Exam number: 14501

Contents

The econometrics packages GiveWin 2.10 and CATS in Rats (CATS 2 test version) will be used throughout this report. If not otherwise stated the estimations have been conducted using CATS. All relevant output from the programs is to be found in the appendix.

Notation follows Juselius (2004): The Cointegrated VAR Model: Econometric Methodology and Macroeconomic Applications.

1. Background

Question 1

We note that relation (2) can be rewritten in the following way (adding y_t^r on both sides):

$$c_t^r = \beta_{20} + \beta_{22} w_t^r + \left(1 - \beta_{22}\right) y_t^r + v_{2,t}$$

Comparing this to relation (1) we see that the following conditions ensure that the two equations are equivalent: $\beta_{10} = \beta_{20}$, $\beta_{11} = 1 - \beta_{22}$ and $\beta_{12} = \beta_{22}$. This confirms that (2) is a restricted version of (1).

Question 2

Assuming that Δp_t is equilibrium correcting to c_t^r and w_t^r means that Δp_t will act so as to (partially) correct a past equilibrium error. We can establish the expected signs of the parameters in relation (4) by using the fact that a real variable is given by its corresponding nominal variable where the price level has been subtracted (all variables are log-transformed). Rewrite (4) as:

$$\Delta p_t - \beta_{40} - \beta_{41}\left(c_t^n - p_t\right) - \beta_{42}\left(y_t^n - p_t\right) - \beta_{43}\left(w_t^n - p_t\right) \sim I(0)$$

$$\Delta p_t = \beta_{40} + \beta_{41}\left(c_t^n - p_t\right) + \beta_{42}\left(y_t^n - p_t\right) + \beta_{43}\left(w_t^n - p_t\right) + stationary\, error$$

It is now easy to see that $\beta_{41} > 0, \beta_{42} > 0, and \beta_{43} > 0$ implies error-correcting behaviour, inflation goes down when prices have increased more than nominal consumption, nominal income, and nominal wealth, respectively.

2. Defining the model and the data

The sample period to be analysed in this report is 1973:1 to 2000:1.[1] The data vector will be referred to as $x_t' = \left\{c_t^r, y_t^r, w_t^r, \Delta p_t, R_{b,t}\right\}$.[2] We have quarterly data but we do not use seasonal dummies since the data is assumed to be seasonally adjusted already.[3]

3. Specification and estimation

The graphs of the data in levels and differences can be found in the appendix. The levels of aggregate real consumption, disposable income and private sector wealth all seem to have followed a linear trend. The long term government bond rate shows sign of a peak around 1983 followed by a level shift (a drop in the mean). Moreover, the inflation rate is quite high in the beginning of the period but seems to settle on a lower level after 1983. The drop in the bond rate is thus likely to be related to the drop in inflation, (cf. the Fisher hypothesis). It is worth noting that inflation is much more volatile than the interest rate over this period. The graphs of the first differenced series all show significant mean-reverting/stationary behaviour. The autocorrelograms show strongly autoregressive behaviours, the partial autocorrelograms have a highly significant coefficient at the first lag and no significant coefficients at longer lags and the spectral density graphs have mass concentrated at zero frequencies (these graphs are not shown). All these observations strongly suggests that data is I(1). If our data vector contained a nominal variable such as the price level we should be aware of I(2) signs. This is not the case here and we will continue our analysis based on the I(1) conclusion.

[1] The data file DKconsumption.xls has been used throughout the analysis (for the time period: 1973:01 to 2000:01).
[2] Note that the data vector has been defined differently from that given in the problem set (inflation and bond rate have changed places), since this turned out to be more convenient in the calculations.
[3] This was assumed since the option season had been set to zero in the prg-file.

Question 3

We start by estimating the VAR model with no dummies included. Then we look at the residuals and study the "economic calendar" to find out whether we should include any dummy variables. Based on the inspection of the levels and differences we estimate the model with a restricted trend[4]. The standard misspecification tests reveal that the residuals are not well behaved; e.g. the Doornik and Hansen (1994) test for multivariate normality strongly rejects the null.

Hence, we take a look at the standardized residuals and pay special attention to those exceeding 3 in absolute value.[5] According to this criterion we find large residuals in 1975:04, 1977:03, 1983:01 and 1987:01. Therefore we choose to include a modified transitory blip dummy taking account of the temporary removal of the VAT in 1975:04 and its gradual re-installation in the following two quarters. We also include a permanent blip dummy to pick up the monetary policy intervention in 1977:04.[6] Moreover we take account of the deregulation of capital movements in 1983:01 and the fiscal policy intervention in 1987:01 by including two shift dummies restricted to the co-integrating (CI) space to allow for the possibility that these events might permanently have changed the mean of the CI relations.

After estimating the model with these dummies the tests for multivariate normality and autocorrelation show well-specified residuals. Importantly, we note that the residuals do not seem to suffer from skewness (the VAR model is sensitive to this assumption). Even though some ARCH effects are present for the bond rate (as is often the case in financial time series) we should not worry "too much" about this feature as the VAR is robust to moderate ARCH.[7]

Question 4

The graphs suggested that a linear trend might be present in the three real variables. Moreover, based on simple economic theory it seems rather natural that these variables have been growing at a positive rate. We therefore specify our model such that we allow for a linear trend in the data and in the CI relations. The ECM representation of the VAR(2) model to be analysed is given as:

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \alpha\beta' x_{t-1} + \mu_0 + \mu_1 t + \Phi D_t + \varepsilon_t$$

where $\mu_0 = \alpha\beta_0 + \gamma_0$ and $\mu_1 = \alpha\beta_1 + \gamma_1$. Excluding the possibility of quadratic trends, we specify the model to have an unrestricted constant and a restricted trend and we thus have $\gamma_1 = 0$, $\beta_0 \neq 0$, $\gamma_0 \neq 0$, $\beta_1 \neq 0$ (the so-called "case 4" in Juselius (2004)). This model specification allows for trend stationary behaviour in the CI relations, i.e. we do not *a priori* assume that the trend in variables cancel in CI relations. Starting out with this specification ensures that we have similarity in the rank test procedure as pointed out by Nielsen and Rahbek (1998). We are then able to test whether the trend cancels in the CI relations afterwards.

