Chapter 16

Extracting Information from the Data: A European View on Empirical Macro.

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Introduction

The last few decades have witnessed a revolution in the use of econometrics in empirical macroeconomics mostly due to the easy access to fast performing computers. Even though the use of new sophisticated techniques has been burgeoning the profession does not seem to have reached a consensus on the principles for good practise in the econometric analysis of economic models. Summers' (1992) critique of the scientific value of empirical models in economics seems equally relevant today.

The basic dilemma is that the reality behind the available macroeconomic data is so *much more rich and complex* than the (often narrowly analyzed) problem being modeled by the theory. How to treat these "additional" features of the data (which often go against the *ceteris paribus* assumptions of the economic model) has divided the profession into the proponents of the so called "specific-to-general" and the proponents of "general-to-specific" approach to empirical economics.

The former, more conventional, approach is to estimate the parameters of a "stylized" economic model, while ignoring the wider circumstances under which the data were generated. These factors are then dumped into the residual term, causing its variance to be large. This practice has important implications for the power of empirical testing, often leading to a low ability to reject a theory model when it is false. As a result, different (competing) theory models are often not rejected despite being tested against the same data. Furthermore, the statistical inference in such models is usually based on a number of untested (and often empirically incorrect) *ceteris paribus* assumptions and the "significance" of estimated parameters may lack scientific meaning.

The "general-to-specific" approach to empirical economics represented by the VAR approach is a combination of induction and deduction. It recognizes from the outset the weak link between the theory model and the observed reality. For example, few, if any, theory models allow for basic characteristics of macroeconomic data such as path dependence, unit-root nonstationarity, structural breaks, shifts in equilibrium means, and location shifts in general growth rates.

These empirical features of economic data are at odds with the prevailing paradigm, which assumes a few (constant) structural parameters describing technology and preferences combined with model based rational expectations (RE) as a description of how economic agents form expectations. These assumptions allow economists to rid their models of free parameters, and in doing so pretend that economics comes close to the precision of the natural sciences. Such models cannot be validly confronted with empirical time-series data as the link is too weak and, therefore, they continue to describe a "toy economy."

We will here focus on some basic principles for empirical research based on the (cointegrated) VAR approach and how it can be used to extract long-run and short-run information in the data by exploiting their integration and cointegration properties. The idea is to replace "simple stylized facts" such as correlations, graphs, etc. (which admittedly have inspired many new advances in theoretical economics) with more sophisticated facts better representing the nonstationary world of economic agents. In such a world, economic behavior is influenced by exogenous shocks pushing economic variables away from equilibrium, thereby activating adjustment forces that gradually pull the system back towards the equilibrium.

As an illustration of the potential strength of the cointegrated VAR approach, we will translate M. Friedman's claim (Friedman 1970) that "inflation is always and everywhere a monetary phenomenon" into a set of testable empirical hypotheses, building partly on Chapter 9, "Inflation and Monetary Policy," in D. Romer (1996). Using Danish monetary data we will then demonstrate that the cointegrated VAR model not just provides more efficient estimates of crucial parameters than conventional regression models, but also gives a conceptual framework for discussing the empirical content of many macroeconomic phenomena, and generates a set of robust empirical regularities characterizing economic behavior, the "new" stylized facts against which the empirical relevancy of theoretical results can be assessed.

Furthermore, the empirical analysis not only delivers a precise answer to whether a hypothesis is rejected (or accepted) but allows the researcher to see why it was rejected and how one can reformulate the hypothesis in the larger framework of the statistical model, thereby suggesting new hypotheses to be tested. Thus, by embedding the theory model in a broader empirical framework, the analysis points to possible pitfalls in macroeconomic reasoning, and at the same time generates new hypotheses for how to modify too narrowly specified theoretical models.

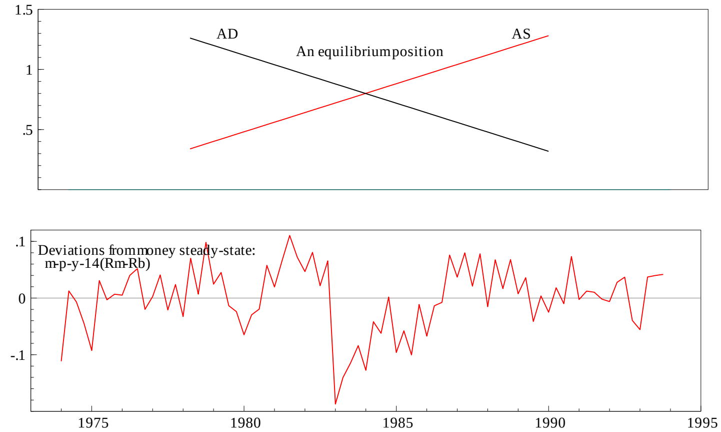


Figure 1: An equilibrium position of the AD and AS curve (upper panel) and deviations from an estimated money demand relation for Denmark: $(m-p-y)_t - 14.1(R_m - R_b)$ (lowerpanel).

We believe that this empirical approach, which is Popperian in spirit, has a large potential for generating new empirically relevant hypotheses for macroeconomic behavior.

Inflation and Money Growth

A fundamental proposition in most macroeconomic theories is that growth in money supply in excess of real productive growth is the cause of inflation, at least in the long run. Here we will briefly consider some conventional ideas underlying this belief as described by Romer (1996).

The well-known diagram illustrating the intersection of aggregate demand and aggregate supply provides the framework for identifying potential sources of inflation as shocks shifting either aggregate demand upwards or aggregate supply to the left. See the upper panel of Figure 1.

As examples of aggregate supply shocks that shift the AS curve to the left Romer (1996) mentions; negative technology shocks, downward shifts in labor supply, and upwardly skewed relative-cost shocks. As examples of aggregate demand shocks that shift the AD curve to the right he mentions: increases in money stock, downward shifts in money demand, and increases in government purchases. Since all these types of shocks, and many others, occur quite frequently there are many factors that potentially can affect inflation. These shocks are isolated or autonomous in assuming that all other factors are kept constant. Some of these shocks may only influence inflation temporarily and are, therefore, less important than shocks with a permanent effect on inflation. Among the latter, economists usually emphasize changes in money supply as the crucial inflationary source. The economic intuition behind this is that other factors are limited in quantity, whereas money in principle is unlimited in supply.

More formally the reasoning is based on money demand and supply and the condition for equilibrium in the money market:

$$M/P = L(R, Y^r), \quad L_R < 0, \quad L_y > 0. \tag{1}$$

where M is the money stock, P is the price level, R the nominal interest rate, Y^r real income, and $L(\cdot)$ the demand for real money balances. Based on the equilibrium condition, i.e. no changes in any of the variables, Romer (1996) concludes, assuming that money causes prices, that the price level is determined by:

$$P = M/L(R, Y^r) \tag{2}$$

The equilibrium condition (1), is a static concept that can be thought of as a hypothetical relation between money and prices for fixed income and interest rate. The underlying comparative static analysis investigates the effect on one variable, say price, when changing another variable, say money supply, with the purpose of deriving the new equilibrium position after the change. Thus, the focus is on the hypothetical effect of a change in one variable (M) on another variable (P), when the additional variables (R and Y^r) are exogenously given and everything else is taken account of by the *ceteris paribus* assumption.

The empirical interest in money demand relations stems from basic macroeconomic theory postulating that the inflation rate is directly related to the expansion in the (appropriately defined) supply of money at a rate greater than that warranted by the growth of the real productive potential of the economy. The policy implication is that the aggregate supply of money should be controlled in order to control the inflation rate. The optimal control of money, however, requires knowledge of the "noninflationary level" of aggregate demand for money at each point of time, defined as the level of money stock, $M/P = L(R, Y^r)$, at which there is no tendency for prices to increase or decrease. Thus, from a practical point of view, the reasoning is based on the assumption that there exists a stable aggregate demand-for-money relation that can be estimated.

Theories of money demand usually distinguish between three different motives for holding money. The transactions motive is related to the need to hold cash for handling everyday transactions. The precautionary motive is related to the need to hold money to be able to meet unforeseen expenditures. Finally, the speculative motive is related to agents' wishes to hold money as part of their portfolio.

A Walrasian economist would require a formal model for agents' willingness to hold money balances based on optimizing behavior. Such a model can be based on (1) theories treating money as a medium of exchange for transaction purposes, so that minimizing a derived cost function leads to optimizing behavior, and (2) theories treating money as a good producing utility, so that maximizing the utility function leads to optimizing behavior.

