University of Copenhagen

“Økonomisk Kandidateksamen 2006 II”

Advanced Econometrics (“Videregående Økonometri")

Project Examination

June 26-28, 2006

- This is an **individual** exam. Cooperation between students is strictly forbidden.
- Please answer all questions in Sections 1-7. In the evaluation of your report the sections have approximately the following weights: Sections 1-3: 10% each, Sections 4-5: 15% each, Sections 6-7: 20% each.
- The report must be written in Danish or in English. It should preferably not exceed 12 pages (15 pages if handwritten). A maximum of 10 pages of supporting material (graphs, program code, estimation output, etc.) can accompany the report as appendices. You can refer to the computer output in the appendices when answering the questions. Note, that you should report your computer output in the font Courier, as it improves readability. You may also add clarifying comments in the computer output as part of your answer.
- The first page is a pre-specified page to be added to your exam. It is available from http://www.econ.ku.dk/okokj/exam2006/First page.doc
- All pages must be numbered consecutively and marked with your **exam number**. You should **not** write your name on the report.
- **2 copies** of your report (including supporting material) must be received at ”Det samfundsvidenskabelige Fakultetskontor, Studie- og eksamenskontoret, 1. sal” (the Office of the Faculty of Social Sciences, 1st floor) before **11.00 noon on June 28**.
- There are many questions in this exam, some of which might seem easier, others more difficult to answer. A good advice is to answer the easiest questions first and then focus on the difficult ones. If you first spend too much time on the difficult questions you may not have enough time to answer the easy ones.
- To save time, you do not have to replicate the formulas and the derivations given in the lecture notes. However, is a good idea to make a reference to the appropriate chapter and formula when answering the questions.

1 Defining the real business cycle model in Ireland (2003)

The purpose of this take-home exam is to econometriclly assess some empirical results on a real business cycle model reported by Peter Ireland, hereafter PI. By solving a nonlinear maximization problem PI obtains estimates of the model parameters, which are claimed to be maximum likelihood estimates. These, however, are only maximum likelihood estimates given that the assumed model is a correct representation of the data. Since, in PI's paper there are hardly any tests of the validity of the underlying assumptions, the idea of this exam is to test as many as possible of these assumptions within the VAR framework.

Thus, you do not have to understand PI's model to pass the exam. The introduction below is only to present some of the basic features and testable assumptions of the model.

A representative agent maximizes expected utility by choosing between consumption, C_t, and total hours worked, H_t

$$E\sum_{t=0}^{\infty}\beta^t(\ln C_t - \gamma H_t) \tag{1}$$

subject to a constant return to scale technology described by the Cobb Douglas production function:

$$Y_t = A_t K_t^{\theta}(\eta^t H_t)^{(1-\theta)} \tag{2}$$

where Y_t is gross output, K_t is capital stock, $\eta > 1$ measures the rate of labor-augmented technological progress and A_t is total factor productivity. Taking the log of (2) gives us:

$$\ln Y_t = \ln A_t + \theta \ln K_t + (1-\theta)(\ln H_t + \ln \eta * t). \tag{3}$$

$\text{Ln}A_t$ is assumed to follow a first order autoregressive model:

$$\ln A_t = (1-\rho)\ln A + \rho \ln A_{t-1} + \varepsilon_t \tag{4}$$

with $|\rho| < 1$ and $\varepsilon_t \sim NI(0, \sigma_\varepsilon^2)$. The following two identities complete the model:

Capital K_t is defined as capital last period, K_{t-1}, corrected for the depreciation rate δ plus investment I_t at time t:

$$K_t = I_t + (1-\delta)K_{t-1}. \tag{5}$$

Gross output is the sum of consumption and investment:

$$Y_t = C_t + I_t. \tag{6}$$

To simplify notation we let $y_t = \ln Y_t$, $a_t = \text{Ln}A_t, a = \text{Ln}A, k_t = \ln K_t, h_t = \ln H_t$, and $c_t = \ln C_t$, so that the the Cobb-Douglas function becomes:

$$y_t = a_t + \theta k_t + (1-\theta)h_t + \underbrace{(1-\theta)\ln\eta t)}_{b_1} \tag{7}$$

and the total factor productivity:

$$a_t = (1-\rho)a + \rho a_{t-1} + \varepsilon_t. \tag{8}$$

The model defined so far is driven by the deterministic trend $b_1 t$ describing labor augmented technological progress and by the random shocks ε_t to total factor productivity a_t. To make the model slightly more flexible, PI assumes that short-run changes in y_t, k_t, h_t and c_t can be approximately described by a VAR described below in moving average form:

$$\begin{bmatrix} y_t \\ k_t \\ h_t \\ c_t \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \\ c_{14} \end{bmatrix} [a_t] + \begin{bmatrix} b_{11} \\ b_{12} \\ b_{13} \\ b_{14} \end{bmatrix} [t] + \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \end{bmatrix} \tag{9}$$

where the residuals $\mathbf{v}_t = \mathbf{D}\mathbf{v}_{t-1} + \boldsymbol{\xi}_t$ and $\boldsymbol{\xi}_t$ is $IN(\mathbf{0}, \mathbf{V})$ and uncorrelated with ε_t.

1.1 Formulate total factor productivity a_t as a function of ε_{t-i}, $i = 0, 1, ..., t-1$, and by replacing $\varepsilon_{t-i} = E(\varepsilon_{t-i}) = 0$ for $i = t, -t+1, t+2...$ We assume that a_t is generated by (8). Does total factor productivity contain a linear trend?

A1.1 We first note that:

$$\begin{aligned} a_t &= (1-\rho)a + \rho a_{t-1} + \varepsilon_t. \\ &= a + u_t \\ \text{where } u_t &= \frac{\varepsilon_t}{1-\rho L} = \rho u_{t-1} + \varepsilon_t \end{aligned}$$

We can now recursively express a_t as a function of the errors and the initial value:

$$\begin{aligned} a_1 &= a + \varepsilon_1 \\ a_2 &= a + \rho(\varepsilon_1) + \varepsilon_2 \\ a_3 &= a + \rho(\rho\varepsilon_1 + \varepsilon_2) + \varepsilon_3 \\ \vdots &\ \ \vdots \qquad \vdots \\ a_t &= a + \rho^{t-1}\varepsilon_1 + \rho^{t-2}\varepsilon_2 + ... + \rho\varepsilon_{t-1} + \varepsilon_t \end{aligned}$$

No there is not a linear trend in total factor productivity.

1.2 PI assumes that $|\rho| < 1$ in (8), obtains an estimate $\hat{\rho} = 0.998$, and concludes that total factor productivity is stationary, though very persistent. Assume that you estimate the VAR model for $x_t' = [y_t, k_t, h_t, c_t]$ and determines the cointegration rank based on the trace test. What would you expect the rank to be for the hypothetical data generating process in (9)?

A.1.2 In a practical situation it would be almost impossible to see the difference between a time series generated with $\rho = 0.998$ and $\rho = 1$. If there is a root of 0.0998 in the process the asymptotic distributions would be better described by Brownian motions than by stationarity. Thus, with four variables in PI's model one would at least expect one unit root, but of course there could be more, so the rank would be three or less. See also A.2.2.

2 Transforming PI's model into a VAR

PI claims that capital, k_t, is unobservable and generates a series for capital by assuming that $\delta = 0.975$ in (5) and that the shocks to capital are the same as the shocks to total factor productivity. Because official US data bases contain a variable for K_t, we will instead use this variable in our VAR analysis. Finally, because identities are generally difficult to account for in a VAR model we will disregard the identity (6). The VAR model that approximately corresponds to PI's model is given by the vector x_t:

$$x_t' = [y_t, c_t, h_t, k_t] \tag{10}$$

where $y_t = y_t^n - p_{y,t}$ = the log of real US GDP, $c_t = c_t^n - p_{c,t}$ = the log of real US aggregate consumption, h_t is the log of total hours worked in US, $k_t = k_t^n - p_{k,t}$ = the log of real US gross capital formation, and $p_{x_1,t}$ is a price deflator for the variable $x_{1,t} = y_t^n, c_t^n, k_t^n$.

2.1 Discuss the adequacy of the nominal-to-real transformation of the variables in the VAR vector (10) under assumption that all nominal variables in levels are (i) $I(2)$ or (ii) I(1).

