rate operating procedure
 
$$y_t = a(p_t - E_{t-1}p_t) + e_t, \quad a > 0 \quad (10.6')$$

$$y_t = -\alpha r_t + u_t, \quad \alpha > 0 \quad (10.7')$$

$$i_t^T = r_t + (E_t p_{t+1} - p_t) \quad (10.8)$$
- The price level appears as
  - An expectational error in AS curve
  - An expected change in Fisher relationship
- Hence, the price level as such is **not determined** by the model
  - If  $(p_t^*, p_{t+1}^*)$  is a solution, then  $(p_t^* + \kappa, p_{t+1}^* + \kappa)$  is a solution for any  $\kappa$
  - The price expectation error and expected price change is *independent* of  $\kappa$
- The endogenous money supply will be indeterminate as well
  - If private agents suddenly expects higher prices, the expectation is self-fulfilling
  - The central bank must increase the nominal money supply to maintain  $i_t^T$
  - . . .  $m_t$  is not determined by the model when  $p_t$  is not
- This is problematic, e.g., when writing nominal contracts, as the price level can suddenly shift for no fundamental reasons

- Is an interest rate operating procedure thus disastrous?
- Not necessarily, as long as the central bank to some extent care about the price level
  - Either by adjusting the interest rate in response to price movements, e.g.,  $i_t = i_t^T + f(p_t)$
  - or, by letting policy to some extent depend on the nominal money supply
- In either case, the central bank's policy will then contain a *nominal anchor*, which will determine the price level

- Example of policy depending on the money supply:

$$m_t = \mu_0 + m_{t-1} + \mu (i_t - i_t^T) \quad (10.9)$$

- This is equivalent to the money base rule considered in last lecture

– Note that for  $\mu \rightarrow \infty$  this corresponds to a pure interest rate operating procedure

- With this rule, the LM curve re-enters the model (to determine actual nominal interest rate):

$$m_t - p_t = -c i_t + y_t + v_t, \quad c > 0 \quad (10.3)$$

- The price level now enters directly, and it will be determined by the model

- For a high value of  $\mu$  the central bank can still be interpreted as acting under an interest rate operating target

– The “aggressive” response of the money supply towards  $(i_t - i_t^T)$  minimizes deviations in actual interest rate from target value  
 – . . . and the price level becomes determinate

- General lesson: Under an interest rate operating procedure, policy design should secure determinacy by letting the interest rate respond to the price level

– Directly . . .  
 – . . . or indirectly

## The term structure of interest rates

- Under an interest rate operating procedure, money demand plays a minor role in transmission of monetary policy
- Money demand depends on short-term interest rates, which are under closest control of the central bank
- Aggregate demand, on the other hand, typically depends on long-term interest rates, which are market determined
- Hence, under and interest rate operating procedure the link between short- and long term interest rates becomes of high importance
- The link is denoted the **term structure**
- Common theory for analyzing the link between long and short interest rates is the **expectations theory** of the term structure
- Simple no-arbitrage condition:
  - The nominal return of holding an  $n$ -period bond to maturity must equal the return of holding an  $n$ -period sequence of one-period bonds
  - Define  $i_{n,t}$  as the nominal yield to maturity on a  $n$ -period bond in period  $t$
  - Define  $i_t$  as the nominal yield on a one-period bond

- The condition (which ignores realistic risk-premia on long bonds) formally:

$$(1 + i_{n,t})^n = \prod_{i=0}^{n-1} (1 + i_{t+i})$$

- Approximately (take logs and assume small yields);

$$i_{n,t} = \frac{1}{n} \sum_{i=0}^{n-1} i_{t+i}$$

- Hence, the long interest rate is an unweighted average of current and future short interest rates
- Note that the condition can be written as a difference equation in the long interest rate:

$$\begin{aligned} & - \text{An } n\text{-period bond becomes an } n - 1\text{-period bond in next period; therefore} \\ & (1 + i_{n,t})^n = (1 + i_t) (1 + i_{n-1,t+1})^{n-1} \quad (*) \end{aligned}$$