A post-Walrasian economist considers such models too specific to qualify as a benchmark for an empirical model based on non-experimental time-series data. Too many untestable restrictions would have to be imposed from the outset, which would invalidate inference if not consistent with the information in the data. Instead, the idea is to start with a general description of the variation of the data from which we try to extract as much information as possible. To secure valid statistical inference all underlying assumptions have to be properly tested.

To illustrate the empirical approach we assume as a starting point that all three motives can affect agents' needs to hold money. Therefore, the initial assumption is that real money stock is a function of the level of real income, Y^r, (assumed to determine the volume of transactions and precautionary money) and the cost of holding money ??, $C = \{R_b - R_m, \pi - R_m\}$, where R_m is the yield on money holdings, R_b is the yield on bonds, and π is the inflation rate. The functional form of the money demand relation is assumed to be log linear signifying the importance of relative rather then absolute changes in a time-series context.

The economic model and the VAR: A dictionary

Discussions between economists and econometricians are often confused by the fact that the wording and the concepts used are often similar, or even identical, even though their economic and econometric meaning differs significantly. This section is, therefore, an attempt to bridge the language gap. We provide a basic dictionary for some of the most crucial concepts used in the formulation of economic hypotheses derived from the selected theory model, and the concepts used in the formulation of testable statistical hypotheses derived from a statistical model describing the data.

The status of variables

A theory model makes a distinction between the *endogenous* and *exogenous* variables, which explicitly enter the model, and the variables outside the model, which are, by assumption, given. Furthermore the endogenous are modeled, whereas the exogenous are given. An empirical model makes the distinction between variables in the model and variables outside the model. Since *all* variables entering the model are allowed to interact, the economic notion of endogenous and exogenous is not useful prior to the empirical analysis. But, since the selection of variables is strongly influenced by the theoretical model for the endogenous variables, the equations of the latter usually make more economic sense. The variables outside the model are not fixed; thus they are likely to influence the variables in the model through the stochastic error term and the lags of the included variables.

For example, in Romer's model for money demand it is assumed that interest rates and income are exogenous and thus given. In the empirical analysis, however, the two variables are included in the model and allowed to interact with the variables of interest (the endogenous variables), money and prices. The *ceteris paribus* assumption of the economic model is taken care of by allowing lags of all variables in every equation. The notion of exogeneity is defined in terms of parameters and can be tested in view of the data. The remaining part of the economy, the economic environment, is assumed to vary freely.

The baseline model is the unrestricted VAR(k) for the p-dimensional vector process $\mathbf{x}_t$

$$\mathbf{x}_t = \mathbf{\Pi}_1 \mathbf{x}_{t-1} + ... + \mathbf{\Pi}_k \mathbf{x}_{t-k} + \mu_0 + \varepsilon_t \tag{3}$$

or its equivalent equilibrium correction form

$$\Delta \mathbf{x}_t = \mathbf{\Pi} \mathbf{x}_{t-1} + \mathbf{\Gamma}_1 \Delta \mathbf{x}_{t-1} + ... + \mathbf{\Gamma}_{k-1} \Delta \mathbf{x}_{t-k+1} + \mu_0 + \varepsilon_t. \tag{4}$$

where $\varepsilon_t \sim N_p(0, \Omega)$. As long as the parameters have been approximately constant over the sample period, and the residuals satisfy the assumptions made, the VAR model is just a convenient way of representing the covariances of the data.

The notion of shocks

Theoretical models often make an important distinction between *unanticipated* and *anticipated* shocks. In the VAR model, Δx_t is the empirical measure of a shock, whereas $E_{t-1}(\Delta x_t \mid \sigma_{t-1}) = \Pi x_{t-1} +$

$\Gamma_1 \Delta x_{t-1} + ... + \mathbf{\Gamma}_{k-1} \Delta \mathbf{x}_{t-k+1} + \mu_0$ is a measure of its anticipated part and ε_t of its unanticipated part given σ_{t-1}, the information available in the process at time $t-1$. The anticipated part can be interpreted as agents' plans for the next period based on σ_{t-1}. As long as the unanticipated part, ε_t, is a white noise process (a testable assumption) the updating of the plans as new information becomes available will be consistent with rational behavior in the sense that agents do not make systematic forecast errors.

A shock in a theoretical model is generally called "structural" if it is a meaningful shock to a variable in a postulated economic structure, or model, keeping all other variables fixed. The estimated residuals have a different interpretation, as they capture partly the unanticipated effect of a change in a variable in the model and partly the unanticipated effect of changes of variables outside the model. For an estimated residual to qualify as a structural shock we assume that it describes a shock, the effect of which is (1) unanticipated (novelty), (2) unique (for example a shock hitting money stock alone), and (3) invariant (no additional explanation by increasing the information set).

The novelty of a shock depends on the credibility of the expectations formation, i.e. whether $\varepsilon_t = \Delta \mathbf{x}_t - E_{t-1}\{\Delta \mathbf{x}_t \mid \sigma_{t-1}\}$ is a correct measure of the unanticipated change in x_t. The uniqueness can be achieved *econometrically* by reformulating the VAR so that the covariance matrix Ω becomes diagonal. For example, by postulating a causal ordering among the variables of the system one can trivially achieve uncorrelated residuals. In general, the VAR residuals can be orthogonalized in many different ways and whether the orthogonalized residuals can be given an *economic* interpretation as unique structural shocks depends crucially on the plausibility of the identifying assumptions. Thus, different schools will claim structural explanations for differently derived estimates based on the same data.

Invariance requires that an estimated structural shock as a function of the VAR residuals $\hat{u}_t = f(\hat{\varepsilon})$ should remain unchanged when increasing the information set. The invariance of the structural shock in the theory model relies on many simplifying assumptions including numerous *ceteris paribus* assumptions. In empirical models the *ceteris paribus* assumption is taken account of by conditioning on the variables in the model. Since essentially all macroeconomic systems are stochastic and highly interdependent, the inclusion of additional *ceteris paribus* variables in the model is likely to change the VAR residuals and, hence, the estimated shocks.

Therefore, a structural interpretation is hard to defend, unless one can claim that the information set is complete, so that the errors in the model are not influenced by other unanticipated shocks from the economic environment.

As exemplified by Romer, theory models also make a distinction between *permanent* and *transitory* shock to a variable. The former is usually defined as a disturbance having a long-lasting effect on the variable and the latter as a disturbance with a short-lived effect. For example, a permanent increase v_t of the oil price is a permanent shock to the price level, whereas it is a transitory shock to inflation. This is because inflation increases in the period of the oil price shock by v_t, but goes back by $-v_t$ to its original level next period.

Thus, a transitory shock disappears in cumulation, whereas a permanent shock has a long-lasting effect on the level of the variable. This gives the rationale for defining a stochastic trend as the cumulation of shocks.

Persistence and the notion of stochastic and deterministic trends

Romer makes a distinction between shocks with either a temporary or a permanent effect on inflation, focusing on the latter as a potential cause of inflation in the economy. In time-series econometrics the notion that inflation has been subject to permanent shocks is translated into the statement that inflation rate is modeled by an $I(1)$ variable

$$\pi_t = \pi_{t-1} + v_t = \sum\nolimits_{i=1}^{t} v_i + \pi_0, \tag{5}$$

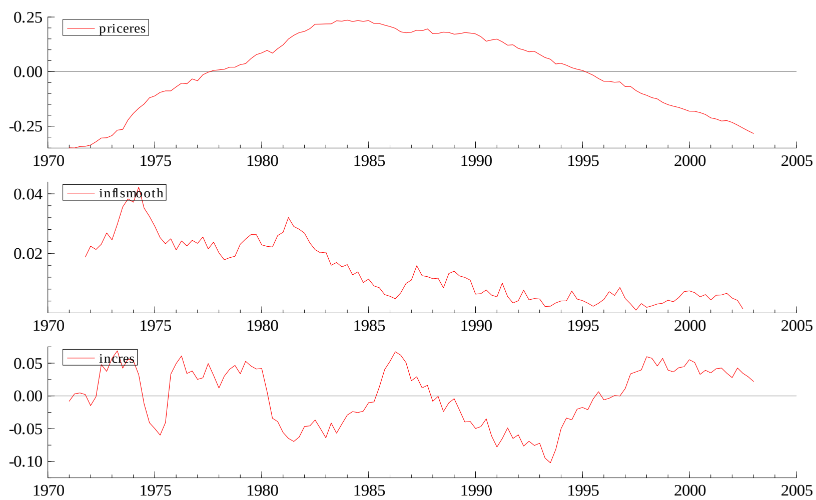
where $\pi_t \equiv \Delta p_t$ and v_t is a stationary disturbance, which can contain both permanent and transitory shocks. In the cumulation $\sum_{i=1}^{t} v_i$, only the effect of the permanent shocks remains. Thus, the difference between a stochastic and a deterministic linear trend is that the permanent increments of a stochastic trend change randomly, whereas those of a deterministic trend are constant over time.