Question 5

We estimate the model with two lags ($k = 2$), a trend restricted to the CI space (CATS: dettrend = cidrift) and the four dummies specified above in the model (the two shift dummies are restricted to the CI space to avoid that they cumulate to broken linear trends in levels). Moreover, we need to include

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[4] I.e. in CATS we use the option for dettrend called "cidrift". This specification choice is discussed in question 4.

[5] This is done using if-then-else-statements in Excel.

[6] Even though the largest residual appears in 1977:03, a rather large residual is present in 1977:04 also, and we therefore choose to specify the dummy at the time of the policy intervention (in agreement with the specification in the prg-file).

[7] The chi-squared test statistic is 12.0140 for the test of ARCH(2) effects (the critical value is 5.991).

differences of the shift dummies unrestricted implying two additional permanent dummies in the model.[8] The ECM model thus takes the specific form:

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \alpha \tilde{\beta}' \tilde{x}_{t-1} + \gamma_0 + \Phi_1 Dtr754_t + \Phi_2 Dp774_t + \Phi_3 Dp83_t + \Phi_4 Dp871_t + \varepsilon_t$$

where $\tilde{\beta}' = \{\beta', \beta_0, \beta_1, \Phi_5, \Phi_6\}$ and $\tilde{x}_t = \{x_{t-1}, 1, t, Ds83, Ds871\}'$.

The estimation results can be found in appendix.

The trace test is conducted using the top-bottom procedure (starting with the null of $p = 5$ common trends and thus $r = 0$ CI relations) since this approach yields asymptotically the most correct results. We use Table 15.4 in Johansen (1996) to obtain the approximately correct critical values for our model specification. Since we have included two (restricted) shift dummies we cannot completely rely on the critical values and we actually ought to simulate these with our exact specification. In practice, it suffices to add $\chi^2_{0.95}(1) = 3.841$ to the critical values for each shift dummy included (here: two). Since our sample consists of only 109 observations, we use the Bartlett small sample correction of the trace statistic and compare this with the "shift corrected" critical value. According to the trace test we have $p - r = 3$ common trends and $r = 2$ stationary CI relations. The Bartlett correction is likely to help us control the size of the test (the type 1 error), but the power of the test (the ability to reject a false null hypothesis) might be very low instead. That points to the importance of using additional information for determination of the rank.

Table 1: Trace test and modulus of the five largest roots of the companion form matrix

r	p-r	Eigenvalues	Trace	Trace Bartlett corrected	Trace95	Trace95 shift corrected	Modulus of five largest roots r=5	r=4	r=3	r=2	r=1
0	5	0.4782	149.05	141.33	86.96	94.64	0.9399	1	1	1	1
1	4	0.3742	79.46	75.78	62.61	70.29	0.8432	0.8816	1	1	1
2	3	0.1209	29.30	26.71	42.20	49.88	0.8432	0.7993	0.8079	1	1
3	2	0.0746	15.51	14.28	25.47	33.15	0.8004	0.7993	0.8079	0.7640	1
4	1	0.0652	7.22	6.54	12.39	20.07	0.3959	0.3918	0.4064	0.4367	0.4642

Question 6

The latter panel of Table 1 shows the modulus of the five largest roots of the companion form matrix. We see that restricting the number of unit roots to two ($r = 3$) ensures that the remaining roots are fairly small (the largest is around 0.8). Setting $r = 2$ makes the largest root only slightly smaller and this suggests that the rank should be set to three.

The graphs of the CI relations should look stationary; plots of these are available in the appendix. The first two relations looks convincingly mean-reverting, but again it is difficult to say whether the third relation could be considered stationary since it has longer swings around the line of zero.

The recursive graphs of the trace statistics are also informative; these are also shown in the appendix.[9] The statistics should grow linearly for $i = 1,...,r$ and stay constant for $i = r+1,...,p$ since they are functions of the eigenvalues.[10] The recursive trace statistics show a clear pattern in both the X-form and

[8] CATS automatically include differences of the shift dummies when the option "shift" is used.

[9] The baseline period was chosen to be 1973:01 to 1979:04. Due to the shift dummies the X-form graphs cannot be calculated before 1987:01.

[10] The eigenvalues will be different from zero for the r stationary relations since the eigenvalues can be interpreted as squared canonical correlations.

the R-form (the latter is "cleaned" for short run effects): the first two lines grow steadily over the whole period, while the remaining three stay almost constant and below the line of one. This would lead to the conclusion that only two CI relations are present.

The t-values of the α-coefficients are also informative, since they reveal whether any of variables are equilibrium correcting to some of the CI relations. One should be aware that the t-statistics are more likely to follow a Dickey-Fuller distribution for the $p-r$ non-stationary relations and we ought to be careful in interpreting the t-values. As can be seen from the unrestricted estimates c_t^r and Δp_t are significantly adjusting to the first CI relation while almost all of the variables seem to adjust to the second relation. The third relation has c_t^r and $R_{b,t}$ adjusting to it. No variables seem to be significantly adjusting to the last two CI relations (although y_t^r does seem to react to the last relation to some extent). Taken together this again suggests two or three CI relations.

Finally, we consider the economic interpretability of the relations, even though one should be careful in interpreting the relations at this early stage of the analysis. Still, the α-coefficients might give a preliminary idea of the underlying mechanisms governing the CI relations. This idea suggests that the first relation could be an inflation relation, the second a consumption-income ratio equation (homogeneity might be satisfied) or alternatively a wealth relation of some kind. Finally, the third relation is likely be a relation governing consumption or possibly the bond rate (even though the latter situation would be surprising given weak exogeneity of this variable).

The trace test and the additional sources of information do not yield completely consistent results. Nevertheless, on the basis of all this information we would conclude that the system contains two CI relations $(r=2)$ and thus three common trends (keeping in mind that there might be an extra relation).

4. Testing

We continue our analysis with $r=3$ and thus the following results are based on this conclusion. This means that we have $p-r=2$ common trends (possibly one real and one nominal trend).