...which approximately is

$$i_{n,t} = \frac{1}{n} i_t + \frac{n-1}{n} i_{n-1,t+1}$$

- Notice how (\*) can be interpreted as a no-arbitrage condition:

$$(1 + i_t) = \frac{(1 + i_{n,t})^n}{(1 + i_{n-1,t+1})^{n-1}}$$

- The right-hand side is the one-period gross return on the long bond. Why:

- Price of bond in period  $t$  is  $P_{n,t} = 1 / (1 + i_{n,t})^n$
- Price of bond in period  $t + 1$  is  $P_{n-1,t+1} = 1 / (1 + i_{n-1,t+1})^{n-1}$
- Gross return is  $P_{n-1,t+1} / P_{n,t} = (1 + i_{n,t})^n / (1 + i_{n-1,t+1})^{n-1}$

- Notice that under uncertainty the relationships are not correct
- The expected one-period return on the long bond is

$$E_t \frac{(1 + i_{n,t})^n}{(1 + i_{n-1,t+1})^{n-1}}$$

- The no-arbitrage condition is therefore

$$(1 + i_t) = E_t \frac{(1 + i_{n,t})^n}{(1 + i_{n-1,t+1})^{n-1}}$$

which does **not** imply (\*) under uncertainty:

$$(1 + i_{n,t})^n = (1 + i_t) E_t (1 + i_{n-1,t+1})^{n-1}$$

- This mistake is ignored here. Important matters are that:

- Long interest rate will be function of current and expected future interest rates

– If aggregate demand depends on long rates, **expectations about future monetary policy, future short rates, becomes of importance**

- Simple example with the “long” bond being a two period bond:

$$(1 + I_t)^2 = (1 + i_t) (1 + E_t i_{t+1})$$

$I_t$  is “long” interest rate

- Approximately:

$$I_t = \frac{1}{2} (i_t + E_t i_{t+1})$$

- The associated simple term structure:

$$I_t - i_t = \frac{1}{2} (E_t i_{t+1} - i_t)$$

- Hence, long rate exceeds short rate if short rates are expected to increase
- If theory is correct, the interest rate spread — the yield curve — is **an indicator** for expectations about future monetary policy; and thus future economic activity!
- Empirically, the difference between long and short rates do have predictive power concerning, e.g., output

- Empirically, the expectations theory of the term structure has, however, had mixed success

- The theory implies

$$\frac{1}{2} (i_{t+1} - i_t) = I_t - i_t + \theta_t$$

$$\theta_t = \frac{1}{2} (i_{t+1} - E_t i_{t+1})$$

- A regression like

$$\frac{1}{2} (i_{t+1} - i_t) = a + b (I_t - i_t) + \theta_t$$

should therefore give  $a = 0$  and  $b = 1$

- Rarely the case! Usually  $b < 1$  (and even  $b < 0$ ) in estimations!
  - However, the theory does not take monetary policymaking into consideration! (Ignores operating procedures)
  - If central bank tends to “smooth” interest rates  **$b$  should not be one**
    - \* Extreme (illuminating) case:
    - \* Policy is  $i_t = i_{t-1}$ . Then, an increase in  $I_t$ , e.g., due to an increase in the risk premium, will not lead to a change in the short rate
  - Also, if the central bank tries to dampen the economic effects of an increased risk premium (by lowering short rates now and in the future), then  $b < 0$  is what theory says
- For lack of better alternative, we stick with the expectations theory of the term structure

- Important implication of the theory concerning **inflation expectations**

- We have

$$\begin{aligned}
 I_t &= \frac{1}{2}(i_t + E_t i_{t+1}) \\
 &= \frac{1}{2}(r_t + E_t r_{t+1} + E_t \pi_{t+1} + E_t \pi_{t+2})
 \end{aligned}$$

where  $r_t$  is the short real interest rate

- Assuming the real rate fluctuates little,  $r_t = E_t r_{t+1} \approx \bar{r}$  we get

$$\begin{aligned}
 I_t &= \frac{1}{2}(\bar{r} + E_t \pi_{t+1} + E_t \pi_{t+2}) \\
 &= \frac{1}{2}(\bar{r} + E_t \bar{\pi}_{t+2})
 \end{aligned}$$

with  $\bar{\pi}_{t+2} = p_{t+2} - p_t$

- Hence, increases in the long rate may reflect increased inflation expectations

- Changes in long rates thus often reflect changes in inflation expectations

- Note if the **short** nominal interest rate is **increased**, the **long** rate may **decrease**, as it can reflect that the market believes that the central bank is trying to bring down future inflation

