Monetary Economics: Macro Aspects, 19/4 2004
Henrik Jensen

Plan for today:

1. The modern "New Keynesian" model of monetary policy analysis
2. Stability properties
3. Optimal monetary policy under discretion

Literature Clarida et al. (1999, *Journal of Economic Literature*, 1661-1675) and Walsh (2003, Chapter 5, pp. 230-247). The latter is "only" supplementary reading and provides the formalities of the microfoundations for the model used by Clarida et. al).

Introductory remarks

- Recent research on monetary policymaking has focused on models with micro-foundations
- Focus is mostly on models with the interest rate as instrument
- Have built a bridge between academics and practitioners
  - Uses models that academics can "accept"
  - Are empirically oriented, and cast in ways that real-life central banks appreciate
- Research has had enormous influence in recent focus (or return?) on the importance of monetary policymaking for business cycles
- New issues are discovered/developed; old results re-emerge in new settings
  - I.e., a healthy mix of progress and confirmation (=science)

The Modern "New Keynesian" model of monetary policy analysis

- Presentation of simplest, and widely applied, variant of micro-founded, small-scale macro model
- Some call models of this type for "New Neoclassical Synthesis"
  - "Neoclassical" due to the reliance on private sector optimization
  - "New" reflecting the underlying assumptions of no instantaneous market clearing
- Clarida et al. (1999) denote the models "New Keynesian"
  - "Keynesian" as the friction providing a role for monetary policy is nominal rigidities
  - "New" as the models opposed to "Old" Keynesian models are derived from first principles
- Hence, the Lucas critique should not apply
- This version of model is a IS/AS variant of great simplicity
  - Despite simplicity, its "yield" in terms of understanding monetary policy problems is far beyond the sum of its (two) parts!

The demand side

- The Keynes-Ramsey rule in log-deviations from steady state:

$$c_t = \mathrm{E}_t c_{t+1} - \varphi \left(i_t - \mathrm{E}_t \pi_{t+1} \right), \quad \varphi > 0$$

- Market clearing:

$$Y_t = C_t + G_t$$

with G_t being exogenous government spending. Rewrite:

$$1 = \frac{C_t}{Y_t} + \frac{G_t}{Y_t}$$

- In logs:

$$\begin{aligned} c_t &= y_t - e_t \\ e_t &\equiv -\ln \left(1 - \frac{G_t}{Y_t} \right) \end{aligned}$$

- Rewritten, log-linearized, Keynes-Ramsey rule in terms of output:

$$y_t = \mathrm{E}_t y_{t+1} - \varphi \left(i_t - \mathrm{E}_t \pi_{t+1} \right) + e_t - \mathrm{E}_t e_{t+1}, \quad \varphi > 0$$

- Define *output gap* as

$$x_t \equiv y_t - z_t$$

where z_t is stochastic, "hypothetical," flex-price output; the *natural rate of output*

- Note that z_t can be interpreted as a technology shock (determining output in a Real Business Cycle model)

- The "IS curve" then provides a dynamic relation for the output gap:

$$x_t = \mathrm{E}_t x_{t+1} - \varphi\left(i_t - \mathrm{E}_t \pi_{t+1}\right) + e_t - \mathrm{E}_t e_{t+1} - z_t + \mathrm{E}_t z_{t+1}$$

or,

$$\begin{aligned} x_t &= \mathrm{E}_t x_{t+1} - \varphi\left(i_t - \mathrm{E}_t \pi_{t+1}\right) + g_t \qquad (2.1) \\ g_t &= \Delta \mathrm{E}_t z_{t+1} - \Delta \mathrm{E}_t e_{t+1} \\ g_t &= \mu g_{t-1} + \widehat{g}_t, \qquad 0 < \mu < 1 \end{aligned}$$

- Note: Although considered a demand-side relationship, it contains supply side elements through z_t
- I.e., g_t is **not a pure demand shock**
  - $g_t < 0$ could be current below-average government expenditures driving output above natural
  - $g_t < 0$ could be current above-average technology, causing output to be above the natural rate

