Monetary Economics: Macro Aspects, 23/2 2004
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Plan for today:

1. Cash-in-Advance models (stochastic)
2. Money and real costs of transactions

Literature: Walsh (Chapter 3, pp. 111-120, 126-131)

3. Plan for next lectures

Cash-in-Advance Models: Stochastic case

- Simple CIA model under certainty exhibited long-run superneutrality
- It is possible that this is not the case, and issues are therefore:
  - What is the potential nature of non-superneutralities in the short and the long run?
  - What are the quantitative implications of inflation and the CIA constraint?
  - Will dynamics match the data in a stochastic version?
  - Can monetary policy play a stabilizing role?
- Issues addressed in stochastic CIA model (solved, calibrated, and simulated — just as the stochastic MIU model)
- Exogenous shocks bringing the economy off steady state are **technology shocks** and nominal money growth shocks
- The channel causing non-superneutrality is (as in the MIU approach) an endogenous labor supply decision

Model and private sector optimization

- Production is given by Cobb-Douglas function

$$y_t = f(k_{t-1}, n_t, z_t) = e^{z_t} k_{t-1}^{\alpha} n_t^{1-\alpha}, \qquad 0 < \alpha < 1, \qquad (3.33')$$

z_t is a technology shock. Model now features endogenous labor

- Assumption about technology shock as in stochastic MIU model:

$$z_t = \rho z_{t-1} + e_t, \qquad |\rho| < 1,$$

with e_t being a mean-zero, white-noise shock

- Nominal money growth rate:

$$\theta_t = \theta^{ss} + u_t$$

where u_t is a shock to the growth rate

- Assumption as in stochastic MIU model:

$$u_t = \gamma u_{t-1} + \phi z_{t-1} + \varphi_t, \qquad 0 \leq \gamma < 1, \quad \phi \lesseqgtr 0$$

with φ_t being a mean-zero, white-noise shock

  – As in MIU model there may or may not be serial correlation in the shocks to nominal money growth

  – As in MIU model, money growth may or may not respond toward past technology shocks, and may be either pro-cyclical ($\phi > 0$) or countercyclical ($\phi < 0$)

- Per-period utility function (like that in stochastic MIU model without money and $a = 1$):

$$u(c_t, 1-n_t) = \frac{(c_t)^{1-\Phi}}{1-\Phi} + \Psi \frac{(1-n_t)^{1-\eta}}{1-\eta},$$

η, Φ, $\Psi > 0$ (Φ, η are coefficients of relative risk aversion).

- Compared to simple CIA model under certainty, leisure provides utility, and a **consumption-leisure decision** will potentially be affected by the CIA constraint

- The CIA constraint is (on consumption goods):

$$c_t \leq \frac{m_{t-1}}{1+\pi_t} + \tau_t \equiv a_t \qquad (3.50')$$

- The budget constraint is (ignoring nominal debt, b, for simplicity):

$$e^{z_t} k_{t-1}^{\alpha} n_t^{1-\alpha} + (1-\delta) k_{t-1} + a_t = c_t + k_t + m_t \qquad (3.51')$$

- Optimization is characterized by (k_{t-1} and a_t are state variables):

$$V(k_{t-1}, a_t) = \max \left\{ \frac{(c_t)^{1-\Phi}}{1-\Phi} + \Psi \frac{(1-n_t)^{1-\eta}}{1-\eta} + \beta \mathrm{E}_t V(k_t, a_{t+1}) \right\}$$

Maximization is over c, m, n, k, and a subject to the CIA constraint, budget constraint and the definition of a.

- Usual trick: Eliminate k_t and a_{t+1} by the budget constraint and definition of a, and one "only" maximizes over c, m and n subject to the CIA constraint

- Let μ_t denote the Lagrange multiplier associated with the CIA constraint
- First-order condition with respect to c_t:

$$c_t^{-\Phi} = \beta \mathrm{E}_t V_k (k_t, a_{t+1}) + \mu_t \tag{3.52'}$$

Marginal utility of consumption equals the marginal losses, which are the expected, discounted marginal value of next-period capital **plus** the "price" of holding cash as measured by μ_t (cost of liquidity services provided by money when nominal interest rate is positive)

  – NB: As in simple CIA model: Marginal cost of consumption is **higher** when the CIA constraint binds

- First-order condition with respect to m_t:

$$\beta \mathrm{E}_t \left[\frac{V_a (k_t, a_{t+1})}{1 + \pi_{t+1}} \right] = \beta \mathrm{E}_t V_k (k_t, a_{t+1}) \tag{3.54'}$$

Expected marginal value in terms of more next-period money wealth, equals the expected marginal value in terms of lower capital holdings