A.2.1 PI assumes that his variables are (trend)stationary. In this case one would expect nominal variables to be I(1) with a deterministic trend and, hence, nominal growth rates to be I(0). However, if the price indices and the nominal variables y_t^n, c_t^n, k_t^n are all I(1) there would not be a need for the nominal-to-real transformation and one could choose to analyse $y_t^n, c_t^n, k_t^n, h_t, p_t$ directly, or alternatively, the equivalent data vector, y_t, c_t, k_t, h_t, p_t. in which the variables are either I(0) around a linear trend or I(1). If, on the other hand, nominal variables are I(2) then the nominal-to-real transformation is needed to make the real variables I(1) with a linear trend. In this case adding inflation to the vector x_t is essential as inflation will be I(1) and, hence, will influence the cointegration properties.

2.2 PI reported a characteristic root of 0.998 which in practise is not distinguishable from a unit root. Thus, we shall assume that there is at least one stochastic (unit root) trend in the VAR model based on (10). Assume that we decide to add an inflation rate to (10). Do you think this would change the number of stochastic trends in the model? Motivate your answer!

A.2.2 Given that inflation rate is considered I(1) it is affected by a stochastic trend which is likely to describe the cumulation of permanent nominal shocks in the system. The latter are likely to derive from different sources than the permanent real productivity shocks assumed by PI. Thus, we would expect to find at least two stochastic trends in the VAR model when $x_t' = [y_t, c_t, h_t, k_t, \Delta p_t]$. However, some or even all of the variables in PI's model are likely to be influenced by nominal as well as real permanent shocks. Therefore, adding inflation rate to PI's model should not change the number of common trends. But, because the nominal stochastic trend cannot be identical to the real productivity trend it means that PI's origninal model would need $r = 2, p - r = 2$ to be consistent with the information in the data.

2.3 Assume that we analyze the VAR model for $x_t' = [y_t, c_t, k_t, h_t, \Delta p_t]$. Which conditions on the VAR model in nominal variables should be satisfied for this transformation to be valid? The inflation rate Δp_t could be measured by any of the three inflation measures $\Delta p_{y,t}$, $\Delta p_{c,t}$, or $\Delta p_{k,t}$. Does it matter which one we choose? Motivate your answer.

A.2.3 If there is just one I(2) trend among the nominal variables and their price deflators, i.e. among $y_t^n, c_t^n, k_t^n, p_{y,t}$, $p_{c,t}$, $p_{k,t}$, then it does not matter for the cointegration results which one we choose. Hence, the nominal-to-real transformation would give us I(1) variables and we can choose any of the three inflation rates $\Delta p_{x_1,t}$ as a measure of the nominal I(1) trend. However, if the nominal variables and the three price deflators share two stochastic I(2) trends, then we would need to include two growth rates representing each of the two differenced I(2) trends.

3 Specification

In the following we shall consider the following VAR(2) model:

$$\begin{gathered}\Delta x_t = \Gamma_1 \Delta x_{t-1} + \Pi x_{t-1} + \Phi D_t + \mu_0 + \mu_1 t + \varepsilon_t, \\ \varepsilon_t \sim IN(0, \Omega\ ),\ t = 1, ..., T, \quad x_{-1}, x_0 \text{ given,}\end{gathered} \tag{11}$$

where

$$x_t' = [(y_t, c_t, , h_t, k_t, \Delta p_{y,t}]. \tag{12}$$

Thus, the VAR model here adds a nominal growth rate to the variables of the PI model. The data are available in the downloadable xls file Ireland.xls for a total sample of 1960:1-2002:1, spanning 42 years of quarterly observations.

However, you are supposed to analyze a slightly different period defined by the period A given under your exam number. In addition there is a downloadable CATS program file, ireland.prg, which reads the data series, defines the sample to be analyzed and contains the basic CATS commands for setting up the cointegration model. Note that you have to change the line: `smpl xxxx:yy zzzz:vv` so that it corresponds to the sample period A given under your exam number. You are free to use any one (or both) of the two software packages CATS and PcGive.

3.1 Assume that the real variables contain a linear deterministic trend. How would you specify the constant term μ_0 and the linear trend $\mu_1 t$ in the cointegrated VAR model (11)? Motivate your choice!