Question 7

We will now test whether the trend can be excluded from the CI relations. We have $p1=8$ variables in the CI space ($p=5$ endogenous variables, a trend and two shift dummies). The test has $m=1$ restriction and $s=p1-m=7$ free parameters. The design and restriction matrices used for formulating this test take the following form (H is $p1\times s$ and R is $p1\times m$):

$$H=\begin{pmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&1&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\\0&0&0&0&0&0&0\end{pmatrix}\qquad R'=\begin{pmatrix}0&0&0&0&0&0&0&1\end{pmatrix}$$

We use CATS Mining to test for exclusion since this yields test results for all possible choice of the rank and thus enables us to see whether the test conclusion is sensitive to the rank choice. The test statistic is chi-square distributed with three degrees of freedom calculated as $\nu=r*m$. The test value is

5.0246 (p-value = 0.1700) and we accept the null that the trend can be excluded from the CI-space. This conclusion is robust to the choice of rank (but borderline for $r = 2$). This means that the trend cancel in the CI space and we can simplify our model accordingly by setting $\gamma_1 = 0$ (the so-called "case 3" in Juselius (2004)) and re-estimate the model with an unrestricted constant only (this will be done in answering question 10 and onwards).

Question 8

The tests for exclusion of each of the shift dummies are readily available from the same CATS Mining exclusion test procedure. These tests also have three degrees of freedom. We see from the results in the appendix that neither of the dummies can be excluded and that this result holds for any choice of rank. For $Ds83$ the test statistic is 17.7731 (p-value = 0.0005) and for $Ds871$ the test statistic is 9.4023 (p-value = 0.0244). It thus seems very important to keep both shift dummies in the CI space.

Question 9

We now test whether the bond rate is weakly exogenous for the long run parameters. This hypothesis corresponds to a zero fifth row in α and this means that the bond rate is not reacting to any equilibrium errors. The design and restriction matrices in this case are given as:

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad R' = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Again we use CATS Mining to test for weak exogeneity. This test has $m = 1$ restriction on each α vector and, thus, three degrees of freedom calculated as $\nu = r * m$. For our case ($r = 3$) the bond rate is a borderline case, but strictly speaking cannot be considered weakly exogenous on the 5 % level based on a test statistic of 7.8645 (p-value = 0.0489). Inspection of the test results for other rank choices show that the weak exogeneity of the bond rate is indeed very sensitive to the choice of r; the bond rate could have been considered weakly exogenous for all other rank choices. Note that if we re-estimate the model with only an unrestricted constant (as will be done for answering the next questions) we get the result that the bond rate could actually be considered weakly exogenous (p-value = 0.0611). We should therefore keep in mind that the cumulated residuals of the bond rate equation might be a candidate for a common driving trend. Note that none of the other variables in the model are weakly exogenous.[11]

For answering the next questions we re-estimate the model with an unrestricted constant since we concluded above that the trend cancels in the CI space (CATS: dettrend = drift) and again we set $r = 3$. The model is specified as before but now we have $\tilde{\beta}' = \{\beta', \beta_0, \Phi_3, \Phi_4\}$ and $\tilde{x}_t = \{x_{t-1}, 1, Ds831, Ds871\}'$. We keep the shift dummies since these are needed to obtain well specified normally distributed residuals (a crucial assumption of the VAR). The results are given in appendix.

Question 10

We estimate the consumption relation (1) using the design and restriction matrices ($s = 5$, $m = 2$):

[11] Real income and real wealth are weakly exogenous for $r = 1$.

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad R' = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

The estimation results reported in the appendix give the following relation:

$$c_t - 0.64 y_t^r - 0.34 w_t^r - 0.02 Ds83 + 0.04 Ds871 \sim I(0) \quad \Rightarrow$$

$$c_t = 0.64 y_t^r + 0.34 w_t^r + 0.02 Ds83 - 0.04 Ds871$$

The coefficients in the relation have the expected signs: $\hat{\beta}_{11} = 0.64 > 0$ and $\hat{\beta}_{12} = 0.34 > 0$ and $\hat{\beta}_{11} + \hat{\beta}_{12} = 0.98 \leq 1$ and thus consumption depends positively on both income and wealth. Since we have only put $r-1=2$ restrictions on the first CI equation it is just-identified and there is no testing involved in this case. Because the estimated vector is in the cointegration space, the only thing we need to do is to check whether the estimated coefficients make economic sense. As always it is a good idea to check whether the relation looks well-behaved by studying the graph of the restricted relation (see appendix). The relation looks fairly stationary but there seems to be problems with reversion to steady state following the deregulation of capital movements in 1983. All in all, interpreting this relation as a plausible consumption function does makes sense.

Looking at the α -coefficients we again see that the bond rate seems to be adjusting to the relation (even though the coefficient is small in magnitude); this speaks, however, against the hypothesis that this variable is be weakly exogenous. Still, if the bond rate is highly correlated with another variable in the model, it could affected implicitly through that other variables response to the CI relations.[12] Real aggregate consumption is also equilibrium correcting to deviations from the long run consumption relation since it has a negative coefficient of -0.1487 (only borderline significant). Still, we might have found a candidate for a CI relation here.

Question 11

We now test whether the restricted version of the consumption relation (2) is stationary. For this purpose we use the design or restriction matrices ($s=4$ and $m=3$):

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad R' = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

This test has $s=4$ free parameters and $m=3$ restrictions (we now have $p1=7$ since the trend has been excluded). The restrictions are over-identifying and the degrees of freedom in the chi-squared distributed LR-test are calculated as $\nu = m-(r-1)$. Hence, we have one degree of freedom in this case.

[12] We return to this issue in section 6 on short run identification.

The hypothesis is accepted with a test statistic of 0.0516. The estimation results yield the following relation:

$c_t^r - 0.67y_t^r - 0.33w_t^r - 0.02Ds83 + 0.04Ds871 \sim I(0) \Rightarrow$ (adding and subtracting y_t^r)

$c_t^r - y_t^r = 0.33\left(w_t^r - y_t^r\right) + 0.02Ds83 - 0.04Ds871$

The coefficient can be identified as $\hat{\beta}_{22} = 0.33$ and thus the relations between the coefficients in equation (1) and (2) established in question 1, is seen to hold almost exactly: $\hat{\beta}_{12} \approx \hat{\beta}_{22}$ and $\hat{\beta}_{11} \approx 1 - \hat{\beta}_{22}$. As wealth increases relative to income, consumption also rises (in real terms), but less than one-for-one. Consumption and the bond rate are significantly adjusting to this restricted consumption relation (the former only borderline). In addition, consumption is equilibrium correcting to the equation due to its negative α -coefficient of -0.1419.