The supply side

- Monopolistic competition in goods markets
- Taylor-Calvo-style staggered price setting (in style of Chapter 5.3.2 in Walsh)
- Calvo set-up:
  - In each period any firm faces a state-independent probability of "being stuck" with its price, $0 < \theta < 1$
  - The probability is independent of when the firm last changed its price
  - Stylistic (and unrealistic?) representation of staggering. The independence of history facilitates aggregation:
    * θ is the fraction of firms not adjusting prices in a period
    * $1 - \theta$ is the fraction of firms adjusting is a period
    * $1 + \theta + \theta^2 + \theta^3 + ... = 1/\left(1 - \theta\right)$ is average duration of a price contract
- When "allowed" to set prices "today" the firms maximize the present value of current and expected future real profits
  - **Expectations about future prices** become of importance (as in simple Taylor two-period staggering)

- Log-linearized, aggregate optimal price setting is characterized by

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \lambda x_t + u_t, \qquad \lambda > 0, \quad 0 < \beta < 1, \quad (2.2) \\ u_t &= \rho u_{t-1} + \widehat{u}_t, \qquad 0 < \rho < 1 \end{aligned}$$

  - Aggregate prices today, depend on prices yesterday (as prices are sticky)
  - Aggregate prices today, depend on aggregate prices for tomorrow
  - Aggregate prices today are a mark-up over real marginal costs; here proportional to the output gap
  - λ is an (inverse) measure of nominal rigidity in the economy: High θ means low λ.
    - (In the limit: $\theta \to 0$, $\lambda \to \infty$ and $x_t = 0$; i.e., $y_t = z_t$)
  - Shock u_t captures variations in prices not captured by output gap (e.g., fluctuations in firms' mark up)
- Note that (2.2) is an expectations-augmented Phillips curve

- Simple monetary transmission mechanism as in earlier models under interest rate operating procedures:
  - The short nominal interest rate affects the real interest rate and aggregate demand and thus the output gap
  - Inflation is then affected by the output gap

- Note: both the IS curve and the Phillips curve are **forward looking:** Current values of x_t and π_t depend on their expected future values, and thus **expected future monetary policy**
- Indeed, forwarding the IS curve successively yields

$$x_t = E_t \sum_{i=0}^{\infty} \{-\varphi (i_{t+i} - \pi_{t+1+i}) + g_{t+i}\} \qquad (2.5)$$

  I.e., current output gap is determined by sum of current and expected future nominal interest rates
- Under the expectations theory of the term structure: Current output gap depends on the **long real interest rate**
- Obviously, credibility of announcements about future policies will be important for macroeconomic performance

- ...much more intricate, however, than in standard Barro-Gordon set up

Stability properties

- Stability properties of system following a shock?
- Note that there are no predetermined state variables in the model
- Both x_t and π_t are endogenous
- We therefore need that the system of expectational difference equations

$$\begin{aligned} x_t &= \mathrm{E}_t x_{t+1} + \varphi \mathrm{E}_t \pi_{t+1} && (*) \\ \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \lambda x_t && (**) \end{aligned}$$

provides **unique, non-explosive,** solutions for x_t and π_t

- To analyze stability properties, one usually formulates the system in matrix form:

$$\begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix}$$

where $\mathbf{A}$ is a 2×2 matrix. Uniqueness of non-explosive solutions requires that the real parts of the eigenvalues of $\mathbf{A}$ are both numerically **greater than one** as system contains two "jump variables" x_t, and π_t; i.e., **two "unstable" eigenvalues** (cf. Blanchard and Kahn, 1980, *Econometrica*)

- Intuition: With two unstable eigenvalues, *any* deviation from fundamentals-based solution lead to explosive paths