A.3.1 The constant term should be unrestricted, i.e. $\mu_0 = \alpha\beta_0 + \gamma_0$, because we need to allow for a constant term in the cointegration relations and a constant term in the equations describing the slope of the linear trends in the data. The trend should be restricted to the cointegration relations, i.e. $\mu_0 = \alpha\beta_1$. This means that we can test whether the linear trend cancel or not in the cointegration relations. The trend cannot be unrestricted as this would imply that we allow for quadratic trends in the data.

3.2 In 1978:1 the Carter administration managed to significantly boost the US economy which resulted in a strong increase in GDP and employment. This can be seen as a positive shock, i.e. an innovational blip outlier in the equations Δy_t and Δh_t. How will you account for this outlier in the VAR model? Motivate your choice!

A.3.2 A blip in Δx_t corresponds to a level shift in x_t. As we do not know whether the level shift cancels or not in the cointegration relations we should include a shift dummy restricted to be in the cointegration relations. Thus, the VAR model should include $\phi_s D_{s,t} + \phi_p D_{p,t}$ where $\phi_s D_{s,t} = \alpha\varphi_0 D_{s,t}$ is restricted to be in the cointegration relations and $\phi_p D_{p,t}$ is unrestricted in the VAR.

3.3 Suppose that the 'Carter' economic boost in 1978:1 changed the equilibrium mean of at least one of the cointegration relations. How would you test this hypothesis?

A.3.3 I would first specify the VAR model with the shift dummy in the cointegration relations, determine the cointegration rank, and then test whether $\varphi_0 = 0$ in all relations.

4 Determination of the rank

You are now supposed to perform a VAR analysis based on your preferred specification of the deterministic components (regarding trends, constant and dummies).

4.1 Determine the rank r of $\Pi = \alpha\beta'$ based on the trace test.

A.4.1 The standard asymptotic tables are based on case 3, i.e. the variables contain a linear, but not a quadratic, trend. The trend is restricted to the cointegration space and a constant enters the equations unrestrictedly. Based on the standard table the choice of rank would be 2.

I(1)-ANALYSIS							
p-r	r	Eig.Value	Trace	Trace*	Frac95	P-Value	P-Value*
5	0	0.35	135.70	127.70	69.61	0.00	0.00
4	1	0.21	64.55	59.10	47.71	0.00	0.00
3	2	0.11	26.03	24.14	29.80	0.13	0.20
2	3	0.04	7.57	6.54	15.41	0.52	0.64
1	4	0.00	0.29	0.24	3.84	0.59	0.62

Q.4.2 Are the standard asymptotic tables valid for your model specification? Motivate your answer!

A.4.2 The total sample is quite large, 166 obs. With so many observations the asymptotic tables should be reasonably close which can be checked by comparing them with the Bartlett corrected ones. The difference is not too bigs $r = 2, p - r = 3$ would be chosen with a p-value of 0.20 when Bartlett corrected and 0.13 when not. However, because of the shift dummy in the cointegration relations, the standard asymptotic tables are no longer valid and the simulated tables should be used instead. The results below show that the results are more supportive of $r = 3$ using the simulated tables without a Bartlett correction. This is surprising as the simulated tables should generally move the distribution to the right. I have to check whether ther is a programming mistake in CATS. However, the economic prior is consistent with two rather three stochastic trends. By choosing $r = 2$ we would achieve a fairly low p-value for the null of 3 unit roots, but the probability of rejecting the economically more plausible hypothesis, $r = 3$, when it is true would probably be high. Because the first three λ_i values are quite large compared to the last two which are quite low it does not seem unreasonable to consider the choice of $r = 3$.

Deterministic specification: Restricted Linear Trend (CIDRIFT)

Level Shifts (1) : 1978:01 (0.422)

Number of Replications (N): 2500
Length of Random Walks (T): 400

The I(1) analysis based on the simulated critical values:

I(1)-ANALYSIS							
p-r	r	Eig.Value	Trace	Trace*	Frac95	P-Value	P-Value*
5	0	0.35	160.75	151.28	97.66	0.00	0.00
4	1	0.22	88.89	81.54	71.58	0.00	0.01
3	2	0.17	47.87	42.97	49.92	0.07	0.18
2	3	0.06	17.21	16.19	30.52	0.66	0.73
1	4	0.04	7.28	6.67	15.12	0.53	0.60

4.3 Check the results of the trace test with other useful information in your estimated model. Is your choice of r consistent with this information? Motivate your answer!

