Monetary Economics: Macro Aspects, 18/2 2004
Henrik Jensen

Plan for today:

1. Shopping-time models
2. Cash-in-Advance Models (certainty)

Literature: Walsh (2003, Chapter 3, pp. 95-111)

3. Plan for next lecture

Shopping-time Models

- In the MIU models, money entered the utility function so as to secure a demand for money
- One argument was that it approximated the utility from saving time from doing transactions on the market
- Shopping-time models formalize this, and help puts restrictions on, e.g., the signs of u_{cm} and u_{lm} under the MIU approach. Note that these signs determined how labor supply and output would react to inflation

A simple model to prove the point

- Leisure defined as:

$$l = 1 - n - n^s$$

where n is fraction of time spent on work, and n^s is fraction of time spent on shopping (buying consumption goods)

- Assumption: Transaction services, ψ, are needed for consumption purchases

$$\psi = c$$

- Assumption: Transaction services are facilitated by real money and are higher the more one shops:

$$\psi = c = \psi(m, n^s) \tag{3.1}$$
$$\psi_m \geq 0, \qquad \psi_{n^s} \geq 0$$

- This is rewritten in terms of shopping time:

$$\begin{aligned} n^s &= g(c, m) \\ g_c &> 0, \qquad g_m \leq 0 \end{aligned}$$

- Per-period utility function:

$$v(c_t, l_t) = v[c_t, 1 - n_t - g(c_t, m_t)] \equiv u(c_t, m_t, l_t)$$

While not an explicit argument in the utility function, money enters *implicitly* — and positively — through its impact on shopping time

Optimal behavior

- Budget constraint as in MIU model with endogenous labor:

$$f(k_{t-1}, n_t) + \tau_t + (1 - \delta) k_{t-1} + \frac{m_{t-1}}{1 + \pi_t} = c_t + k_t + m_t$$

(again, bonds are ignored)

- Optimization is characterized by

$$V(a_t, k_{t-1}) = \max \{ v[c_t, 1 - n_t - g(c_t, m_t)] + \beta V(a_{t+1}, k_t) \}$$

with

$$a_t \equiv \tau_t + \frac{m_{t-1}}{1 + \pi_t}$$

Optimization is over c, n, m, k, and a subject to the budget constraint and the definition of a_t.

- The budget constraint and definition of a_t are used to substitute out a_{t+1} and k_t, to get an unconstrained maximization problem

- To asses how money affects the consumption-leisure choice, consider the first-order conditions w.r.t. c_t and n_t

- First-order condition with respect to c_t:

$$\begin{aligned} v_c(c_t, 1 - n_t - g(c_t, m_t)) &= v_l(c_t, 1 - n_t - g(c_t, m_t)) g_c(c_t, m_t) \\ &+ \beta V_k(a_{t+1}, k_t) \end{aligned}$$

Marginal utility of consumption is equated to the marginal losses, which consist of lost leisure due to more time spent on shopping and the marginal value of lower next-period capital

- First-order condition with respect to n_t:

$$v_l(c_t, 1 - n_t - g(c_t, m_t)) = \beta V_k(a_{t+1}, k_t) f_n(k_t, 1 - n_t)$$

Marginal loss of labor is equated to the marginal gain which is the addition to next-period capital (of a magnitude determined by the marginal product of labor = real wage)

- Using these first-order conditions gives the condition for the consumption-leisure choice:

$$\frac{v_l}{v_c - v_l g_c} = f_n(k_t, 1 - n_t).$$

The marginal rate of substitution between leisure and consumption equals the real wage

- This expression holds in each and every period, and therefore also in the steady state. So, how is it affected by, e.g., a raise in m_t? This will be indicative for how superneutrality fails in the long run

- The numerator is affected according to

$$v_{lm} = -v_{ll} g_m < 0$$

More money makes the marginal utility of leisure smaller, as it increases leisure for given consumption (by reducing shopping time). This will tend to increase labor and output

- We get that the denominator is affected according to

$$u_{cm} = (v_{ll} g_c - v_{cl}) g_m - v_l g_{cm} \lesseqgtr 0 \qquad (3.2')$$

This is ambiguous (and corresponds to determining the sign of u_{cm} in the MIU model).