We now turn to estimation of (3) which is a relation between the consumption/income ratio and the real rate of interest defined by the Fisher hypothesis as the (nominal) bond rate net of inflation. The design and restriction matrices for this test are given below ($s = 4$ and $m = 3$):

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad R' = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

This test also has one degree of freedom and the test is again clearly accepted (with a test statistic approximately equal to zero!). The estimation results give the following equation:

$c_t^r - y_t^r + 2.04\left(R_{b,t} - \Delta p_t\right) - 0.03Ds83 + 0.07Ds871 \sim I(0) \Rightarrow$

$c_t^r - y_t^r = -2.04\left(R_{b,t} - \Delta p_t\right) + 0.03Ds83 - 0.07Ds871$

The coefficient of the real rate is estimated as $\hat{\beta}_{31} = -2.04$. This means that consumption is reduced when the real rate of interest increases. It indeed seems very likely that a higher real rate *ceteris paribus*, makes it more attractive to invest in e.g. government bonds and thus wealth goes up and consumption decreases. Inspection of the α -coefficients reveals that (in particular) wealth and inflation react to this relation – both with highly significant coefficients (income and bond rate also react to some extent). All variables (except for wealth) are error-correcting to this relation. Comparing the results of estimating (2) and (3) in terms of economic plausibility, (3) seems more likely to be a component in the long run structure (given that the two are mutually exclusive).[13]

Question 12

We now test whether the dynamic adjustment equation for inflation can be considered stationary. We specify the design and the restriction matrices as ($s = 6$ and $m = 1$):

[13] The fact that the bond rate seems to react to relation (2) again contradicts our weak exogeneity result and this also points to the use of relation (3) instead of (2) in the long run structure.

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad R' = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Since $m < r-1$ the equation is not identified with the single zero restriction.[14] To be able to test the hypothesis we can, nevertheless, estimate the unidentified relation without testing and check whether the coefficients are plausible. In this case there would be no testing involved. We can also choose to impose an additional just identifying restriction, for example, a zero restriction on the 1987 shift dummy, or we can impose additional overidentifying (and therefore testable) restrictions. In this case we have chosen to put zero restrictions on the two shift dummies.[15] This leaves us with one degree of freedom and a test statistic of 2.9643 (p-value = 0.0851) and we accept the null. The estimates lead to the following relation:

$$-5.02c_t^r + 2.03y_t^r + 2.48w_t^r + \Delta p_t \sim I(0) \Rightarrow$$
$$\Delta p_t = 5.02c_t^r - 2.03y_t^r - 2.48w_t^r$$

The signs predicted in question 2 hold only for $\hat{\beta}_{41} = 5.02 > 0$. The fact that $\hat{\beta}_{42} = -2.03 < 0$ and $\hat{\beta}_{43} = -2.48 < 0$ means that inflation is not equilibrium error correcting to income and wealth. However, the results could be sensitive to the fact that we have restricted both shift dummies to zero. The α-coefficients reveal that only Δp_t is significantly error correcting to this relation (as expected).

An irreducible CI relation looses the co-integrating property if just one variable is dropped from the equation, see Davidson (2001). To find out whether the inflation adjustment relation is irreducible, we put zero restrictions on c_t^r, y_t^r and w_t^r in turn. The result of this experiment (each test has $m = 4$ and thus two degrees of freedom) is that y_t^r and w_t^r can be left out (test statistics are 5.3458 and 5.8824, respectively), while c_t^r cannot be excluded as this renders the relation non-stationary (with a test value of 7.3633 and a p-value = 0.0252). This relation is therefore irreducible and it could be a "building block" for the long run structure. Note, however, that the result is likely to depend on whether the exclusion of both the shift dummies was justified.

Question 13

We now turn to identification and estimation of a complete long run structure. We put the results from the previous testing of individual equations together and test whether the relations (1), (3) and (4) can be used as the starting point for the identification process.

First, we estimate a just-identified system. Relation (1) was a just-identified equation, so we leave it as it is. Relations (3) and (4) (the latter with zero restrictions on the shift dummies) were over-identified so we need to impose fewer restrictions on these two. With respect to (3) we choose to include w_t^r also and concerning (4) we restrict only the shift dummy $Ds871_t$ to zero (since the exclusion tests showed that this was more likely to be superfluous). The result of this estimation is available in the appendix. The

[14] The order condition for identification says that we must impose at least $m \geq r-1$ restrictions on each equation to obtain identification (this is however just a necessary and not a sufficient condition for formal identification).

[15] The question has been interpreted to require that a formal test should be carried out. Estimation of the (unidentified) relation was also done (incl. the shift dummies) and the graphs of the CI relation looked "fairly stationary".

graphs of all relations look stationary (not reported in the appendix). The estimated result shows that *Ds83* is not very significant in the first relation, that wealth is not significant in the second relation, and that wealth and the 1983 shift dummy are insignificant in the third relation. Thus, imposing just-identifying restrictions can be a useful way of checking whether previously estimated relations can be simplified or improved.

We can also estimate an over-identified system consisting of the relations that were estimated in questions 10 to 12. The results of this joint estimation are available in the appendix. The number of degrees of freedom in this test is given by

$$\nu = \sum_{i=1}^{r} \left[m_i - (r-1) \right]$$

Here we have $m_1 = 2$, $m_2 = m_3 = 3$ and thus $\nu = 2$. This system is accepted with a test statistic of 5.2091 (p-value = 0.0739).

This ends the identification of the long run structure and from now on we can treat the $\hat{\beta}$'s as fixed (predetermined) stationary variables as will be done during identification of the short run structure.

Finally, we discuss different types of identification. Identification can exist on three levels. Generic identification refers to a situation where the formal rank conditions are satisfied.[16] Empirical identification requires that insignificant estimates can be set to zero without violating the formal rank conditions. Finally, economic identifications refers to a situation where the restricted model satisfies the first two types of identification and in addition can be given a relevant economic interpretation along the lines of economic theory.

To find out whether the estimates are empirically identified we calculate t-statistics of the β-estimates; results are reported in appendix. From these we see that (almost) all coefficients are highly significant and thus the structure indeed seems to be empirically identified.[17]

With respect to economic identification the qualitative features of relation (1) and (3) are not changed by the joint of all three relations, but the signs of the coefficients of the last relations have been reversed.