An expression for price levels can be obtained by integrating the inflation rate:

$$p_t = \sum_{i=1}^{t} \Delta p_i + p_0 = \sum_{i=1}^{t} \pi_t + p_0 = \sum_{s=1}^{t} \sum_{i=1}^{s} v_i + \pi_0 t + p_0, \tag{6}$$

and the notion of permanent shocks to inflation rate can be translated to the econometric statement that the DGP of prices contains a second order stochastic trend together with a first order linear trend. We note that the unit root is a *statistical approximation* of persistent behavior over the period of investigation. It should be stressed that it can rarely be given a direct interpretation as a *structural economic parameter,* i.e. it is not a generic property of an economic model (Juselius 1999).

As an illustration, the upper panel of Figure **??** shows a second order stochastic trend, $lnP_t - b_1 t$, the middle panel shows the corresponding first order stochastic trend, ΔlnP_t, and the lower panel the first order stochastic trend in the log of real aggregate income, $lnY_t^r - b_2 t$.



Stochastic trends in Danish prices, real income, and inflation, based on quarterly data 1973:1-2003:4.

The fact that macroeconomic variables typically have been (and continue to be) subject to permanent shocks explains why such data in levels are generally found to be strongly time dependent, whereas changes in variables, i.e. the shocks, are less so. From an empirical perspective it is, therefore, useful to distinguish between:

• stationary variables with a short time dependence, i.e. with significant mean reversion, and

• (unit root) nonstationary variables with a long time dependence, i.e. with insignificant mean reversion.

A further distinction is to classify the variable according to the degree of integration (persistence), for example into $I(-1)$, $I(0)$, $I(1)$, and $I(2)$ variables (Johansen 1996). The cointegrated VAR model exploits this feature of the data as a means to classify variables with a similar persistency profile, i.e. variables which share a similar time path of persistent shocks.

The notion of equilibrium

The theoretical concept of a monetary equilibrium is defined as a point where money demand equals money supply. This definition is generally not associated with a specific point in time. In a VAR model an equilibrium point is a "resting position," i.e. a value of the process at which there are no adjustment forces at work. The empirical counterpart of the theoretical equilibrium (1), with the opportunity cost of holding money, $R = (R_b - R_m)$, is a cointegrating relation in the VAR model, i.e.:

$$\ln(M/P)_t - \ln(Y^r)_t - L(R_b - R_m)_t = v_t \tag{7}$$

where v_t is a stationary equilibrium error. The econometric condition for this to be the case is either that

1. the liquidity ratio, $\ln(M/P)_t - \ln(Y^r)_t$ and the interest rate spread $(R_b - R_m)_t$ are both stationary, or that

2. the liquidity ratio, $\ln(M/P)_t - \ln(Y^r)_t$ and the interest rate spread $(R_b - R_m)_t$ are both nonstationary, but cointegrating.

In the first case, real money stock and real income have experienced the same cumulated permanent shocks (i.e. they share the same stochastic trend). The same is true for the long-term bond rate and the short-term deposit rate. Thus, v_t is the sum of two stationary errors.

In the second case real money stock and real income have experienced different permanent shocks that have cumulated to a stochastic trend in the liquidity ratio, $\ln(M/P)_t - \ln(Y^r)_t$. If the interest rate spread $(R_b - R_m)_t$ has been subject to the same permanent shocks, then the stochastic trends cancel in the linear combination $\ln(M/P)_t - \ln(Y^r)_t - L(R_b - R_m)_t$, so that v_t becomes stationary.

The second case is illustrated in Figure 1 (lower panel) by the graph of the deviations from an estimated money demand relation based on Danish data with the opportunity cost of holding money being measured by $(R_b - R_m)_t$ (Juselius 2005). The stationarity of v_t implies that whenever the system has been shocked, it will adjust back to equilibrium, but this adjustment need not be (and often is not) fast. In some cases it is sluggish enough to make v_t look more like a nonstationary process.

The graphs in Figure 2 illustrate a situation where the stationary equilibrium error, v_t, is the sum of two nonstationary error processes (or possibly two stationary processes with a shift in the equilibrium mean). It also illustrates that the equilibrium point, $v_t = 0$, is essentially never observed. This is because when a shock has pushed the process away from its previous equilibrium position, the economic adjustment forces begin to pull the process back towards its new position, but this

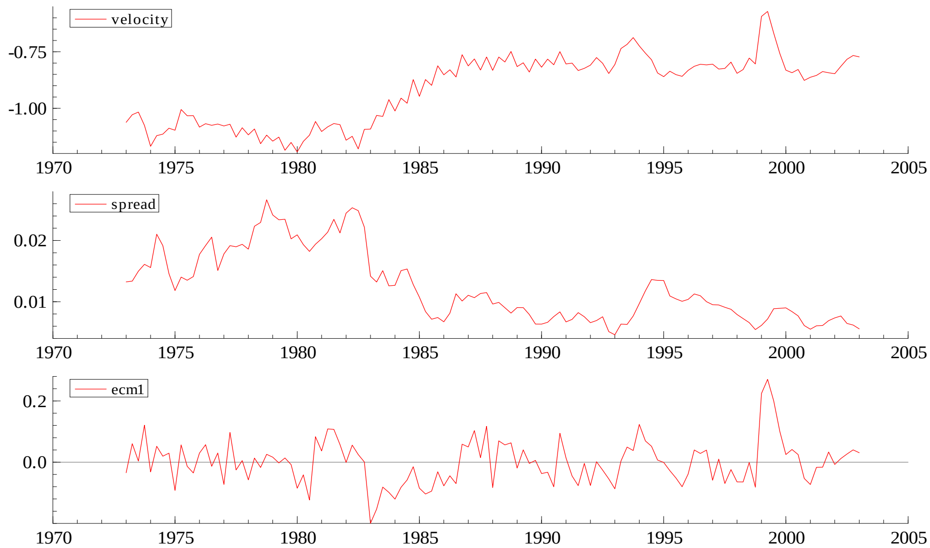


Figure 2: Money velocity (upper panel), the interest rate spread (middel panel), and money demand (lower panel) for Danish data.

adjustment is disturbed by new shocks and the system never comes to rest. Therefore, we will not be able to empirically observe an equilibrium position, except as a resting point towards which the process gravitates.

Interpretation of coefficients

Empirical investigation of (7) based on cointegration analysis poses several additional problems. Although in a theoretical exercise it is straightforward to keep some of the variables fixed (the exogenous variables), in an empirical model none of the variables in (1), i.e. money, prices, income, or interest rates, can be assumed to be given. The stochasticity of all variables implies that the equilibrium adjustment can take place in either money, prices, income, or interest rates. Therefore, the magnitude of the long-run equilibrium error v_t is not necessarily due to a money supply shock at time t, but can originate from a long-run change in any of the other variables.

The interpretation of the coefficients in (7) is similar to the interpretation of usual regression coefficients, in the sense that the coefficient to lnY_t^r is the long-run effect of lnY^r on lnM^r under the assumption of long run *ceteris paribus* (see Johansen 2005).

Causality

In (1) the money market equilibrium is an exact mathematical expression and it is straightforward to invert it to determine prices as is done in (2), provided one assumes causality from money to prices. The observations from a typical macroeconomic system are adequately described by a stochastic vector time series process. If the relation (2) is interpreted as a statement about a conditional expectation, and estimated by regression methods, then inversion is no longer guaranteed (see for instance Hendry and Ericsson 1991). The cointegrating relation (7) is a relation *between* variables and can be normalized on any variable without changing the relationship. The lagged cointegrating relations influence the current changes of the process through the adjustment coefficients to the disequilibrium errors. Thus, the causality in the reduced form VAR goes from lagged values to current values, not between current values.