## A model for monetary policy analysis and the impact of interest rate rule parameters

- The simple one-period sticky-wage variant of the MITU model, did not provide adequate output and inflation dynamics
- Here focus is on extended version of the model, which
  - allows for richer dynamics
  - allows for a distinction between long and short interest rate
  - allows for an analysis of the performance of various (short) interest rate policy rules
- Model is due to Fuhrer and Moore
- Simple closed economy IS/AS model
- IS curve:

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} - a_3 r_{t-1} + u_t \quad (10.35)$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

- More lags (empirically, not theoretically explained)
- Delayed response of demand to a change in the long real interest rate  $r_t$

- Term structure equation formulated as a difference equation in the long rate:

$$r_t - D(E_t r_{t+1} - r_t) = i_t^f - E_t \pi_{t+1} \quad (10.36)$$

- $i_t^f$  is the one period nominal interest rate — the policy instrument
- $D$  is a measure of the duration of the long bond (equals the term for conventional discount bonds)
- Note if  $r_t > i_t^f - E_t \pi_{t+1}$ , agents must be expecting a capital loss on the long bond, i.e.,  $E_t r_{t+1} > r_t$
- Note that a given expected capital loss drives up the long rate by more, the longer term the bond has (an expected increase in the interest rate is more costly when the bond has a long term)

- AS curve: A Fuhrer-Moore **inflation inertia equation**:

$$\pi_t = \frac{1}{2}(\pi_{t-1} + E_t \pi_{t+1}) + \gamma q_t + \eta_t \quad (10.40)$$

where  $q_t$  summarizes the business cycle stance (lagged, current and expected future output)

- Price adjustment structure like staggered price setting, but here in terms of inflation rather than prices
- Empirically plausible; theoretically an ongoing issue to model it satisfactorily (Fuhrer and Moore is just one attempt)

- Model is closed by a specification of the monetary policy rule (an interest rate rule):

$$i_t^f = b_1 i_{t-1}^f + b_2 (\pi_t - \pi^T) + b_3 y_t + b_4 (y_t - y_{t-1}) + \varphi_t, \quad ((10.41))$$

$\varphi_t$  is “policy shock”

- Model is calibrated, and simulated (with estimated parameters; cf. Table 10.1)

- The baseline show that a contractive interest rate shock give rise to output and inflation dynamics which reasonable mimics what is found in VAR analyses

- The long rate becomes expansive after a while, in anticipation of future short interest rate cuts to bring back the economy to initial equilibrium

- Implications of demand shocks:

- Hump-shaped output response as one “would like”

- Changing the policy rule parameters:

- Increasing  $b_3$  substantially lowers output variance and causes a faster return to initial equilibrium

- Increasing  $b_2$  also lowers output variance (a reflection of the fact that the demand shock pushes output and inflation in same directions; so does the interest rate)

- The long rate will now respond less, in anticipation of the faster adjustment — hence a more aggressive policy rule stabilize the long interest rate

- Very simple model captures empirical regularities, and is nice laboratory for analyzing **performance of alternative policy rules**
  - Models of this basic type play large role for real-life central banks (US Fed uses — larger — version of such model for policy analyses)
- Care must be taken: We have gone further and further away from micro-founded models, and the Lucas critique may start to “bite”!
- Looking positively at this: The good performance of the model invites a search for micro-founded models which, e.g., generate the lags in the IS equation
  - Some recent progress on this: Habit formation in utility function
  - Then, utility of consumption today depends on consumption yesterday
  - Implication: Current consumption will depend on past consumption, which translates into output persistence
- Immediate issue: Given that model is “good,” what is then **the best** ( $\equiv$ optimal) interest rate policy?

## Plan for next lecture

Wednesday, March 31

Interest rate policies (II)

1. Optimal interest rate rule in simple model for policy analysis
2. International evidence on interest rate rules

Literature: Walsh (Chapter 10, pp. 508-515); Clarida et al. (1998); Taylor (1993)

Monday, April 4 (last lectures before Easter)

Exercises (to be announced)