- First-order condition with respect to n_t:

$$\Psi (1 - n_t)^{-\eta} = (1 - \alpha) \beta \mathrm{E}_t V_k (k_t, a_{t+1}) \mathrm{e}^{z_t} k_{t-1}^{\alpha} n_t^{-\alpha} \tag{3.55'}$$

Marginal loss in terms of less leisure equals the expected value of higher future capital (which is higher the higher is the marginal product of labor)

- Relationships between partial derivatives of the value function from the envelope theorem:

$$V_k (k_{t-1}, a_t) = \beta \mathrm{E}_t V_k (k_t, a_{t+1}) \left[\alpha \mathrm{e}^{z_t} k_{t-1}^{\alpha-1} n_t^{1-\alpha} + 1 - \delta \right] \tag{3.57'}$$

The marginal value of current capital equals the expected marginal value of future capital "corrected for" the net marginal product of current capital (Keynes-Ramsey rule "in disguise")

$$V_a (k_{t-1}, a_t) = \mu_t + \beta \mathrm{E}_t V_k (k_t, a_{t+1}) \tag{3.56'}$$

The marginal value of real balances per se equals the marginal costs in terms of the "price" of the CIA constraint and the expected value of lower capital

Steady state, and the form of non-superneutrality

- Let $\lambda_t \equiv \beta \mathrm{E}_t V_k(k_t, a_{t+1})$ be discounted, expected the marginal value of capital
  - We get (f.o.c. for c)
$$c_t^{-\Phi} = \lambda_t + \mu_t \tag{3.59}$$
  - We get (f.o.c. for m)
$$\beta \mathrm{E}_t \left[\frac{\lambda_{t+1} + \mu_{t+1}}{1+\pi_{t+1}}\right] = \lambda_t \tag{3.60}$$
    Remember the marginal value of money, $V_a(k_t, a_{t+1}) / (1+\pi_{t+1})$ indeed are μ and λ [from (3.56')].
  - We also get (from relationship between the derivatives of the value function):
$$\lambda_t = \beta \mathrm{E}_t R_t \lambda_{t+1} \tag{3.62}$$
    where $R_t = \alpha \mathrm{e}^{z_{t+1}} k_t^{\alpha-1} n_{t+1}^{1-\alpha} + 1 - \delta = \alpha(y_{t+1}/k_t) + 1 - \delta$
  - Finally, we get (from f.o.c. for n):
$$\Psi(1-n_t)^{-\eta} = \lambda_t (1-\alpha)\left(\frac{y_t}{n_t}\right)$$
- In steady state we have $\beta R^{ss} = 1$. This determines y^{ss}/k^{ss} **independently of monetary factors**
- From resource constraint, $y^{ss} = c^{ss} + \delta k^{ss}$ one identifies $(c^{ss}/k^{ss}) = (y^{ss}/k^{ss}) - \delta$
- From production function, one gets $(n^{ss}/k^{ss}) = (y^{ss}/k^{ss})^{1/(1-\alpha)}$

- What then determines n^{ss}?
- Essentially, the consumption-leisure decision:
$$\frac{(c^{ss})^{-\Phi}}{\Psi(1-n^{ss})^{-\eta}} = \frac{\lambda^{ss} + \mu^{ss}}{\lambda^{ss}(1-\alpha)(y^{ss}/n^{ss})}$$
- A higher μ^{ss} tends to make consumption more costly relative to leisure (the nominal interest rate is positive implying positive costs of holding money for transactions); hence, less labor is supplied. The CIA constraint **implies a distorting tax**
- More specifically, in steady state
$$\beta\left[\frac{\lambda^{ss} + \mu^{ss}}{1+\pi^{ss}}\right] = \lambda^{ss}$$
$$\beta\left[\frac{1+\mu^{ss}/\lambda^{ss}}{1+\pi^{ss}}\right] = 1$$
$$\frac{1+\mu^{ss}/\lambda^{ss}}{1+\pi^{ss}} = \frac{1}{\beta} = R^{ss}$$
- So (as in simple CIA model)
$$i^{ss} = \mu^{ss}/\lambda^{ss}$$
  and consumption leisure choice becomes
$$\frac{(c^{ss})^{-\Phi}}{\Psi(1-n^{ss})^{-\eta}} = \frac{1+i^{ss}}{(1-\alpha)(y^{ss}/n^{ss})}$$
- I steady-state, higher money growth and inflation will **raise the nominal interest rate** and induce a **substitution away from the cash good** (consumption) towards the "non-cash" good, leisure:
$$\boxed{\frac{\partial n^{ss}}{\partial \theta^{ss}} < 0}$$

- Note in contrast with the MIU model with leisure, the non-neutrality is non-ambiguous and thus independent of Φ
- In MIU model with leisure, a higher nominal interest rate reduced m, and depending upon $u_{cm} \gtrless 0$ it reduced or increased n
- In CIA model, the effect of money growth is "direct": Consumption is being **taxed** by a positive nominal interest rate, while leisure is not