  - One effect that tends to increase work is that if $v_{cl} > 0$, then $-v_{cl} g_m > 0$ implying that the marginal utility of consumption increases with more money: More money frees up leisure, increasing the marginal utility of consumption

- Effects that tend to increase work arises also through the effect on shopping costs affecting deduction from marginal utility of consumption. I.e., changes in the term $v_l g_c$:

$$v_{ll} g_c g_m - v_l g_{cm}.$$

These are both positive if $g_{cm} < 0$

  - First part, unambiguously positive, and captures the fall in marginal utility of leisure induced by the increased leisure freed up by money. This reduces the marginal shopping costs of higher consumption
  - Second part is positive for $g_{cm} < 0$ and captures the lower shopping time needed for given consumption, which means a smaller deduction from marginal utility of consumption
  - Both parts make, all things equal, households substitute away from leisure and work more to get more consumption

- So, unless $v_{lc} << 0$ — consumption and leisure are strong substitutes — higher m will lead to more work

  - Hence, higher money growth and inflation reduces employment and output as in benchmark MIU model calibration

- A shopping time model is therefore equivalent to the MIU approach, but....

  - ....formalizes the idea of utility from transaction services of money
  - ....one knows better how and why superneutrality fails

- Welfare implications of inflation and nominal interest rates? Just as in MIU model, as we get a first-order condition like in the MIU model governing optimal money holdings:

$$-v_l g_m + \beta \frac{V_a(a_{t+1}, k_t)}{1+\pi_{t+1}} = \beta V_k(a_{t+1}, k_t)$$

Marginal gains of money in terms of more current leisure and next-period money wealth are equated to the marginal cost in terms of lower next-period capital.

- Using the value function relationships (by the envelope theorem)

$$\begin{aligned} V_k(a_t, k_{t-1}) &= \beta V_k(a_{t+1}, k_t)\left[f_k(k_{t-1}, n_t) + 1 - \delta\right] \\ &= \beta R_{t-1} V_k(a_{t+1}, k_t) \qquad (*) \end{aligned}$$

and

$$V_a(a_t, k_{t-1}) = \beta V_k(a_{t+1}, k_t) \qquad (**)$$

to get

$$-v_l g_m + \beta \frac{V_a(a_{t+1}, k_t)}{1+\pi_{t+1}} = V_a(a_t, k_{t-1})$$

Hence,

$$-v_l g_m = V_a(a_t, k_{t-1})\left[1 - \beta \frac{V_a(a_{t+1}, k_t)}{(1+\pi_{t+1})\, V_a(a_t, k_{t-1})}\right]$$

But

$$\begin{aligned} \frac{V_a(a_{t+1}, k_t)}{V_a(a_t, k_{t-1})} &= \frac{V_k(a_{t+2}, k_{t+1})}{V_k(a_{t+1}, k_t)} \qquad \text{(using (**))} \\ &= \frac{1}{\beta R_t} \qquad \text{(using (*))} \end{aligned}$$

- So,

$$-v_l g_m = V_a(a_t, k_{t-1})\left[1 - \frac{1}{(1+\pi_{t+1})\, R_t}\right]$$

- As the Fisher relationship implies $1 + i_t = R_t(1 + \pi_{t+1})$,

$$-v_l g_m = V_a(a_t, k_{t-1}) \frac{i_t}{1+i_t}$$

- As in MIU model, it is optimal to have $i_t = 0$ so the private marginal product of real money balances **equals zero**; namely at $g_m = 0$.
  - Friedman rule is optimal again
  - Money balances are then at a level high enough to minimize shopping time

Cash-in-Advance Models

Basic model and optimal choices under certainty

- Takes the transactions purpose of money literally:
  - Having cash, is *by assumption needed* to purchase some (or all) goods
  - A "Cash-in-Advance" constraint is introduced
- Certainty case ("simple")
  - Uncertainty involves further complications: One may suddenly hold too little or too much cash (former case leads to suboptimal low consumption; latter case leads to suboptimal low savings)
- Utility (endogenous leisure dropped for simplicity)

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1. \tag{3.12}$$

- Budget constraint:

$$\begin{aligned} \omega_t &\equiv f(k_{t-1}) + \tau_t + (1-\delta) k_{t-1} + \frac{m_{t-1} + (1+i_{t-1}) b_{t-1}}{1+\pi_t} \\ &= c_t + k_t + m_t + b_t \end{aligned} \tag{3.15'}$$

(b_t is real bond holdings per capita).