Finally, recursive estimation is done to check the over-identified model for parameter non-constancy (the VAR is sensitive to this assumption). The appendix reports a recursive graph for beta constancy.[18] The scaled test statistic stays below the line of one for almost the whole sample period and we thus conclude that the structure is acceptable for the whole sample period.

5. The moving average representation

The moving average (MA) representation of the VAR model is given by:

$$x_t = \tilde{\beta}_\perp \alpha_\perp' \sum_{i=1}^{t} \varepsilon_i + \text{ stationary and deterministic components}$$

Note that $\alpha_\perp' \sum_{i=1}^{t} \varepsilon_i$ can be interpreted as the $p-r$ common stochastic trends in the system while $\tilde{\beta}_\perp$ constitute the loadings. A perfect duality is seen to exist between the pulling forces, $\beta' x_t$ (with loadings α) of the AR representation and the pushing forces of the MA representation.

Table 2: The unrestricted MA representation[19]

[16] The rank condition for identification is given in Juselius (2004) section 12.2. CATS automatically checks whether the rank conditions are satisfied.

[17] If some of the estimated coefficients were insignificant, we could experience lack of empirical identification provided that setting these coefficients to zero would result in violation of the rank condition.

[18] The baseline period is 1973:03 to 1981:01.

[19] t-values in brackets. The number of decimals has been reduced to ease the row operations.

	$\tilde{\beta}_{\perp,1}$	$\tilde{\beta}_{\perp,2}$		ε_c	ε_y	ε_w	$\varepsilon_{\Delta p}$	ε_{R_b}
c_t^r	-0.70	-0.62	$\alpha'_{\perp,1}$	0.10	0.03	-0.15	-0.11	0.98
y_t^r	1.02	-0.70	$\alpha'_{\perp,2}$	-0.14	-0.68	-0.52	0.49	0.01
w_t^r	-3.99	-0.51						
Δp_t	-0.19	0.49						
$R_{b,t}$	0.65	0.01						

C-matrix	ε_c	ε_y	ε_w	$\varepsilon_{\Delta p}$	ε_{R_b}	Trend
c_t^r	0.01 (0.08)	0.40 (4.33)	0.43 (2.56)	-0.2258 (-0.87)	-0.69 (-0.50)	0.00
y_t^r	0.20 (0.99)	0.51 (4.36)	0.22 (1.03)	-0.46 (-1.39)	0.99 (0.56)	0.00
w_t^r	-0.33 (-1.71)	0.23 (2.00)	0.85 (4.16)	0.19 (0.61)	-3.90 (-2.26)	0.00
Δp_t	-0.02 (-1.83)	-0.23 (-3.48)	0.01 (0.95)	0.04 (1.71)	-0.19 (-1.66)	-0.00
$R_{b,t}$	0.07 (2.27)	0.03 (1.55)	-0.09 (-2.90)	-1.60 (-1.60)	2.44 (2.44)	-0.00

Question 14

We start out estimating the (unrestricted) MA representation of the model. The results are shown in Table 2 as well as in the appendix.[20] The columns in C show how the residuals load into each variable, while the rows in C provide the weights to these residuals. Since we can impose $p-r-1=1$ restriction on each vector without changing the likelihood function, $\tilde{\beta}_\perp$ and $\alpha_\perp$ are not generically identified and only the C-matrix has standard errors. Still, we can use linear row manipulations to impose just-identifying restrictions on $\alpha_\perp$, since these will not change $C=\tilde{\beta}_\perp\alpha_\perp{}'$. We choose to normalize on the bond rate and the real wealth variable and after doing a few row manipulations we obtain the identified version of the $\alpha_\perp$-matrix shown in Table 3. Note that we now have a one in each row and a corresponding zero in the other row.

A unit column in $\alpha_\perp$ corresponds to a zero row in α and thus this means that the variable in question is weakly exogenous. A test of a unit vector in $\alpha_\perp$ is therefore just a test for weak exogeneity. From the definition of the common trends in the MA representation, $\alpha_\perp{}'\sum_{i=1}^{t}\varepsilon_i$, we see that a unit vector in $\alpha_\perp$ means that the cumulated residuals of the equation for that variable constitute a common driving trend. Note that zero restrictions on $\alpha_\perp$ based on weak exogeneity imply that part of C and $\tilde{\beta}_\perp$ becomes identical. This makes sense since one of the vectors in $\tilde{\beta}_\perp$ then represents loadings to the shocks to the weakly exogenous variable. And by definition, the column of C corresponding to the variable in question gives the long run impact of shocks to exactly that variable.

Table 3: The $\alpha_\perp$-matrix identified by linear row manipulations

	ε_c	ε_y	ε_w	$\varepsilon_{\Delta p}$	ε_{R_b}
$\alpha'_{\perp,1}$	0.14	0.23	0	-0.24	1
$\alpha'_{\perp,2}$	0.27	1.31	1	-0.94	0

[20] The β-matrix has been normalized on c_t^r in the first two relations and on Δp_t in the third relation.

Question 15

The coefficients in the C-matrix show the way each of the variables are affected by shocks to the different variables in the system. The diagonal elements of C thus show the effect of shocks to the variables themselves while the off-diagonal elements give the effects from all other variables. To see whether a consumption shock has a significant influence on inflation in the long run, we need to find the effect of ε_c on Δp_t: the coefficient is -0.02 with a t-value of -1.83. This means that a consumption shock is likely to cause inflation to decrease (not a significant effect, however). This stands in contrast to what standard economic theory would predict as we would expect a consumption boom to drive inflation above normal.

Question 16

Assuming that the bond rate is weakly exogenous for the long run parameters means that the bond rate does not react on the deviations from the long-run relations. If, in addition, all row elements corresponding to the bond rate in the short run matrix, Γ_1, are zero, we will obtain a unit row vector in C. This means that the variable itself is a common trend (not just its cumulated residuals as in question 14). Since in this case there are neither long run nor short run effects from other variables in the system, we call such a variable strongly exogenous. As can be seen from the short run matrix reported in the appendix the bond rate is not likely to be strongly exogenous in our case, since significant effects from lagged consumption are present.