Dynamics

- Method as in stochastic MIU model:
- Calibration: Assign plausible values the parameters of the model. Values chosen to conform with basics of MIU model
- Simulation:
  - Perform a linearization of the model's dynamic equations (everything is expressed as percentage deviations from steady state)
  - Solve this system by numerical methods (various simulation programs are available on the internet)
  - Create artificial time series data from the system
- From the artificial data one evaluates the statistical properties of the model

- **Main results**
  - As in MIU model, if money shocks, φ_t–shocks, shall pay a role, persistence in money growth is necessary ($\gamma > 0$). Otherwise, the shock will *not* affect expected next-period inflation, and thus — through the Fisher relationship — period t nominal interest rate. In effect, the "consumption tax" does not vary!
    - ∗ Hence, only "anticipated money" matters
  - The effects of money shocks on labor and output are stronger the more persistence in money growth, and the effects are stronger than in MIU model
    - ∗ Reason: Effects of variations in the nominal interest rate are having a **direct** effect on the consumption-leisure choice; in MIU model the effect were **indirect** through money demand and u_{cm}
  - If technology shocks are met with procyclical money, output is more stable (as in MIU model with $\Phi > b$). The magnitude is small (but stronger than in MIU model)
    - ∗ Reason: When a positive technology shock is met by an increase in money growth, the nominal interest rate increases and discourages labor supply
  - No "liquidity effect of monetary shocks" (as in IS/LM models):
    Positive money shock *increases* nominal interest rates

Real resource costs of transactions

- An alternative way of modelling money as a means of facilitating transactions
- In shopping-time models, transactions took *time* (which is a valuable commodity)
- In CIA models, transactions *must* be carried out using money
- Another possibility is that transactions involve direct resource costs. I.e., *transaction costs*.
- More volume of goods traded in the market, more resources are being "wasted"
- Idea is that presence money can reduce transaction costs
- Thus money does not provide utility directly or indirectly, but frees up resources spent on transactions
- Transaction costs:

$$\Upsilon(c,m) \quad \Upsilon_c > 0, \quad \Upsilon_m < 0$$

- The function will show up in the budget constraint:

$$\begin{aligned} & f(k_{t-1}) + (1-\delta)k_{t-1} + \tau_t + \frac{m_{t-1}}{1+\pi_t} + \frac{(1+i_{t-1})b_{t-1}}{1+\pi_t} \\ = \; & c_t + m_t + b_t + k_t + \Upsilon(c_t, m_t) \end{aligned}$$

- Feenstra (1986) demonstrated equivalence of the MIU approach and the transaction cost approach (analogy to showing that the shopping-time approach also implies money-in-the-utlity implicitly)
- With
  a) plausible restrictions on Υ,
  b) a definition of a money-in-the utility function $W(x,m)$, (with x to be specified)
  c) Introduction of a standard consumption-based utility function $U(c)$,
  – it turns out that the following two problems have the **same solution** (ignore capital):

$$\max \quad U(c) \quad \text{s.t.} \quad y = c + \Upsilon(c,m) + b + m \qquad (3.37')$$

$$\max \quad V(x,m) = U(W(x,m)) \quad \text{s.t.} \quad y = x + b + m \qquad (3.38')$$

where $x \equiv c + \Upsilon(c,m)$

- I.e., the transactions cost problem has solution c^*, b^*, m^* and money-in-the utility function has solution $x^* = c^* + \Upsilon(c^*, m^*)$, b^*, m^*.
- Hence, transactions cost idea is another candidate behind the money in the utility approach (note however, for equivalence to hold, one has consumption **plus** transactions cost as an argument in the MIU model)

Summary

- MIU-models, shopping time models, CIA models and other models of money, are . . . . just models
- Models, nevertheless, are useful, consistent abstractions to use for thinking about economics
- The micro-founded flex-price models analyzed so far are:
  - Suitable for long run-analyses of links between money and inflation, and potential real allocation
  - Suitable for thinking about why money exists and what is the value of money (direct utility, liquidity service, saved leisure....)
  - Suitable for thinking about the optimal rate of inflation (robustness of Friedman rule)
  - Less suitable for analyzing the short run implications of monetary shocks as the models, by nature, exhibits monetary neutrality (although not necessarily superneutrality)
  - To remedy the short-run failure of such models, one *must* introduce incomplete nominal adjustment

Plan for next lectures

Wednesday, February 25

1. Public budget accounting, inflation and debt
2. Equilibrium seigniorage

Literature: Walsh (Chapter 4, pp. 135-164)

Monday, March 1

1. Optimal taxation and seigniorage
2. Robustness of the Friedman rule?

Literature: Walsh (Chapter 4, pp. 172-187; pp. 192-195)