Path dependence

In a static theory model the variables assume constant values. When dynamics are introduced the variable follow trajectories set out by the dynamics of the model and the initial values. Thus the development of a variable depends on which path it is on. In an empirical, or stochastic, model the trajectory of a variable is influenced both by the initial values and the dynamics, but furthermore by the stochastic shocks. Hence the development over time changes at each time point as a function of the past and the new shock that hits the system. In this sense the variables show path dependence.

Pulling and Pushing Forces in the Cointegrated VAR

The purpose of this section is to illustrate that the cointegrated VAR model can provide a precise description of the pulling and pushing forces. For illustrative purposes we consider here the simple VAR model (4) with lag length $k = 1$, which we write in the form

$$\Delta x_t - \gamma = \alpha(\beta' x_{t-1} - \beta_0) + \varepsilon_t \tag{8}$$

where $E\Delta x_t = \gamma$, $E\beta' x_t = \beta_0$, and $\beta'\gamma = 0$, and we define $\mu_0 = \gamma - \alpha\beta_0$. The long-run information in the data is summarized in the reduced rank restriction:

$$\Pi = \alpha\beta' \tag{9}$$

and α and β are $p \times r$, $r \leq p$.

Inversion of (8) yields the moving average representation or, as it is often called, the common trends representation:

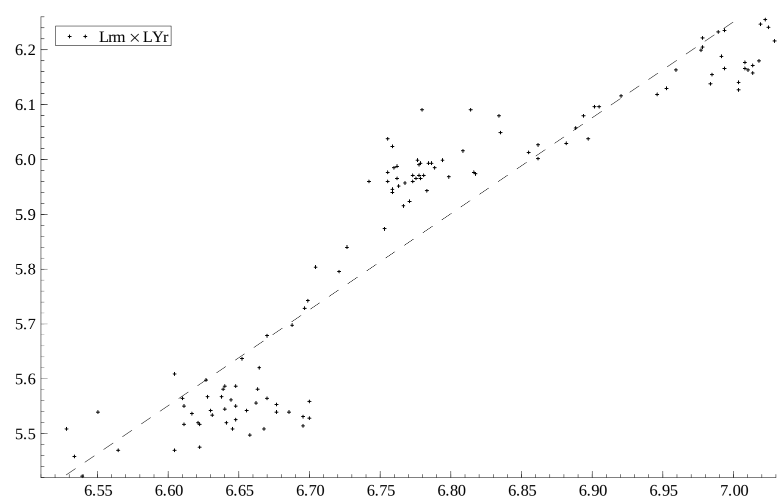
$$x_t = C \sum_{i=1}^{t} \varepsilon_i + \gamma t + C^*(L)(\varepsilon_t + \mu_0) \tag{10}$$

where C is of reduced rank $p - r$.

$$C = \beta_\perp(\alpha'_\perp \beta_\perp)^{-1}\alpha'_\perp = \tilde{\beta}_\perp \alpha'_\perp \tag{11}$$

where $\tilde{\beta}_\perp = \beta_\perp(\alpha'_\perp \beta_\perp)^{-1}$ and $\beta_\perp, \alpha_\perp$ are the orthogonal complements of β and α.

We now discuss how the two reduced rank conditions, (9) and (11), can be used to describe the forces pulling towards equilibrium versus the forces pushing the process along the attractor set defined by the relation $\beta' x = \beta_0$. We assume for illustrative purposes that the process is $x'_t = [m^r_t, y^r_t]$, where m^r_t is the log of real money stock and y^r_t is the log of real aggregate income. Figure 4 shows a crossplot of m^r_t against y^r_t. The straight line measures the attractor set $sp(\beta_\perp) = sp(1, 1)$, and the distance from the dots to the line the disequilibrium error $\beta' x_t = m^r_t - y^r_t$.



A crossplot of real aggregate income and real money stock (M3)

(12)

The geometry of the cointegrated VAR model is illustrated in Figure 5. A constant liquidity ratio $m^r - y^r = \beta_0$ describes an equilibrium

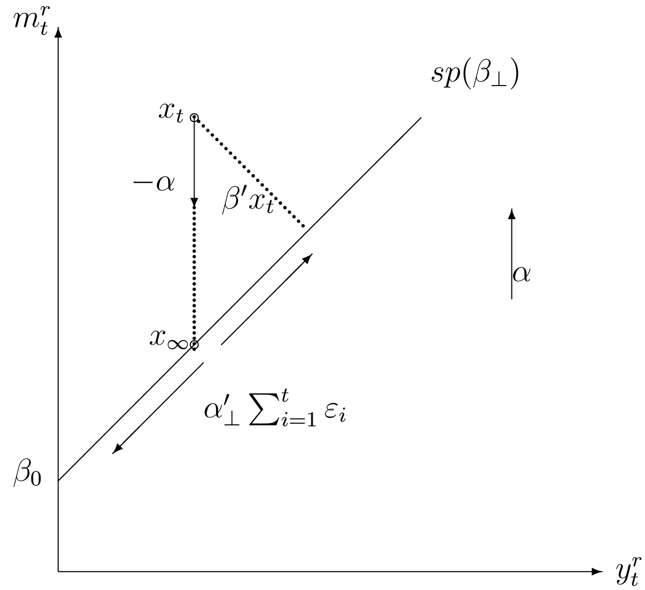


Figure 3: The process $x_t' = [y_t^r, m_t^r]$ is pushed along the attractor set by the common trends and pulled towards the attractor set by the adjustment coefficients

position between money stock and income. In the notation of (9) this implies that $\beta' = [1, -1]$, so that $\beta' x_t - \beta_0 = m_t^r - y_t^r - \beta_0 \neq 0$ measures a stationary disequilibrium error. The steady-state positions or attractor set $m_t^r - y_t^r - \beta_0 = 0$ describes a system at rest. If the errors were switched off starting from a point x_t, the trajectory of the process would be a straight line from x_t along the vector α until it hits the attractor set. The speed of the process is proportional to the length of the vector α and to the distance from the attractor set as measured by the disequilibrium error, $\beta' x_t - \beta_0$. At the (long-run) equilibrium point, x_∞, there is no economic adjustment force (incentive) to change the system to a new position.

When there are stochastic shocks to the system, the picture is almost the same. The process x_t is pulled towards the attractor set along the vector α and proportional to the disequilibrium error $m_t^r - y_t^r - \beta_0$ see (8), but is now disturbed by the random shocks ε_t. The process x_t never hits the attractor set, but fluctuates around it due to the shocks. The force with which it is being pulled along the attractor set is proportional to $\alpha'_\perp \sum_{i=1}^t \varepsilon_i$. The latter measures the cumulative effect of autonomous shocks hitting the system.

The pulling forces can be translated into the equilibrium correction model:

$$\begin{bmatrix} \Delta m_t^r \\ \Delta y_t^r \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} (m_t^r - y_t^r - \beta_0) + \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix},$$

and the pushing forces into the corresponding common trends model:

$$\begin{bmatrix} m_t^r \\ y_t^r \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \alpha'_\perp \sum_{i=1}^{t} \varepsilon_i + \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} t + C^*(L) \begin{bmatrix} \varepsilon_{1,t} + \mu_{01} \\ \varepsilon_{2,t} + \mu_{02} \end{bmatrix},$$

with $\alpha'_\perp = \frac{1}{\alpha_1 - \alpha_2}[-\alpha_2, \alpha_1]$. Assume now that $\alpha' = [\alpha_1, 0]$, i.e. only money stock is equilibrium correcting when $m_t^r - y_t^r - \beta_0 \neq 0$. In this case $\alpha'_\perp = [0, 1]$, implying that the common stochastic trend driving this system originates from (real productivity) shocks to aggregate income. The cumulated sum of these shocks determines where on the attractor set (the 45^0 line) the system is located.

Note, however, that the interpretation of the equilibrium relation, $m_t^r = y_t^r + \beta_0$, is not that this relation will be satisfied in the limit as $t \to \infty$. An equilibrium position is something that exists at all time points as a resting point towards which the process is drawn after it has been pushed away.

The reliability of the empirical results, i.e. whether the structuring of the economic reality using the pushing and pulling forces of the cointegrated VAR model is empirically convincing, depends on how well the unrestricted baseline model describes the data. As long as a first order linear approximation of the underlying economic structure provides an adequate description of the empirical reality, the VAR model is essentially a convenient summary of the covariances of the data. Provided that further reductions (simplifications) of the model are based on valid statistical inference, the final parsimonious model would essentially reflect the information in the covariances of the data.