A.4.3 The t-values of α_2, α_3 and α_4 shows that the first two have large t-values in several equations, whereas there are hardly any vaguely significant ones in α_4 based on standard p-levels for the t test. This is support for the choice of $r = 3$. However, there seems to be a fairly big root in the model both for $r = 2$ and 3. This might suggest that the nominal-to-real transformation may not have been completely adequate in our case. (As a matter of fact long-run price homogeneity was rejected). Whatever the case, the characteristic root is smaller for $r = 2$ than for $r = 3$ but the difference is not that large. The graphs of the cointegration relations suggest that the first two relations are strongly mean reverting, whereas the third relation exhibits longer swings (slightly more persistence in the equilibrium errors. The graphs of the recursively calculated trace tests suggest that $r = 3$ might be a good choice, even though the results also indicate some degree of non-constancy in the model. Altogether, the evidence is somewhat inconclusive but, nevertheless, seems reasonable consistent with the preferred choice of $r = 3$.

4.4 Is your choice of r consistent with the assumptions in PI's model? Motivate your answer.

A.4.4 If we assume that the near unit root (0.998) in PI's model is a unit root and if we assume that inflation contains a unit root and that the inflationary shocks are different from the total factor productivity shocks, then we should expect two stochastic trends in our extended VAR model. In this sense the results are consistent with PI's model.

Independently of your choice of rank above, continue with $r = 3$.

5 Testing hypotheses

PI made the following assumptions on his model:

1. y_t and c_t are stationary around a linear trend

2. h_t is stationary around a constant mean

3. k_t is trend-stationary and influenced by the same shocks that have generated total factor productivity a_t. As already mentioned PI found a root of 0.998 in a_t.

4. a_t and, hence k_t, act as the main driving forces in the model.

These assumptions were imposed without being first tested in PI's paper, so you are now supposed to do it instead using your VAR model.

5.1 Test PI's assumptions 1 and 2 above.

A.5.1 Based on Table 2 in the Appendix it appears that the trend-stationarity of both income and consumption is rejected. However, if we also allow the shift dummy to enter, then income, but not consumption can be accepted as trend-stationary as seen from Table 1. Based on Table 3 in the Appendix it appears that h_t cannot be accepted as stationary.

5.2 Test the hypothesis that k_t is trend-stationary. A direct test of a_t is more difficult as a_t not directly observable. However, if we consider PI's total factor productivity to be a unit root process ($0.998 \simeq 1.0$), then $a_t = y_t - \theta k_t - (1-\theta)h_t - b_1 t$ should be nonstationary. Test this hypothesis and comment on the result!

A.5.2 Table 2 shows that gross capital can be accepted as trend-stationary with a p-value of 0.65. Also, the Cobb-Douglas production function, $y_t - \theta k_t - (1-\theta)h_t$, can be accepted as trend-stationary based on the test value $\chi^2(2) = 4.02$ (0.13). This means that the finding in PI of a root of 0.998 in total factor productivity hypothesis must be the result of generating capital and total factor productivity from the same shocks ε_t. By using the observed value of gross capital stock, a homogeneous Cobb-Douglas production function can be accepted without assuming a stochastic trend in a_t.

5.3 Test the hypothesis that one of the main driving stochastic forces in this system is described by the cumulated shocks to k_t.

A.5.3 None of the variables can be accepted as weakly exogenous based on Table 4. Capital stock is definitely not weakly exogenous and the shocks to capital stock cannot be one of the driving stochastic trends.

5.4 PI did not include inflation rate in his model. Thus, the second stochastic driving trend might very well represent cumulated inflationary shocks. Test the hypothesis that cumulated shocks to capital stock and inflation represent the two stochastic trends.

A.5.4 Even though inflation rate was not accepted as weakly exogenous, it was not too far from the acceptance level. However, the joint w.e hypothesis of inflation and capital stock was strongly rejected $\chi^2(6) = 43.77$ (0.00).