- Remark difference from analysis in a model with two predetermined variables
  - Stability requires two stable eigenvalues
- Had system had one predetermined variable and one jump variable, uniqueness of a non-explosive rational expectations equilibrium would require one stable and one unstable eigenvalue
  - Analogy with Ramsey growth model formulated in consumption (the jump variable) and capital (the predetermined variable): Unique equilibrium when system has a stable and unstable eigenvalue (in continuous time: one below zero and one above zero)
- Find $\mathbf{A}$. From (**) and (*) we get

$$\mathrm{E}_t x_{t+1} = x_t - \varphi \beta^{-1} [\pi_t - \lambda x_t]$$

$$\mathrm{E}_t x_{t+1} = \left(1 + \varphi \beta^{-1}\right) x_t - \varphi \beta^{-1} \pi_t$$

- With $\mathrm{E}_t \pi_{t+1} = \beta^{-1} [\pi_t - \lambda x_t]$ from (**), we get the system as

$$\begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 + \varphi \lambda \beta^{-1} & -\varphi \beta^{-1} \\ -\lambda \beta^{-1} & \beta^{-1} \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix}$$

- The eigenvalues δ_1 and δ_2 are computed from:

$$\begin{vmatrix} 1 + \varphi \lambda \beta^{-1} - \delta & -\varphi \beta^{-1} \\ -\lambda \beta^{-1} & \beta^{-1} - \delta \end{vmatrix} = 0$$

- This gives a second-order polynomial in δ:

$$\delta^2 - \beta^{-1}\left[1 + \beta^{-1}\left(1 + \varphi\lambda\beta^{-1}\right)\right]\delta + \beta^{-1} = 0$$
$$\beta\delta^2 - \left[1 + \beta^{-1}\left(1 + \varphi\lambda\beta^{-1}\right)\right]\delta + 1 = 0$$

- The solutions are:

$$\delta = \frac{1 + \beta^{-1}\left(1 + \varphi\lambda\beta^{-1}\right) \pm \sqrt{\left(1 + \beta^{-1}\left(1 + \varphi\lambda\beta^{-1}\right)\right)^2 - 4\beta}}{2\beta}$$

- Check for following parameter values
  - Aggregate demand's real interest rate sensitivity; $\varphi = 0.1$
  - Inflation's sensitivity to aggregate demand; $\lambda = 0.14$
  - Discount factor; $\beta = 0.99$
- With these values we get

$$\delta_1 = 0.8347$$
$$\delta_2 = 1.2101$$

- System **does not provide unique solutions for** x_t and π_t! For a fixed nominal interest rate, the economy will feature **infinitely many** non-explosive output and inflation paths
  - Rational expectations equilibrium is not unique $\approx$ indeterminacy
- (Note: result does **not** rely on particular parameter values; can be proven generally.)

- Hence, the price level indeterminacy under interest rate operating procedure of earlier models is replaced by inflation and output gap indeterminacy — **real indeterminacy**
  - Intuition: An **arbitrary** increase in inflation expectations will — for a given nominal interest rate — **decrease** the real interest rate, and **increase output** and **increase inflation**
  - Self-fulfilling prophecy!
  - The economy will gradually return to steady-state following this "sun-spot driven" burst of inflation and output
  - The economy may thus be subject to non-fundamental-based fluctuations in output and inflation
- The purpose of the deriving guidelines for good monetary policy is therefore again two-fold:
  - Secure that the economy will not be subject to self-fulfilling bursts of inflation and output
    - * I.e., secure a determinate — unique — equilibrium for inflation and output gap (and thus the price level, as p_{t-1} enters explicitly in the Phillips curve)
  - Secure the optimal manner by which the output gap and inflation fluctuates in response to fundamental shocks

Optimal monetary policy under discretion

- The criterion of monetary policy is to minimize the expected discounted sum of deviations of output gap and inflation from their long-run equilibrium values
- In each period t, the **utility function** is assumed to be

$$-\frac{\alpha}{2}x_t^2 - \frac{1}{2}\pi_t^2, \qquad \alpha > 0.$$

- Recent research has shown that such a function can be derived as a second-order Taylor approximation of the utility function of the representative agent in the economy! (see Walsh, Chapter 11, but this is **really** supplementary!)
- Note zero inflation target; and output target equal to natural rate (reasonable under monopolistic competition? — see later)
- Due to forward looking nature of model, there is difference between solution under commitment to a policy path, or period-by-period optimization (discretion). We examine discretion here.