Embedding Romer's Money Demand Model in the VAR

This section will illustrate how to address Romer's model of money demand and supply based on the time series vector $x'_t = [m, p, y^r, R_m, R_b]_t$, $t = 1, ..., T$, where m is the log of M3 money stock, p the log of the implicit GNE price deflator, y^r the log of real GNE, R_m the interest rate yield on M3, and R_b the interest rate yield on 10-year bonds. All variables are stochastic and, thus, need to be statistically modeled, independently of whether they are considered endogenous or exogenous in the economic model.

The idea is now to translate the implications of the monetary inflation model into a set of testable hypotheses on the unrestricted VAR model. These will be formulated as restrictions on α and β, i.e. the pulling forces of the vector equilibrium correction model (8), and on $\tilde{\beta}_\perp$ and $\alpha_\perp$ i.e. the pushing forces of the common stochastic trends model (10).

The pushing forces

Based on an AD - AS system it seems natural to assume at least two *autonomous* empirical shocks $u_t^{nom} = \alpha'_{\perp,1}\varepsilon_t$ and $u_t^{real} = \alpha'_{\perp,2}\varepsilon_t$, where u_t^{nom} describes a nominal shock and u_t^{real} a real shock, each of them measured as a linear combination of the VAR errors. The unrestricted common trends model with two autonomous shocks can be formulated as:

$$\begin{bmatrix} m_t - p_t \\ y_t^r \\ \Delta p_t \\ R_{m,t} \\ R_{b,t} \end{bmatrix} = \underbrace{\begin{bmatrix} d_{11}\, d_{12} \\ d_{21}\, d_{22} \\ d_{31}\, d_{32} \\ d_{41}\, d_{42} \\ d_{51}\, d_{52} \end{bmatrix}}_{\tilde{\beta}_\perp} \underbrace{\begin{bmatrix} \sum u_i^{nom} \\ \sum u_i^{real} \end{bmatrix}}_{\alpha'_\perp \sum \varepsilon_i} + \begin{bmatrix} \gamma \\ \gamma \\ 0 \\ 0 \\ 0 \end{bmatrix} t + Z_t, \qquad (13)$$

where Z_t consists of stationary components and the inflation and interest rates have been restricted *a priori* to have no deterministic trends.

The formulation (13) is just a convenient description of the information in the data. The only prior restrictions entering the model are the number of stochastic trends and the three exclusion restrictions on the deterministic trends. These low-level economic priors are testable and, thus, can act as a first sorting device between theory models that are consistent with the basic information in the data and those that are not. For example, many rational expectations models predict fewer stochastic trends than actually found in the data, suggesting that actual economic behavior is less informed than predicted by the RE assumption.

The next level of testable economic priors is associated with restrictions on the vectors determining the stochastic trends $\alpha_\perp$ and on their loading matrix $\tilde{\beta}_\perp$. Assuming that inflation is a pure monetary phenomenon, it is natural to call an unanticipated empirical shock to money stock a *nominal* shock, so that $\alpha'_{\perp,1} = [1, 0, 0, 0, 0]$ and, similarly, an unanticipated empirical shock to real aggregate income a real shock, so that $\alpha'_{\perp,2} = [0, 1, 0, 0, 0]$.

The prior restrictions on the loadings matrix should reflect the statement that "inflation is always and everywhere a monetary phenomenon." We will interpret this as a statement that a permanent increase in money stock in excess of real GNE will only result in an equivalent increase in prices, with no changes in real money stock and real aggregate income. Similarly, the stochastic trend in the inflation rate should only influence nominal but not real interest rates. Only permanent real (productivity) shocks would have a lasting impact on any of the real variables.

The data-generating process consistent with these priors can be formulated as a restricted version of (13):

$$\begin{bmatrix} m_t - p_t \\ y_t^r \\ \Delta p_t \\ R_{m,t} \\ R_{b,t} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & d_{12} \\ 0 & d_{12} \\ d_{31} & 0 \\ d_{31} & 0 \\ d_{31} & 0 \end{bmatrix}}_{\tilde{\beta}_\perp} \underbrace{\begin{bmatrix} \sum u_i^{nom} \\ \sum u_i^{real} \end{bmatrix}}_{\alpha'_\perp \sum \varepsilon_i} + \begin{bmatrix} \gamma \\ \gamma \\ 0 \\ 0 \\ 0 \end{bmatrix} t + Z_t. \tag{14}$$

It describes an economy where real aggregate income and real money stock have been subject to the same series of real shocks, $\sum u_i^{real}$, without any lasting impact from the nominal shocks, $\sum u_i^{nom}$. The nominal stochastic trend, $\sum u_i^{nom}$, is common for the inflation rate and the two interest rates, so that real interest rates and the interest rate spread are stationary variables.

The pulling forces

The specification of the pulling forces has to be consistent with the pushing forces as the moving average and the autoregressive representation are two sides of the same coin. Two autonomous shocks are consistent with three cointegration relations, $\beta_i' x_{t-1}$, $i = 1, 2, 3$, where β_i defines a stationary linear combination of the nonstationary variables. The unrestricted cointegrated VAR model can be formulated as:

$$\begin{bmatrix} \boldsymbol{\Delta} m_t^r \\ \boldsymbol{\Delta} y_t^r \\ \boldsymbol{\Delta}^2 p_t \\ \boldsymbol{\Delta} R_{mt} \\ \boldsymbol{\Delta} R_{b,t} \end{bmatrix} = \begin{bmatrix} a_{11}\, a_{12}\, a_{13} \\ a_{21}\, a_{22}\, a_{23} \\ a_{31}\, a_{32}\, a_{33} \\ a_{41}\, a_{42}\, a_{43} \\ a_{51}\, a_{52}\, a_{53} \end{bmatrix} \begin{bmatrix} \beta_1' x_{t-1} - \beta_{0,1} \\ \beta_2' x_{t-1} - \beta_{0,2} \\ \beta_3' x_{t-1} - \beta_{0,3} \end{bmatrix} + \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \gamma_5 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \end{bmatrix} \tag{15}$$

In (15) α and β are unrestricted and we need to impose some prior restrictions consistent with the representation (14). Given (14) $(m - p - y^r)$, $(R_b - R_m)$, and $(R_b - \Delta p)$ define stationary relations and can, therefore, be chosen as our economic priors for $\beta' x_{t-1}$. However, any linear combinations of them would also be stationary, so they are unique only in the sense of uniquely defining the cointegration space.

The following restrictions on α are consistent with the restrictions on $\alpha_\perp$ in (14):

$$\alpha'_\perp = \begin{bmatrix} 1\,0\,0\,0\,0 \\ 0\,1\,0\,0\,0 \end{bmatrix} \Longleftrightarrow \alpha = \begin{bmatrix} 0\,0\,0 \\ 0\,0\,0 \\ *\,*\,* \\ *\,*\,* \\ *\,*\,* \end{bmatrix}$$

The assumption that inflation rate is positively related to money expansion in excess of money demand suggests that $a_{31} > 0, a_{32} > 0$. The sign of the coefficient a_{41} can be negative (lower interest rate when there is excess liquidity) or positive (if inflationary expectations, due to an increase in m, make the central bank increase its interest rate). The expectations hypothesis predicts that the long-term interest rate is determined by the sum of appropriately weighted interest rates of shorter maturity and the Fisher parity that the nominal bond rate should be related to the expected inflation rate. This leads to the hypothetical vector equilibrium correction model:

$$\begin{bmatrix} \mathbf{\Delta} m_t^r \\ \mathbf{\Delta} y_t^r \\ \mathbf{\Delta}^2 p_t \\ \mathbf{\Delta} R_m \\ \mathbf{\Delta} R_{b,t} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_{31} & a_{32} & 0 \\ -a_{41} & 0 & a_{43} \\ 0 & -a_{52} & 0 \end{bmatrix} \begin{bmatrix} (m-p-y^r)_{t-1} - \beta_{0,1} \\ (R_b - R_m)_{t-1} - \beta_{0,2} \\ (R_m - \Delta p)_{t-1} - \beta_{0,3} \end{bmatrix} + \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \gamma_5 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \end{bmatrix} \tag{16}$$

Thus, in order to explain monetary transmission mechanisms empirically (accounting for the fact that all variables are stochastic) it is not enough to formulate hypotheses about money demand and money supply. We also need to formulate empirically testable implications of the Fisher hypothesis and of the term structure of the interest rates.