- Cash-in-advance (CIA) constraint on consumption goods:

$$c_t \leq \frac{m_{t-1}}{1+\pi_t} + \tau_t \tag{3.13}$$

- Note, as opportunity cost of holding money is i_t, the CIA constraint will *always hold with equality for* $i_t > 0$
  - I.e., why would one then hold more money than needed?
  - (Not necessarily the case with uncertainty — one could "end up with too much cash")
- Optimization is characterized by (ω_t and m_{t-1} are state variables):

$$V(\omega_t, m_{t-1}) = \max \{u(c_t) + \beta V(\omega_{t+1}, m_t)\}$$

Maximization is over c, m, b, k and subject to budget constraint and CIA constraint.

- From $\omega_t = c_t + k_t + m_t + b_t$, one can eliminate b_t from budget constraint:

$$\omega_{t+1} = f(k_t) + \tau_{t+1} + (1-\delta) k_t + \frac{m_t}{1+\pi_{t+1}} + R_t (\omega_t - c_t - k_t - m_t)$$

with $R_t \equiv (1+i_t)/(1+\pi_{t+1})$ being the real interest rate (NOTE: Walsh does not make this substitution)

- Let μ_t denote the Lagrange multiplier associated with the CIA constraint
  - Note, if $\mu_t > 0$ the constraint binds with equality; if $\mu_t = 0$ the constraint does not bind

- The optimization problem is thus

$$V(\omega_t, m_{t-1}) = \max \left\{ u(c_t) + \beta V(\omega_{t+1}, m_t) - \mu_t \left(c_t - \frac{m_{t-1}}{1+\pi_t} - \tau_t \right) \right\},$$

over c, k, and m.

- First-order condition with respect to c_t:

$$u_c(c_t) = \beta R_t V_\omega(\omega_{t+1}, m_t) + \mu_t$$

Marginal utility of consumption equals the marginal losses, which are the discounted marginal value of next-period wealth **plus** the "price" of holding cash as measured by μ_t (cost of liquidity services provided by money)

  – NB: Marginal cost of consumption **higher when the CIA constraint binds,** $\mu_t > 0$

- First-order condition with respect to k_t :

$$\beta V_\omega(\omega_{t+1}, m_t)[f_k(k_t) + 1 - \delta] = \beta R_t V_\omega(\omega_{t+1}, m_t)$$

Marginal gain in terms of more next-period wealth equals the marginal loss in terms of less next-period wealth due to lower bond holdings

  – (implies familiar $R_t = f_k(k_t) + 1 - \delta$)

- First-order condition with respect to m_t:

$$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) = \beta R_t V_\omega(\omega_{t+1}, m_t)$$

Marginal gains in terms of more next-period wealth and money per se (for transactions), equals marginal loss in terms of less next-period wealth due to lower bond holdings

- Relationships between partial derivatives of the value function from the envelope theorem:

$$V_\omega(\omega_t, m_{t-1}) = \beta R_t V_\omega(\omega_{t+1}, m_t)$$

– In optimum, equality between the period t marginal value of wealth and the discounted next-period loss in terms of less next-period wealth due to lower bond holdings

$$V_m(\omega_t, m_{t-1}) = \mu_t \frac{1}{1+\pi_t}$$

– Marginal value of money carried into period t equals their marginal cost in terms of the "price" of holding cash as measured by $\mu_t / (1+\pi_t)$

- What is the nominal interest rate, and does the CIA constraint bind?
  - Let $\lambda_t \equiv V_\omega(\omega_t, m_{t-1})$ define the marginal value of wealth (= the Lagrange multiplier on the budget constraint in Walsh)
  - From $V_\omega(\omega_t, m_{t-1}) = \beta R_t V_\omega(\omega_{t+1}, m_t)$ one can write

$$\lambda_t = \beta R_t \lambda_{t+1}$$

  - From
$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) = \beta R_t V_\omega(\omega_{t+1}, m_t)$
one can write
$$\beta \frac{1}{1+\pi_{t+1}} \lambda_{t+1} + \beta \mu_{t+1} \frac{1}{1+\pi_{t+1}} = V_\omega(\omega_t, m_{t-1}) = \lambda_t$$
  - Hence,

$$\begin{aligned} \beta \frac{1}{1+\pi_{t+1}} (\lambda_{t+1} + \mu_{t+1}) &= \beta R_t \lambda_{t+1} \\ \frac{1}{1+\pi_{t+1}} (\lambda_{t+1} + \mu_{t+1}) &= \frac{1+i_t}{1+\pi_{t+1}} \lambda_{t+1} \\ \Longrightarrow i_t &= \frac{\mu_{t+1}}{\lambda_{t+1}} \end{aligned} \tag{3.29}$$