Question 17

If all estimated coefficients in a column in C are zero this implies that a column in α is proportional to a unit vector. This means that the variable in question is exclusively adjusting to one CI relation while the other variables are exclusively adjusting to the remaining CI relations. Moreover, we would have a zero row in $\alpha_{\perp}$ and thus there would be no permanent (only transitory) effects of the empirical shocks of the variable on the other variables in the system. Therefore, testing the joint insignificance of the coefficients in the column of C is equivalent of testing whether the variable associated with the these coefficients has a unit vector in α.

6. The identification of the short-run structure[21]

We start by estimating a parsimonious short run adjustment structure with the three over-identified CI relations fixed at their estimated values. This approach can be justified by the fact that $\hat{\beta}$ is super-consistent.[22] Up to this point we have been considering the reduced form VAR. However, this model does not take account of simultaneous effects between the system variables and potentially relevant current effects are left in the residuals. We should therefore check the off-diagonal elements of the residual covariance matrix to find out whether such effects ought to be modelled explicitly. The structural form representation of the VAR is obtained by pre-multiplying by a non-singular $p \times p$-matrix A_0, see Juselius (2004):

$$A_0\Delta x_t = A_0\Gamma_1\Delta x_{t-1} + A_0\alpha\tilde{\beta}^{c\prime}\tilde{x}_{t-1} + A_0\Phi D_t + A_0\gamma_0 + A_0\varepsilon_t$$

where all variables have defined previously (except D_t which includes the three permanent and the one transitory dummy). Note that $\tilde{\beta}$ is now fixed at its estimated value $\tilde{\beta}^c$. To start with, we disregard the

[21] All estimations in this section are done using GiveWin.
[22] In addition, we normalize the CI relations such that they become mean zero. See the appendix for details on code.

possibility of simultaneous effects and thus consider the case where $A_0 = I$ which means that the model is exactly identified by the $p(p-1) = 20$ zero restrictions on A_0.

The result of estimating the unrestricted ECM model is shown in the appendix. We have included an unrestricted constant to pick up the non-zero mean present in the model.[23] Moreover we should remember to include differences of the two shift dummies. Note that the residual correlation seems to be rather moderate, i.e. the largest correlation (0.30730) is found between Δc_t^r and Δw_t^r.

The F-tests on retained regressors provide information on whether any variables are insignificant in all equations and in this case, they can probably be excluded. We see that Δw_{t-1}^r and Δy_{t-1}^r can be excluded[24] and the formal joint test of this is clearly accepted (the F-test statistic is 0.4743).

Next we turn to the individual equations: careful examination of which variables are insignificant in each equation leads us to reduce the model stepwise and finally obtain a system with few and highly significant coefficients. The result of this procedure is seen in appendix. All in all, it was possible to impose 28 zero restrictions (in addition to the exclusion of Δw_{t-1}^r and Δy_{t-1}^r) and these were accepted with a test statistic of 31.999 (p-value = 0.2745).[25] All coefficients are significant on a 5 percent level[26].

Question 18

We now re-estimate the system as a partial system by conditioning on $R_{b,t}$ and w_t^r. The bond rate was found to be weakly exogenous for the long run parameters (for exogeneity to hold for the short run parameters also, we would need strong exogeneity). In the partial system only the equations for c_t^r, y_t^r and Δp_t are modelled, but we now allow for some of the simultaneous effects that appears to be important from the correlation matrix. The starting point should be the unrestricted system in which we would need to test whether some of the variables could be omitted altogether and then search for a parsimonious model. A 'trial and error" approach led to the results reported in the appendix (the structure was accepted with a p-value of 0.0857). First of all, we see that residual correlation has only improved a little. Second, it is worth noting that some of the variables previously present have now disappeared from the system, i.e. $ecm1_{t-1}$ and $R_{b,t}$ have been tested out. On the other hand we have very significant current effects from w_t^r on both c_t^r and Δp_t and from Δp_t on c_t^r. From the estimation of the parsimonious structure we saw that the w_t^r - and $R_{b,t}$ -residuals were highly negatively correlated. Taken together this suggests that wealth picks up some of the same effects that the bond rate. When current effects of wealth are allowed the bond rate is therefore no longer needed.[27]

Conditioning on $R_{b,t}$ and w_t^r could be rather dangerous since none of the two variables are strongly exogenous. We are likely to loose information on the feed-back effects from bond rate and real wealth to consumption, income and inflation. On the other hand, conditioning on weakly exogenous variables might improve the estimation results and make them more robust, especially if the conditioning variables pick up the effects that the dummies were supposed to take care of (note that the differences of the shift dummies are tested out in the partial system). Given the vague results on weak exogeneity

[23] The non-zero mean in due to the dettrend = drift specification in CATS.

[24] It also appears that $ecm3_{t-1}$ could be excluded but we would be reluctant to do this to start with.

[25] Even though the (unrestricted) constant is not significantly different from zero in all equations, we choose to leave it in the model to allow for a non-zero mean.

[26] Except the constant and $ecm3_{t-1}$ in the equation for y_t^r which is only significant at a 10 percent level.

[27] Cf. the discussion of the rather "puzzling" result in question 10 that the supposedly weakly exogenous bond rate seemed to be equilibrium error reacting.

we should probably refrain from relying on the partial system and stay with the parsimonious structure as the system seems to contain only weak instruments.