The role of expectations

In a world where real growth rates are stationary variables (as usually found), the Fisher parity assumes that real interest rates are constant (or stationary with a constant mean) so that a nominal interest rate, R_t^m, of maturity m can be decomposed into a real interest rate, r^m, and the average expected inflation, $\frac{1}{m}\mathcal{E}_t\sum_{i=1}^m \Delta p_{t+i}$:

$$R_t^m = r^m + \frac{1}{m}\mathcal{E}_t(p_{t+m} - p_t) + \varepsilon_t = r^m + \frac{1}{m}\mathcal{E}_t\sum_{i=1}^m \Delta p_{t+i} + \varepsilon_t \tag{17}$$

where r^m denotes a constant real interest rate of maturity m. We note that a stationary real interest rate is logically consistent either with nominal interest rate and expected inflation rate both being nonstationary or, alternatively, both being stationary. Based on arbitrage arguments one would expect the yield of a financial asset to be unpredictable and, thus, nominal interest rates to approximately follow a random walk, i.e. to be nonstationary. In this case, a stationary real interest rate would be econometrically consistent with the expected inflation rate being a nonstationary variable.

Forecasting a nonstationary variable with a stationary forecast error is generally not feasible except possibly over very short periods. This is because many "extraordinary" future events are truly unpredictable and, thus, violate the probability formulation of the sample period (Clements and Hendry 1999).

A further complication in the interpretation of the VAR is that the Fisher parity is defined in terms of the expected inflation rate, which is unobservable, whereas (14) is specified in terms of the actual inflation rate. Nevertheless, as long as the stochastic trend in the actual inflation rate is the same as in the expected inflation rate, the statistical inference regarding the long-run pulling and pushing forces is robust to this problem. For example, under the plausible assumption that agents (1) form expectations according to $E_t^e(\Delta p_{t+m}) = f(x_t)$ and (2) use forecasting models which have performed well historically, say $\Delta p_{t+1} = f(x_t) + v_t$, where v_t is stationary with mean zero, then Δp_t and $E_t(\Delta p_{t+m})$ would be cointegrating and we can substitute one for the other. Thus, under fairly weak assumptions on agents' expectations behavior, we can expect the *long-run VAR results* to be robust to the problem of unobservable inflationary expectations.

From a theoretical point of view the Fisher parity condition (17) should hold independently of whether m denotes a short or a long maturity. When Fisher parity holds the interest rate spread would be related to the difference between expected average inflation rates over the short and the long maturity period, respectively:

$$R_t^l - R_t^s = r^l - r^s + \frac{1}{l}\mathcal{E}_t \textstyle\sum_{i=1}^{l} \Delta p_{t+i} - \frac{1}{s}\mathcal{E}_t \textstyle\sum_{i=1}^{s} \Delta p_{t+i}. \qquad (18)$$

Given that the Fisher parity holds, a stationary spread is achieved when the difference between the average expected inflation rates is a stationary variable.

To sum up: We have found that the hypothesis of inflation being a purely monetary phenomenon is econometrically consistent with stationary real interest rates, a stationary spread, and a stationary liquidity ratio. But we have also found that the logic of the reasoning may need to be modified to allow for persistent inflationary shocks and for 'non-RE' expectations. Thus, both nonstationarity and expectations may potentially necessitate a reformulation of the theoretical priors.

Deductive Inference: Extracting Information from the Data

The previous section illustrated how the data can be structured based on their integration properties and how this can be used to discriminate between the empirical logic of different economic hypotheses. We will provide an empirical example of pulling and pushing forces in a VAR analysis applied to Danish money demand data over the last three decades. To economize on space only the most important results will be reported and the interested reader is referred to the more detailed analyses in Juselius (2005).[1]

The baseline model is a five-dimensional VAR of real M3, real GNE, inflation rate, and a short- and a long-term interest rate. Furthermore, the VAR model includes a shift dummy $Ds83_t$ in the cointegration relations and a permanent intervention impulse dummy, $\Delta Ds83_t$, in the equations. This is to account for a significant shift in the equilibrium mean of the long-run money demand relation and in the spread between the two interest rates in 1983:1 as a result of deregulating the Danish capital markets.

Table 1 reports the cointegration results of testing the stationarity of a number of possible relations, among others the hypothetical relations in (16). The hypothesis $\mathcal{H}_1$ shows that the liquidity ratio can be considered stationary when allowing for an equilibrium mean shift at 1983:1. $\mathcal{H}_2$ and $\mathcal{H}_3$ test the stationarity of an IS-type relationship between real aggregate income and real (ex post) short-term and long-term interest rate, respectively. The stationarity of both is rejected, whereas a negative relation between the inflation rate and the real aggregate income, $\mathcal{H}_5$, can be accepted. It essentially reflects the steadily declining inflation rate and the steadily increasing real aggregate income over this period and should at most be interpreted as a "reduced form" relation. A more structural interpretation will be suggested in the next section.

$\mathcal{H}_6$ shows that inflation rate is nonstationary over this period, even if we allow for a mean shift at 1983. $\mathcal{H}_7$ and $\mathcal{H}_8$ show that real interest rates are found to be nonstationary, even if we allow the mean to be different before and after capital deregulation. $\mathcal{H}_9$ accepts the stationarity of the interest rate spread when allowing for a change in the mean (the risk premium) after deregulation. $\mathcal{H}_{10}$ and $\mathcal{H}_{11}$ test whether stationarity can be achieved by relaxing the unitary coefficient between nominal interest rates and inflation rates, but nonstationarity prevails. $\mathcal{H}_{13}$ is a test of (18), i.e. whether the stationarity of the spread is improved by including the inflation rate. No such evidence was found, but when testing the stationarity of a homogeneous relation between the inflation rate and the two interest rates in $\mathcal{H}_{16}$, stationarity was strongly supported.

Table 2 report the joint estimation of the identified long-run structure $\{\mathcal{H}_1$, $\mathcal{H}_5$, and $\mathcal{H}_{14}\}$ together with the estimated adjustment coefficients.

[1] All analyses in this and subsequent sections have been performed using the software package CATS for RATS (Dennis et al. 2005).

Table 1: Testing the stationarity of single relations when r=3.

	m_t^r	y_t^r	Δp_t	R_{m_t}	R_{b_t}	$Ds83_t$	$\chi^2(v)$	*p-value*
				Tests of liquidity ratio relations				
$\mathcal{H}_1$	1	-1	0	0	0	-0.34	1.5(2)	**0.47**
$\mathcal{H}_2$	0	1	-48.0	48.0	0	-0.70	8.8(1)	0.00
$\mathcal{H}_3$	0	1	-43.3	0	43.3	0.12	12.8(1)	0.00
$\mathcal{H}_4$	0	1	0	-163	-163	2.0	3.5(1)	0.06
$\mathcal{H}_5$	0	1	34.3	0	0	0.42	0.9(1)	**0.34**
				Tests of inflation, real interest rates and the spread				
$\mathcal{H}_6$	0	0	1	0	0	0.021	9.0(2)	0.01
$\mathcal{H}_7$	0	0	1	-1	0	0.012	10.6(2)	0.00
$\mathcal{H}_8$	0	0	1	0	-1	-0.008	14.4(2)	0.00
$\mathcal{H}_9$	0	0	0	1	-1	-0.014	4.2(2)	**0.12**
				Tests of combinations of interest rates and inflation rates				
$\mathcal{H}_{10}$	0	0	1	-0.45	0	0.016	4.9(1)	0.03
$\mathcal{H}_{11}$	0	0	1	0	-0.27	0.014	6.7(1)	0.01
$\mathcal{H}_{12}$	0	0	0	1	-0.82	-0.009	1.7(1)	**0.19**
				Tests of homogeneity between inflation and the interest rates				
$\mathcal{H}_{13}$	0	0	0.10	1	-1	-0.016	4.4(1)	0.04
$\mathcal{H}_{14}$	0	0	-0.30	1	-0.70	-0.012	0.02(1)	**0.89**

The four overidentifying restrictions were accepted with a p-value of 0.40. All three relations have experienced a shift in the equilibrium mean at 1983:1, signifying the major impact of the deregulation of the Danish capital market. The second relation suggests that the short-term interest rate has been homogeneously related to the long-term bond rate (0.80) and the inflation rate (0.20).