  - The nominal interest rate is positive, *only* when the CIA constraint binds, $\mu_{t+1} > 0$, i.e., when there is a cost of "liquidity services" provided by real money holdings

- Note how λ_t / P_t, the utility value of *nominal* money, equals the present value of marginal values of money, cf. (3.28)
  - Money is "priced" like a conventional asset

- Note then that the first-order condition guiding consumption choice can be rewritten as

$$\begin{aligned} u_c(c_t) &= \lambda_t + \mu_t \\ &= \lambda_t \left(1 + \frac{\mu_t}{\lambda_t}\right) \end{aligned}$$

- With the expression for the nominal interest rate:

$$u_c(c_t) = \lambda_t (1 + i_{t-1})$$

Hence, a positive interest rate increases the marginal cost of consumption *above* the marginal value of wealth

- The "price" of consumption goods in terms of output has increased by a positive i_{t-1} due to the need for holding cash (foregoing interest income) to purchase goods
- The nominal interest rate is equivalent to a "consumption tax." However, it is a **non-distorting tax in the long run** as it:

  a) Does not affect long-run capital accumulation

  b) Does not distort any intratemporal trade-offs

- Note with $\mu_t > 0$ — a binding CIA constraint — the model features a strong version of the quantity theory of money:

$$c_t = m_t = \frac{M_t}{P_t}$$

Constant consumption-based velocity — unrealistic and due to certainty and the simplicity of the model

Steady-state properties: Superneutrality or not?

- From the steady-state condition $R^{ss} = 1/\beta$ and the capital accumulation condition one gets the familiar condition:

$$f_k(k^{ss}) + 1 - \delta = 1/\beta$$

  Hence, long-run capital and output per capita are **invariant** w.r.t. monetary factors

- Steady-state consumption follows from the national account as

$$c^{ss} = f(k^{ss}) - \delta k^{ss}$$

  I.e., **long-run superneutrality holds**

- As usual nominal money growth affects inflation and inflation affects the nominal interest rate (through the Fisher relationship):

$$\begin{aligned} \pi_t &= \theta^{ss} \\ i^{ss} &\approx R^{ss} + \pi^{ss} \end{aligned}$$

- Analogy with MIU model concerning relative marginal values of real money balances (in terms of liquidity services) and consumption:

$$\frac{\mu}{u_c} = \frac{\mu}{\lambda(1+i)} = \frac{i}{1+i}$$

- Difference with MIU approach (and shopping time approach), **no steady-state welfare costs of inflation**; only c^{ss} matters for utility, and c^{ss} is **independent of inflation and the nominal interest rate**

Extensions yielding non-superneutrality and a well-defined optimal inflation rate

- Introduction of consumption-leisure trade-off
  - As leisure can be "purchased" without money, the CIA constraint "taxes" consumption relative to leisure (**distorts** the trade-off)
  - Households choose more leisure and less labor supply; output is lower
- Cash and credit goods
  - Subset of consumption goods can be bought on credit; i.e., the CIA constraint does not apply
  - The CIA constraint "taxes" cash goods, but not credit goods (**distorts** relative demand)
  - Can account for time-varying velocity as $m = c^{\text{cashgoods}}$, such that m/c^{allgods} varies with expected inflation and the nominal interest rate (higher i likely to reduce $m/c^{\text{allgods}} \approx$ higher velocity)
- CIA restriction on investment in physical capital
  - Then, accumulation of capital becomes "taxed," and steady state capital will be lower (investment decision is **distorted**)
- All cases strongly qualifies the "any inflation rate goes" result of simple CIA model. Now it will be optimal to have $i^{ss} = 0$, i.e., to eliminate any distortion arising from the CIA constraint.
  $= >$ Implement the Friedman rule!

Summary

- MIU models can be rationalized by shopping-time models
- Most properties are the same, but shopping-time models help restricting the cross-derivatives of the utility function in MIU models
- Cash-in-advance constraint is a direct portrait of money's role as a means of transactions
- Simple model exhibits superneutrality, but no well-defined optimal inflation rate (a positive nominal interest rate is a non-distorting "consumption tax")
- Amending the simple model will cause non-superneutrality and a well-defined optimal inflation rate. The Friedman rule again.....
- Business cycle properties of CIA models? Next time; with model including endogenous labour supply

Plan for next lecture

Monday, February 23

1. Cash-in-Advance models (stochastic)
2. Money and real costs of transactions

Literature: Walsh (Chapter 3, pp.126-131)