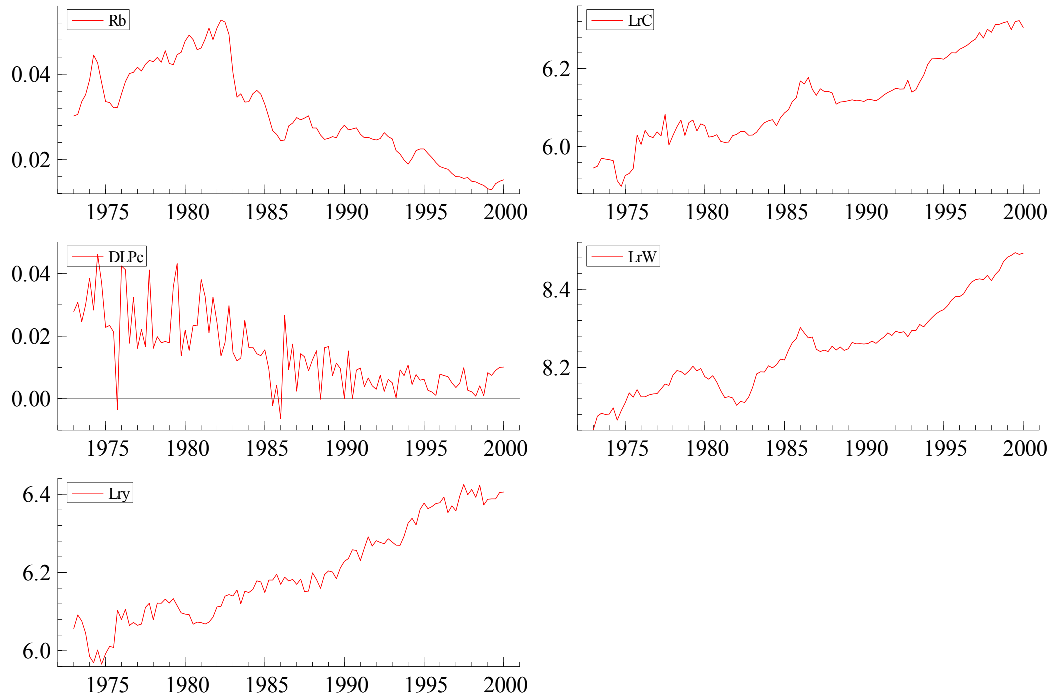
Question 19

In the single equation model there is just one cointegration relation, hence r=1 by assumption, and just equation which is adjusting, hence α is proportional to a unit vector. Therefore, the single equation model is equivalent to the multivariate model if $r=1$ and α is proportional to a unit vector with the significant coefficient corresponding to the consumption equation.

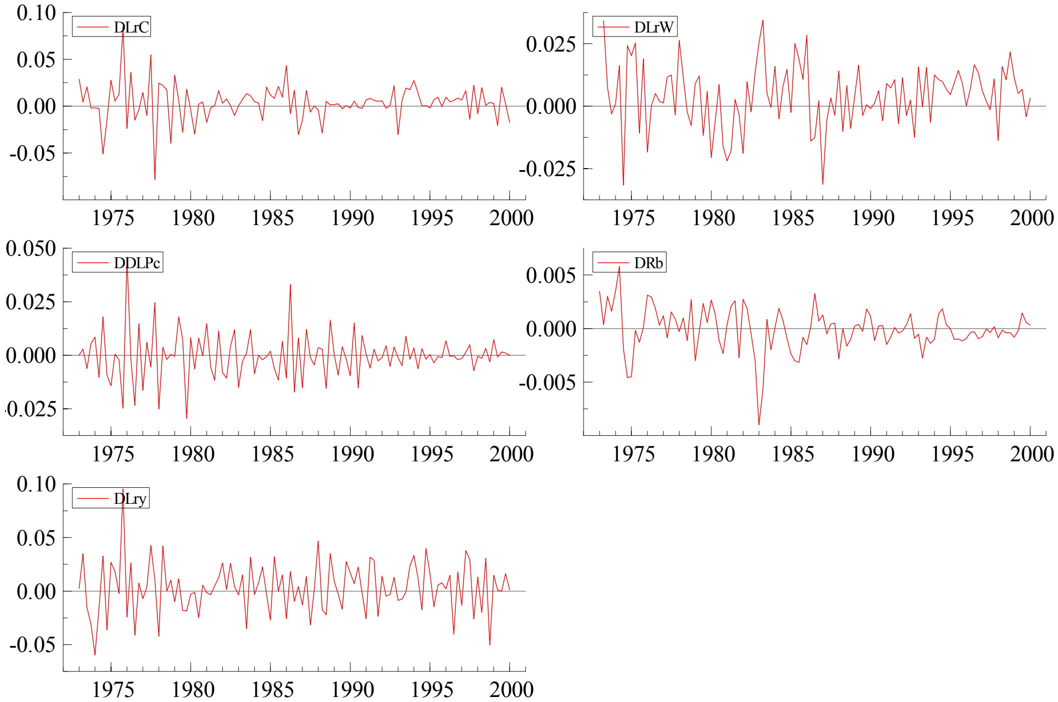
Appendix

Section 3: Specification and estimation

Graphs of data in levels



Graphs of data in first differences



Question 5 and 6

The unrestricted VAR model with a restricted trend

The unrestricted estimates:

BETA(transposed)

	LRC	LRY	LRW	DLPC	RB	DS83	DS871	TREND
Beta1	4.2955	3.2669	4.5356	204.3947	89.8652	2.6387	1.4929	-0.0158
Beta2	-8.3094	-26.3377	22.0127	-27.5955	143.1559	-0.0524	1.3491	0.0704
Beta3	55.6513	-27.2231	-35.3496	-16.4731	-125.9880	-2.2930	0.4669	0.0222
Beta4	23.4263	11.0489	-5.1789	-23.3107	151.4780	3.1066	1.1895	-0.1020
Beta5	2.6800	-13.1985	-32.9717	3.7221	-172.1680	-1.4556	-3.2854	0.1846

ALPHA

	Alpha1	Alpha2	Alpha3	Alpha4	Alpha5
DLRC	-0.0049	-0.0031	-0.0026	-0.0026	0.0013
	(-3.5876)	(-2.2686)	(-1.9135)	(-1.8803)	(0.9837)
DLRY	-0.0026	0.0070	0.0019	-0.0018	0.0036
	(-1.4700)	(3.8751)	(1.0358)	(-1.0219)	(2.0252)
DLRW	0.0003	-0.0046	-0.0002	0.0007	0.0017
	(0.3304)	(-5.2266)	(-0.2590)	(0.8238)	(1.9101)
DDLPC	-0.0042	0.0014	0.0008	0.0007	-0.0006
	(-7.0628)	(2.3772)	(1.3523)	(1.1708)	(-0.9455)
DRB	0.0001	-0.0003	0.0003	-0.0002	-0.0002
	(1.0158)	(-2.5560)	(2.2303)	(-1.7981)	(-1.1416)