5.5 Test the hypothesis that any of the VAR variables has a unit vector in α. Interpret the result. Does the result support the basic assumptions in PI.

A.5.5 The test results in appendix, Table 5, shows that the only variable that can be accepted is capital stock with a fairly high p-value of 0.46. This is a result which is very much against the assumptions of the PI model, which assumed that capital stock (or total factor productivity) was the driving stochastic trend, implying that consumption, hours worked and GDP were the purely adjusting variables. The finding that capital stock is exlusively adjusting is not very surprising considering that it was found to be trend-stationary.

Because of the change in monetary policy regime around 1979, PI tested parameter constancy by comparing the parameter of his (structural) model in the period prior to 1979:4 with those in the period after 1981:1. The test rejected parameter constancy but, nevertheless, he continued the analysis based on the full sample period.

5.6 To check the PI's result, test the hypothesis that the long-run parameters estimated for the period defined by the beginning of your sample to 1979:4 remain constant when gradually extending the sample period. Assume that the test rejects parameter constancy. Would you still have confidence in your test results obtained in Sections 4 and 5? Motivate your answer!

A.5.6 Figure 5.6.1 in the Appendix shows that the recursively calculated test of whether the estimated $\tilde{\beta}$ based on the sample 1960:4-1979:4 is the confidence band of the recurively calculated β, i.e.whether $\tilde{\beta} \in \text{sp}(\beta_{(t_1)}),\ t_1 = T_1, \ldots, T$, was rejected for almost all recursively calculated sample periods. On the other hand Figure 5.6.2 shows that the max test of β constancy does not reject parameter constancy (even though the tests are close to the rejection line). However, the latter test generally suffers from very low power and usually rejects parameter constancy when there is massive evidence of non-constancy. All the previous results were derived under the assumption of a constant parameter VAR model. Since parameter constancy seemed to be rejected we simply do not know which, if any, of the previously obtained results we can trust. However, by carefully checking the other recursive tests it might be possible to isolate the parameter non-constancy to be a problem only in some part of the model. In this case we might be able to trust the part of the model which has been reasonably constant, though not the full model.

6 Identification of the long-run structure

Continue your VAR analysis based on sub-sample B indicated under your exam number. You need not test for cointegration rank, but continue with $r = 3$ as before. Also, we assume that the specification of the deterministic terms is the same as for the full model.

Assume that one of the long-run relations describes the Cobb-Douglas production function (3), another one the savings ratio, $c_t - y_t$, and the third one a relation for the inflation rate, $\Delta p_{y,t}$.

6.1 Estimate an identified and empirically acceptable long-run structure that contain these three relations. If any of the above defined relations is not stationary as such you should add additional variables until it is accepted as stationary.

A.6.1 The identified structure shown below was accepted with a p-value of 0.21 based on $\chi(5) = 7.15$. The first relation describes a plausible trend-stationary Cobb-Douglas function, the second the consumption-income ratio as a function of gross capital stock, and the third inflation to be dynamically adjusting to $y^n - p_y$ in an equilibrium correcting way. The interpretation of the second relation is that capital stock has been poitively co-moving with the savings rates, which seems plausible. Howver, the α coefficients are not very close to the expected dones. For example, Gross income is not significantly adjusting to the Cobb-Douglas production function and consumption is overshooting to the savings ratio relation.

β'						
	LY	LC	LH	LCAPG	DLPY	TREND
Beta(1)	1.00 [NA]	0.00 [NA]	−0.45 [−14.63]	−0.55 [−17.56]	0.00 [NA]	0.00 [3.24]
Beta(2)	−1.00 [NA]	1.00 [NA]	0.00 [NA]	0.05 [6.87]	0.00 [NA]	0.00 [NA]
Beta(3)	0.01 [6.21]	0.00 [NA]	0.00 [NA]	0.00 [NA]	1.00 [NA]	0.00 [NA]