- To solve model note first:
  - In period t, policy cannot affect expectations about future variables (no persistence in equations)
  - Hence, when optimizing, expected future variables are taken as given
- Trick: Treat x_t as the policy instrument, and find i_t compatible with the solution afterwards

- Maximizing

$$-\frac{1}{2}\mathrm{E}_t \sum_{i=1}^{\infty} \beta^i \left[\alpha x_t^2 + \pi_t^2\right], \qquad 0 < \beta < 1$$

w.r.t. i_t subject to (2.1) and (2.2) is thus equivalent of maximizing

$$-\frac{\alpha}{2}x_t^2 - \frac{1}{2}\pi_t^2 + F_t \tag{3.1}$$

w.r.t. x_t subject to

$$\pi_t = \lambda x_t + f_t \tag{3.2}$$

taking **as given** F_t and f_t

- Subsequently, one finds the nominal interest rate compatible with the solution
- The problem becomes a sequence of single-period problems
- Simple first-order condition:

$$-\alpha x_t = \lambda \pi_t \qquad \text{(3.3')}$$

- "Lean against the wind" policy: If inflationary pressures arise, contract output ($x_t < 0$) such that the marginal cost (left-hand side) equals the marginal gain (the right hand side)
- Note that with more nominal rigidity, lower λ, **the inflation-output trade-off** is more unfavorable: A given reduction in output reduces inflation by less

- Use the first-order condition in the Phillips curve to eliminate the output gap:

$$\begin{aligned}\pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \lambda x_t + u_t \\ \pi_t &= \beta \mathrm{E}_t \pi_{t+1} - \left(\lambda^2/\alpha\right) \pi_t + u_t \\ \pi_t &= \frac{\beta}{1+\lambda^2/\alpha} \mathrm{E}_t \pi_{t+1} + \frac{1}{1+\lambda^2/\alpha} u_t\end{aligned}$$

- Autonomous first-order expectational difference equation in π_t (notice one unstable eigenvalue securing a unique non-explosive solution for inflation)
- Solve by method of undetermined coefficients
- Conjecture solution

$$\pi_t = X u_t$$

- Forward conjecture and take expectations:

$$\mathrm{E}_t \pi_{t+1} = X \mathrm{E}_t u_{t+1} = X \rho u_t$$

- Insert into difference equation:

$$X u_t = \frac{\beta}{1+\lambda^2/\alpha} X \rho u_t + \frac{1}{1+\lambda^2/\alpha} u_t$$

- Identifies X by

$$X = \frac{\beta\rho}{1+\lambda^2/\alpha} X + \frac{1}{1+\lambda^2/\alpha}$$

- Hence,

$$X = \frac{1}{1+\lambda^2/\alpha - \beta\rho}$$

- Solution for inflation is therefore

$$\begin{aligned}\pi_t &= \frac{1}{1+\lambda^2/\alpha - \beta\rho} u_t \\ &= \alpha \frac{1}{\lambda^2 + \alpha\left(1-\beta\rho\right)} u_t \qquad (3.5)\end{aligned}$$

- Solution for output gap follows as

$$x_t = -\lambda \frac{1}{\lambda^2 + \alpha\left(1-\beta\rho\right)} u_t \qquad (3.4)$$

- Implications:
  - No impact of demand and technology shocks; these pose no trade-offs
  - Impact of a "cost-push" shock is "spread out" on inflation and output gap; i.e., there *is* a trade-off in monetary policy
    - * Higher α, relatively higher inflation variability
    - * Higher λ, lower inflation variability; ambiguous effect on output gap variability
    - * Higher ρ, higher macroeconomic variability
    - * Higher β, higher macroeconomic variability