The hypothesis that the cumulated residuals from a specific VAR equation measure a common driving trend can be specified as a zero row in α. This implies that the variable in question is weakly exogenous for the long-run parameters. The opposite hypothesis that the residuals of an equation have transitory but no permanent effects on the variables of the system can be specified as a unit vector in α. Both hypotheses are tested for each equation and the results reported in Table 2. We note that the bond rate can be assumed weakly exogenous with a p-value of 0.20, whereas real aggregate income is a borderline case with a p-value of 0.08. Thus, the estimated results suggest that cumulated empirical shocks to real aggregate income constitute one of the common stochastic trends consistent with the hypothetical scenario, whereas the second trend seems to be generated from cumulated empirical shocks to the long-term bond rate, rather than to money stock.

Table 2: An identified structure of long-run relations.

	m_t^r	y_t^r	Δp_t	$R_{m,t}$	$R_{b,t}$	$Ds83_t$
Test of overidentifying restr. $\chi^2(4) = 4.05[0.40]$						
$\hat{\beta}_1'$	**1.00**	**−1.00**	0.00	0.00	0.00	**−0.34** [−13.60]
$\hat{\beta}_2'$	0.00	0.00	**−0.20**	**1.00**	**−0.80** [−15.65]	**−0.01** [−10.67]
$\hat{\beta}_3'$	0.00	**0.03** [3.67]	**1.00**	0.00	0.00	**0.01** [5.46]

The adjustment coefficients				Zero row in α	Unit vector in α
	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\chi^2_{0.95}(3) = 7.81$	$\chi^2_{0.95}(2) = 5.99$
Δm_t^r	**−0.21** [−4.74]	**3.38** [3.21]	0.24 [0.53]	17.56 [0.00]	**3.81** [0.15]
Δy_t^r	**0.06** [2.27]	**−1.40** [−2.21]	−0.44 [−1.59]	**6.84** [0.08]	10.32 [0.01]
$\Delta^2 p_t$	−0.00 [−0.11]	−0.29 [−0.79]	**−0.84** [−5.33]	26.65 [0.00]	**3.96** [0.14]
$\Delta R_{m,t}$	−0.00 [−0.28]	−0.07 [−1.54]	0.02 [1.00]	8.88 [0.03]	**5.09** [0.08]
$\Delta R_{b,t}$	0.00 [0.69]	**0.13** [2.04]	0.05 [1.87]	**4.64** [0.20]	6.88 [0.03]

The unit vector hypothesis is accepted for real money stock with a p-value of 0.15, for inflation rate with p-value 0.14, and for the short-term interest rate with a p-value of 0.08. Thus, consistent with the weak exogeneity tests, these tests suggest that unanticipated empirical shocks to real money stock, inflation, and the short-term interest rate have only exhibited small, if any, permanent effects on the system. However, the residuals in the short-term and the long-term interest rate equations are simultaneously correlated (0.42) and, therefore, not uniquely defined. For example, a Cholesky decomposition of the residual covariance matrix might very well change the interpretation.

Whatever the case, the hypothetical scenario for the determination of money and inflation needs to be modified to account for the estimated α dynamics. We note that the first two adjustment coefficients in the money stock equation are highly significant and the combined effects can be shown to describe a long-run money demand relation toward which money stock has been equilibrium correcting:

$$\begin{aligned}\Delta m_t^r &= ... - 0.21(m^r - y^r) + 3.38(R_m - 0.2\Delta p - 0.8R_b) + ... \\ &= ... - 0.21(m^r - y^r + 3.2\Delta p - 16.1R_m + 12.9R_b)_{t-1} + ...\end{aligned}$$

At the same time the estimated α coefficients in the inflation equation show that inflation is not affected by excess money nor by the homogeneous interest rate relation. This suggests that money stock has been purely accommodating to money demand without any inflationary effects. Instead, the inflation rate is significantly error-correcting to the third cointegration relation, the implication of which will be discussed in the next section.

Even though the estimated cointegration results seemed to provide some support for our prior hypotheses, the picture changed quite significantly when the estimated dynamic adjustment mechanisms were taken into consideration. Thus, to exclusively focus on the cointegration implications of a theoretical model (as often done in the literature) is not sufficient for valid inference.

The empirical estimates of the corresponding moving average model (13) with $u_t^{\text{real}} = \alpha'_{\perp,1}\hat{\varepsilon}_t = \hat{\varepsilon}_{y,t}$ and $u_t^{nom} = \alpha'_{\perp,2}\hat{\varepsilon}_t = \hat{\varepsilon}_{Rb,t}$[2] are given by:

$$\begin{bmatrix} m_t^r \\ y_t^r \\ \Delta p_t \\ R_{m,t} \\ R_{b,t} \end{bmatrix} = \underbrace{\begin{bmatrix} \underset{[3.75]}{\mathbf{0.70}} & \underset{[-5.07]}{\mathbf{-11.84}} \\ \underset{[6.59]}{\mathbf{1.13}} & \underset{[-2.51]}{\mathbf{-5.40}} \\ \underset{[-7.00]}{\mathbf{-0.04}} & \underset{[1.27]}{0.08} \\ \underset{[1.11]}{0.01} & \underset{[5.51]}{\mathbf{0.93}} \\ \underset{[1.93]}{\mathbf{0.04}} & \underset{[5.31]}{\mathbf{1.39}} \end{bmatrix}}_{\tilde{\beta}_\perp} \underbrace{\begin{bmatrix} \sum_{i=1}^t \varepsilon_{y,i} \\ \sum_{i=1}^t \varepsilon_{Rb,i} \end{bmatrix}}_{\alpha'_\perp \sum \hat{\varepsilon}_i} + \begin{bmatrix} 0.026 \\ 0.029 \\ 0.000 \\ 0.000 \\ 0.000 \end{bmatrix} t + ... \quad (19)$$

According to (14) the real stochastic trend should influence real money stock and real income with the same coefficients. The estimated coefficients are 0.70 to money stock and 1.13 to real income. Both are positive and not significantly different from the prior hypothesis in (14) of a unitary coefficient. However, the prior assumption that the nominal stochastic trend should not influence real money stock nor real aggregate income was clearly violated, in the sense that the second nominal trend (defined here as the cumulated empirical shocks to the long-term bond rate) has significantly affected both real money stock and real income negatively, but the former much more so.[3] Note, however, that the interpretation of the empirical results relying on (14) is no longer straightforward as one of the basic hypotheses, that the nominal stochastic trend derives from empirical shocks to excess money, was already rejected.

Finally, while the empirical shocks to real aggregate income are found to generate the second stochastic trend consistent with our prior, the way it effected the system variables is not.

[2]Even though the joint weak exogeneity of the bond rate and real aggregate income was only borderline acceptable, the subsequent conclusions are robust to whether weak exogeneity is imposed or not.

[3]This suggests that money demand is more interest rate elastic than investment demand.

Inductive inference: A European view on empirical macro

Based on the first deductive part of the analysis we found that the empirical evidence did not fully support for the hypothetical structure (16). By comparing the theoretical model with the corresponding empirical results we may get some understanding of why and where the theory model failed to be an adequate description of the empirical behavior.

We will first investigate why the stationarity of some of the prior hypotheses $\{\mathcal{H}_7, \mathcal{H}_8, \mathcal{H}_{10}, \mathcal{H}_{11}, \mathcal{H}_{13}\}$ in Table 1 was violated using the estimates in (19). We note that the inflation rate is significantly affected only by $\sum\varepsilon_y$ and the short-term interest rate only by $\sum\varepsilon_{Rb}$, whereas the bond rate is affected by both stochastic trends. Hence, cointegration between the inflation rate and either of the two interest rates is not possible. The fact that the inflation rate and the two interest rates share two common trends explains why the homogeneous interest rate/ inflation rate relation, $\mathcal{H}_{14}$, was "more stationary" than the interest rate spread relation, $\mathcal{H}_9$. The 'stationarity' of the latter was only achieved by suppressing the small but significant effect of $\sum\varepsilon_y$ on the bond rate. Thus, *the discrepancy between the Fisher parity and the empirical evidence is because the stochastic trend in inflation rate originates from different empirical shocks than the ones in interest rates.*

The discrepancy between the expectations hypothesis and the empirical evidence is because the bond rate, but not the short rate, has been affected by the empirical shocks to real aggregate income. Furthermore, the latter effect is similar to the effect on inflation rate but with opposite sign suggesting that the large public deficits (mostly to finance the large unemployment in this period) had a positive effect on the government bond rate and a negative effect on inflation rate.