α			
	Alpha(1)	Alpha(2)	Alpha(3)
DLY	−0.03 [−0.40]	0.18 [2.13]	−0.59 [−2.31]
DLC	0.06 [1.22]	−0.15 [−2.32]	−0.80 [−4.15]
DLH	0.03 [0.96]	0.07 [1.72]	−0.27 [−2.37]
DLCAPG	0.48 [5.34]	−0.60 [−5.06]	−0.13 [−0.38]
DDLPY	−0.05 [−2.16]	0.04 [1.32]	−0.44 [−4.82]

6.2 Is your structure empirically identified? Motivate your answer!

A.6.2 The structure fails to be empirically identified if we loose generic identification by setting an insignificant coefficient to zero. Since all coefficients are strongly significant we can conclude that the structure seems empirically identified. However, the normalization coefficients can in principle correspond to an insignificant variable. To check this, we can have a look at the significance of the corresponding α coefficient. If it is significant it is a sign that the normalization variable is significant. If it is insignificant then we can re-normalize on anther variable to be able to get standard errors. This could be relevant for the first relation, where the α coefficient in the income equation is insignificant. If we normalize on the stock of capital instead income will be shown to be very significant. Thus we have empirical identification.

6.3 Is your structure irreducible? Motivate your answer!

A.6.3 A structure is irreducible, if we cannot remove any variables from the relations without loosing cointegration. This is more or less the case here, except for the Cobb-Douglas relation which in this period could be accepted as stationary without a trend. But, the trend would then have been excludable altogether which was not meant to be an option in this exam question. (As a matter of fact, long-run exclusion of the trend could be accepted based on a p-value of 0.24.)

Assume the following β structure:

$$\begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \\ a & 0 & 0 \\ -a & d & 0 \\ b & 0 & f \\ 0 & e & 0 \\ c & -e & 0 \end{bmatrix} \tag{13}$$

6.4 Is β_1 identified, just identified, or overidentified with respect to β_2 and β_3? Motivate your answer!

A.6.4 β_1 is not identified with respect to β_3, because it is not possible to distinguish β_1 from $\tilde{\beta}_1 = \beta_1 + \omega\beta_3$ for arbitrary values of ω ($r_{1.3} = 0$)

6.5 Is β_3 identified, just identified, or overidentified with respect to β_2 and β_1? Motivate your answer!

A.6.5 β_3 is overidentified with respect to β_1 with two identifying restrictions of which one is overidentifying. This is because the pair (c,0) is identifying and the coefficient a in the pair $a(1-1, 0-0)$ is identifying. β_3 is identified with respect to β_2 with two identifying restrictions of which one is overidentifying. This is because the pair (d,0) is identifying and the coefficient e in the pair $e(1-1, 0-0)$ is identifying. Thus, $r_{3.2} = 2$ and $r_{3.1} = 2$ and

7 The identification of the short-run structure

7.1 Estimate a parsimonious short-run adjustment structure (i.e. with few and significant coefficients) for your equation system by keeping the three cointegration relations fixed at their estimated values (Hint: use the option 'save the data' in CATS or the 'calculator' in GiveWin to create the estimated cointegration variables). You may disregard the simultaneous effects.

A.7.1 Sys(1) in the Appendix contains the estimated short-run adjustment structure for the VAR model and Mod(4) a parsimonious simplification where the test of the 22 overidentifying restrictions are accepted based on a p-value of 0.32. However, based on the F-tests of regressors in the VAR system it appears that the lagged change in inflation rate can be deleted altogether from the system. By re-estimating the system and testing the same parsimonious structure as in Mod(4) we arrive at Mod(5) where the 18 overidentifying restrictions can be accepted with a p-value of 0.29.

7.2 Consider the two equations in the system which have the highest residual correlation. Let us call them $\Delta x_{1,t}$ and $\Delta x_{2,t}$. Would it be possible to obtain uncorrelated residuals by adding $\Delta x_{2,t}$ as explanatory variable in the equation for $\Delta x_{1,t}$, or $\Delta x_{1,t}$ as explanatory variable in the equation for $\Delta x_{2,t}$, or both. Motivate your answer!