- Solution for x_t and π_t and the associated solution for $\mathrm{E}_t x_{t+1}$ and $\mathrm{E}_t \pi_{t+1}$ (e.g., $\mathrm{E}_t \pi_{t+1} = \rho \pi_t$) can be used in IS-curve to find associated solution for the nominal interest rate
- The optimal value of the nominal interest rate can be written in many ways
- If written as a function of expected next-period inflation one gets:

$$\begin{aligned} x_t &= \mathrm{E}_t x_{t+1} - \varphi\left(i_t - \mathrm{E}_t \pi_{t+1}\right) + g_t \qquad (2.1) \\ -\frac{\lambda}{\alpha}\pi_t &= -\frac{\lambda}{\alpha}\mathrm{E}_t \pi_{t+1} - \varphi\left(i_t - \mathrm{E}_t \pi_{t+1}\right) + g_t \\ -\frac{\lambda}{\alpha\rho}\mathrm{E}_t \pi_{t+1} &= -\frac{\lambda}{\alpha}\mathrm{E}_t \pi_{t+1} - \varphi\left(i_t - \mathrm{E}_t \pi_{t+1}\right) + g_t \end{aligned}$$

and thus

$$\begin{aligned} i_t &= \left(1 + \frac{1}{\varphi}\left[\frac{\lambda}{\alpha\rho} - \frac{\lambda}{\alpha}\right]\right)\mathrm{E}_t \pi_{t+1} - \frac{1}{\varphi}g_t \\ &= \left(1 + \frac{\lambda\left(1-\rho\right)}{\varphi\alpha\rho}\right)\mathrm{E}_t \pi_{t+1} - \frac{1}{\varphi}g_t \qquad (3.6) \end{aligned}$$

- Hence, written like this, an increase in expected inflation is met by a larger increase in the nominal interest rate, the real interest rate increases
- Such an interest rate rule typically secures determinacy
  - Self-fulfilling burst of inflation and output increases are **ruled out**
  - An arbitrary increase in inflation expectations increases the real interest rate, decreases output gap and inflation
  - ....invalidates the increase in inflation as a rational expectations equilibrium

- Empirical analysis of interest rate rules by Clarida et. al (2000, *Quarterly Journal of Economics*)
- For US, in 1970s the estimated coefficient on expected inflation was below one
- In 1980s and onwards, the estimated coefficient is significantly above one

- Combining the empirical results with theory:
  - The high and persistent inflation rates in the 1970s could have been expectations driven; monetary policy did not respond sufficiently aggressive towards inflation expectations
  - In the 1980s and onwards, less fluctuations and lower average inflation; consistent with the fact that the possibility of self-fulfilling inflationary bursts are ruled out

- Note, the empirics are not "proof" of self-fulfilling fluctuations; but "only indicative evidence"

- Other implications of optimal monetary policy under discretion:
  - After a cost-push shock, inflation gradually moves back towards target; in accordance with **inflation targeting**
  - Caveat: so does the output gap; so is it output gap targeting?
    - Note: we cannot tell from the interest rate rule, what is in the loss function of the central bank
    - (3.6) could look as if output gap did not enter.....
  - If technology shocks are random walk, $z_{t+1} = z_t + \widehat{z}_t$, then the nominal rate does **not** respond to technology shocks
    - If $z_t > 0$ current output gap falls, but for given expected future output gap, expected future output increases and current output increases, leaving the output gap unchanged
    - Inflation does not change either
      => Increasing output, no inflation, no central bank response is compatible with optimality (US in the 1990s?

- Distinguishing the source of disturbances is therefore **very** important for monetary policy conduct

Plan for next lectures

Wednesday, April 21

1. Optimal monetary policy under discretion:
   Credibility problems and "Rogoff-conservatism"
2. Optimal monetary policy under commitment: Policy inertia
3. Practical implications and extensions

Literature Clarida et al. (1999, *Journal of Economic Literature*, 1675-1707). As supplementary reading, I recommend Woodford (1999, see link on course web page).

Monday, April 26

1. Delegation solutions to credibility problems in the "New Keynesian" model of monetary policy analysis
2. Nominal income growth targeting and other approaches

Literature: Jensen (2002, *American Economic Review*).