We will now take a closer look at the common trends implications of the three cointegration relations reported in Table 2. The liquidity ratio is described by the common trends

$$\begin{aligned} m^r - y^r &= (0.70 - 1.13)\textstyle\sum\varepsilon_y - (11.8 - 5.4)\sum\varepsilon_{Rb} \\ &= -0.43\textstyle\sum\varepsilon_y - 6.4\sum\varepsilon_{Rb}, \end{aligned}$$

and the homogeneous inflation rate - interest rates relation by

$$\begin{aligned} R_m - 0.8R_b - 0.2\Delta p &= -(0.032 - 0.008)\textstyle\sum\varepsilon_y + (0.93 - 1.12)\sum\varepsilon_{Rb} \\ &= -0.024\textstyle\sum\varepsilon_y - 0.19\sum\varepsilon_{Rb}. \end{aligned}$$

Combining the two gives:

$$m^r - y^r - 16.1(R_m - 0.8R_b - 0.2\Delta p) = 0.04\textstyle\sum\varepsilon_y - 3.2\sum\varepsilon_{Rb}.$$

While the two stochastic trends almost cancel in the liquidity ratio the results become much more stationary when combining the latter with the opportunity cost of holding money relative to holding bonds or real stock (the second cointegration relation).[4]

Finally, the cointegration implications underlying the less interpretable inflation - income relation can be inferred from:

$$\begin{aligned}\Delta p + 0.03y^r &= -(0.04 - 0.04)\sum\varepsilon_y + (0.08 - 0.16)\sum\varepsilon_{Rb} \\ &= 0.0\sum\varepsilon_y - 0.08\sum\varepsilon_{Rb}\end{aligned}$$

From (19) we note that inflation rate has been significantly (and negatively) influenced by $\sum\varepsilon_y$ but positively by $\sum\varepsilon_{Rb}$, whereas the opposite is the case with real aggregate income explaining why these two variables are negatively related over this sample. While a negative effect of a bond rate shock on aggregate income is completely plausible, the negative effect of an income shock on inflation is less so. As the negative co-movement between inflation and real aggregate income is very pronounced and, therefore, statistically significant, it would be against the Popperian spirit to ignore this finding. It is an example of an important piece of information in the data signalling the need to dig deeper in order to understand more.

Altogether, the empirical analysis of our VAR model has found support for some prior hypotheses, but also detected a number of surprising results. First of all, the finding that the empirical shocks to excess money, inflation, and the short-term interest rate exclusively, had a transitory effect on each other and the other variables of the system strongly suggests that in the post-Bretton Woods period *the Danish inflation is not explained by monetary factors.* The question is whether this surprising result is related to the finding of a negative relation between inflation and real aggregate income.

This prompted us to investigate the possibility that in this period the Danish price inflation had its origin in wage inflation and/or imported inflation (Juselius 2005). This study indicated that many of the surprising results in the monetary model can be explained by some institutional changes which strongly influenced macroeconomic mechanisms in this period. In particular, the creation of the European Community and more generally the deregulation of international capital markets seem to have been crucial in this respect. Two important explanations can be mentioned:

[4] The two stochastic trends do not cancel completely because (18) was estimated under the two weak exogeneity restrictions, and the β coefficients were estimated without this restriction.

1. The increased global price competitiveness significantly weakened the labor unions and put a downward pressure on nominal wage claims.

2. The increased internationalization of the capital market moved the determination of the Danish bond rate away from the domestic to the international capital markets and, thus, made the Danish bond rate exogenously determined in the present model.

The increased price competitiveness seemed foremost to have resulted in an adjustment of labor productivity (increase in labor intensity combined with new technology) with a parallel increase in unemployment (firing part of the labor force, hiring fewer people, and outsourcing). Thus, the cumulated empirical shocks to real aggregate income are essentially measuring shocks to trend-adjusted productivity. Positive shocks to the latter have often been associated with improvements in labor intensity (producing the same output with less labor and working harder for the same pay). Evidence supporting this is, for example, that unemployment rate and trend-adjusted productivity have been positively cointegrated in this period.

The finding that $\sum \varepsilon_y$ loaded negatively into the inflation rate can be explained by the downward pressure on prices as a result of the high unemployment rates in this period. Similarly, the finding that $\sum \varepsilon_y$ loaded positively into the bond rate can be explained by the large increase in the supply of government bonds to finance huge unemployment compensation in this period.

The finding that the bond rate, rather than the short rate, was weakly exogenous[5] points to the importance of international capital markets as an increasingly important driving force in the domestic macro economy. Closely related is our finding of nonstationary Fisher parities, an (almost) nonstationary interest rate spread and in an extended study (Juselius 2005) nonstationary real exchange rates and uncovered interest rates parities. As similar empirical results have been found in a variety of empirical studies based on other countries data it is a strong signal that we need to understand why the international parity conditions are so persistent and how this lack of fast adjustment or market clearing has influenced the mechanisms of our domestic economy.

As demonstrated above empirical puzzles detected in the VAR analyses often suggest how to proceed to make empirical evidence and theory

[5] Even though the distinction between empirical shocks to the short-term and the long-term interest rate was not unambiguous, the weak exogeneity finding of the bond rate has been confirmed in other studies based on different sets of information and different temporal aggregation

fit together more closely. This includes modifying the theoretical model by extending the information set or changing the model altogether. In either case the analysis points forward, which is why we believe the cointegrated VAR methodology has the potential of being a progressive research paradigm.

Conclusions

The purpose of this paper was to illustrate how to extract information from the data based on the cointegrated VAR model and its decomposition into pulling and pushing forces. Methodologically the approach combines deduction and induction. The deductive part of the analysis is based on a theory model, the testable implications of which have been translated into a set of hypotheses on the parameters of the VAR model describing long-run relations, adjustment dynamics, driving trends, and their effects on the variables of the system. Since the theory model (by necessity) is based on numerous simplifying assumptions, the VAR analysis usually detects discrepancies between empirical and theoretical behavior. The inductive part of the analysis treats such discrepancies as a useful piece of information helping us to adjust our intuition of how the economic and the empirical model work together, sometimes leading to modifications of a too narrowly specified theoretical model, in other cases to the generation of new hypotheses.

Thus, by embedding the theory model in a broader empirical framework, the analysis of the statistically based model often provides evidence of possible pitfalls in macroeconomic reasoning. It also generates a set of relevant "stylized facts", such as the number of autonomous shocks and how they affect the variables of the system. This is in contrast to the more conventional graphs, mean values, and correlation coefficients, of which the latter are inappropriate when data are nonstationary. Finally, it provides a check on how sensitive theory based conclusions are to the *ceteris paribus* assumption.

The empirical application of a model for monetary inflation demonstrated that many aspects of the theory model can be translated into a set of testable hypotheses, all of which should be accepted for the theory model to have full empirical validity. This is in contrast to many empirical investigations, where inference is based on test procedures that only make sense in isolation, but not in the full context of the empirical model.

Econometrically, the approach follows the principle of general-to-specific, starting essentially from the covariances of the selected data and then sequentially imposing more and more restrictions on the model, some of which might be consistent with the theoretical priors, others not. In the latter case the rich structure of pulling and pushing forces provides a wealth of information which should help to generate new hypotheses and guide the user to follow new paths of enquiry.

Thus, a careful analysis of the empirical results might at an early stage suggest how to modify either the empirical or the economic model. This is one way of translating the notion of a design of experiment and the link between theory and empirical evidence when the latter is based on data collected by passive observation suggested in Haavelmo (1944):

> "In the second case we can only try to adjust our theories to reality as it appears before us. And what is the meaning of a design of experiment in this case. It is this: We try to choose a theory and a design of experiments to go with it, in such a way that the resulting data would be those which we get by passive observation of reality. And to the extent that we succeed in doing so, we become masters of reality — by passive agreement."

T he alternative, which is to force the chosen economic model on the data, thereby squeezing an exuberant reality into "all-too-small-size clothes," is a frustrating experience that often makes the desperate researcher choose solutions that are not scientifically justified.

Macroeconomic data have often been found to be quite informative about hypothetical long-run relationships and the propagation mechanism driving them. Therefore, we are convinced that the policy usefulness of empirical macro models can be vastly improved by properly accounting for the dynamic adjustment of feedback and interaction effects to long-run steady-states.

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