A.7.2 The highest residual correlation, 0.66, in Mod(4) or Mod(5) is between the residuals from the income and the worked hours equations. Comparing the estimated coefficients from the two equations show that they contain exactly the same regressors. Thus, there are three pair of significant coefficients that can be used to check whether the corresponding variable can act as a valid instrument for the current effect. To begin with we will check whether the current effect of hours worked can be identified in the income equation by imposing a zero restriction on anyone of the regressor variables. If any of the latter is a valid instruments then the residual correlation coefficient should become insignificant. We first check whether lagged consumption can be used as a valid instrument for the current effect of hours worked using the formula:

$$\tilde{b}_{ii} = \frac{r_{ij} s_{ii} s_{jj}}{s_{jj}^2} b_{ji}$$

where b_{ij} is the j^{th} coefficient in the i^{th} equation (when the regressors are the same inneach equatiion) and s_{ii} is the residual standard error in equation i.

We then check whether $b_{ij} - \tilde{b}_{ij}$ is significantly different from zero. The effect of adding the current change in hours worked to the income equation on the lagged consumption variable is now calculated as:

$$\tilde{b}_{11} = \frac{0.66 \times 0.00719 \times 0.00346}{0.00346 \times 0.00346} \times 0.39 = 0.53$$

In a similar way we obtain $\tilde{b}_{12} = 0.89$ and $\tilde{b}_{13} = 0.10$. We can check whether $b_{1j} - \tilde{b}_{1j} \simeq 0$. We obtain $b_{11} - \tilde{b}_{11} = 0.15$ which is not significantly different from zero using the estimated standard error of 0.12, $b_{12} - \tilde{b}_{12} = -0.47$ which is significantly different from zero using the se of 0.11, and $b_{13} - \tilde{b}_{13} = 0.026$ which is not significantly different from zero using the se of 0.05. Thus, we should be able to achieve two overidentifying restrictions in the income equation with respect to current change in hours worked. Since both lagged consumption and $ecm2$ seemed to be valid instruments we would expect the residual correlation to become much smaller in this case. Mod(6) in the Appendix estimates this model where the first equation contains the current effect of hours worked and lagged consumption and the $ecm2_{t-1}$ has been set to zero. The remaining equations are identical to Mod(5). We notice that the current effect is estimated with a t-value of 6.9 and that residual correlation is now -0.19 compared to 0.66 in Mod(5).

In a similar way we can check whether the current effect of income in the hours worked equation can be identified by using lagged hours worked as an instrument. We obtain $b_{32} - \tilde{b}_{32} = 0.52$ which is significantly different from zero using the estimated standard error of 0.06. Thus, the condition for lagged hours to be a valid instrument is not satisfied. Mod(7) reports the estimates of the jointly simultaneous effects in income and hours worked equations. However the residual correlation is now -0.81 compared to 0.66 in the VAR model.

7.3 Assume that you are primarily interested in the estimated dynamic adjustment towards the long-run relations, i.e. in $\alpha\beta' x_{t-1}$. Would it be possible to re-specify the system as a partial system by conditioning on any of the variables without loosing information about this adjustment towards the long-run relations? Motivate your answer based on the results obtained in 7.1.

A.7.3 The parameters of interest are the adjustment coefficeints α. The argument for when we can estimate a partial model is based on a partitioning of the joint density into the conditional and marginal densities:

$$\begin{aligned} &D(\Delta x_t|\Delta x_{t-1}, \beta' x_{t-1}; \theta) = \\ &= D(\Delta x_{1,t}|\Delta x_{2,t}, \Delta x_{t-1}, \beta' x_{t-1}; \theta_1) \times D(\Delta x_{2,t}|\Delta x_{t-1}; \theta_2) \end{aligned} \tag{14}$$

where $x_t' = [x_{1,t}', x_{2,t}']$, $x_{2,t}$ are the conditioning variables and $x_{1,t}$ are the variables explained by the partial model θ_1, θ_2 are variation free and only θ_1 contains the parameters of interest α. The estimates of Mod(5) shows that all equations contain at least one ecm. Hence it is not possible to partition the joint model into a conditional and a marginal model, such that the conditional model only contains the parameters of interest, i.e. the coefficients to the ecm terms whereas the marginal model does not. A partial model will therefore imply a loss of information regarding the short-run adjustment towards the long-run relations.

Reference: Ireland, P. (2003): A method for taking the model to the data. *Journal of Economic Dynamics and Control*, 28, 1205-1226.