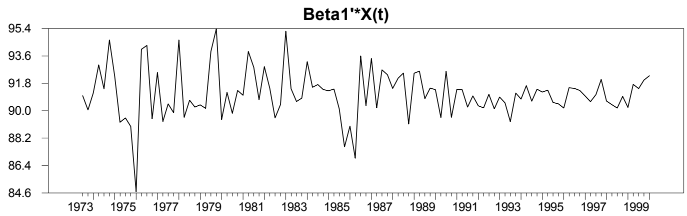
PI

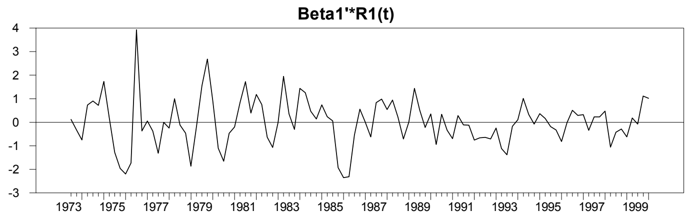
	LRC	LRY	LRW	DLPC	RB	DS83	DS871	TREND
DLRC	-0.1969	0.0904	-0.0290	-0.8064	-1.1725	-0.0167	-0.0201	0.0003
	(-2.3620)	(1.5901)	(-0.3970)	(-2.8412)	(-2.7617)	(-2.4975)	(-3.6408)	(1.0168)
DLRY	0.0011	-0.3116	-0.0350	-0.7076	-0.3804	-0.0226	-0.0078	0.0014
	(0.0100)	(-4.1472)	(-0.3633)	(-1.8877)	(-0.6784)	(-2.5682)	(-1.0723)	(3.5591)
DLRW	0.0487	0.1150	-0.1523	0.1809	-0.7895	0.0013	-0.0106	-0.0001
	(0.8969)	(3.1073)	(-3.2047)	(0.9793)	(-2.8571)	(0.3100)	(-2.9501)	(-0.4940)
DDLPC	0.0294	-0.0571	-0.0014	-0.9188	-0.0733	-0.0099	-0.0013	0.0000
	(0.8185)	(-2.3262)	(-0.0447)	(-7.5021)	(-0.4001)	(-3.4397)	(-0.5437)	(0.0680)
DRB	0.0141	0.0007	-0.0113	0.0377	-0.0852	-0.0008	0.0001	-0.0000
	(1.7093)	(0.1227)	(-1.5598)	(1.3402)	(-2.0271)	(-1.2707)	(0.1777)	(-0.7755)

Log-Likelihood = 2663.1400

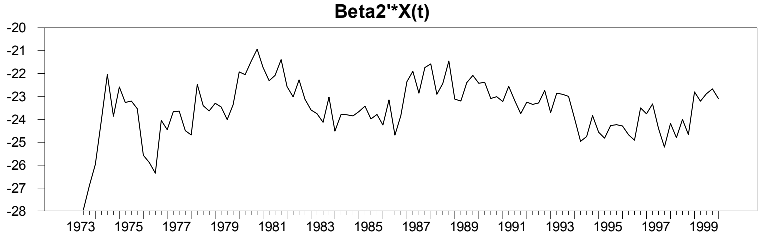
Question 6

Graphs of the CI relations

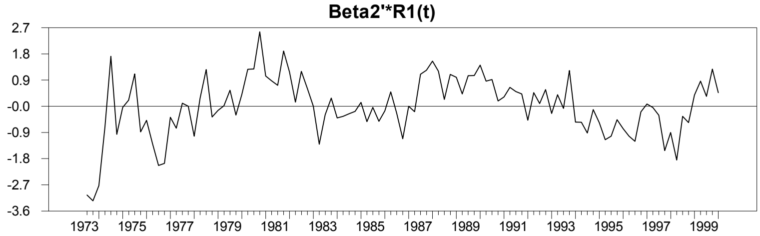




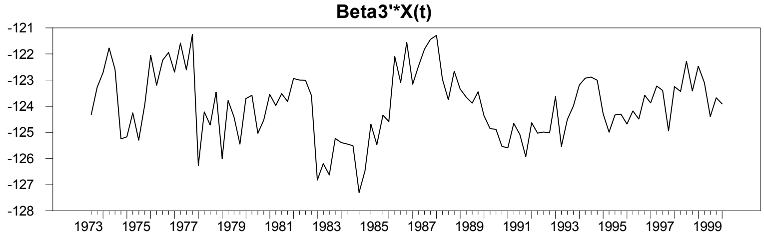
Beta2'*X(t)



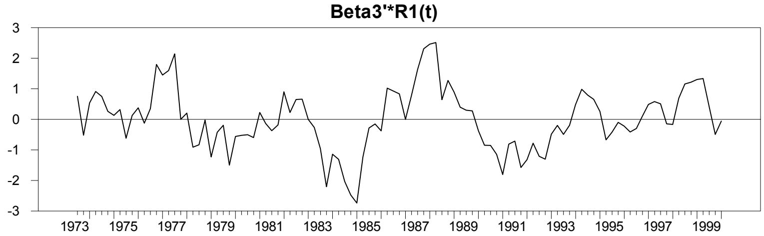
Beta2'*R1(t)



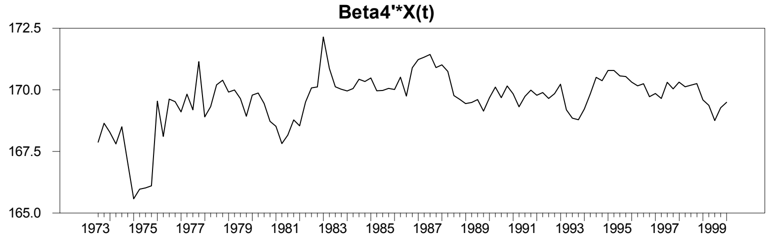
Beta3'*X(t)



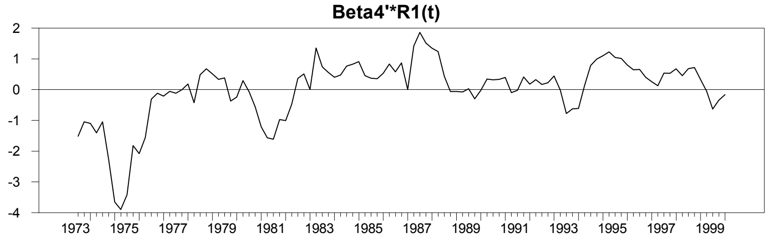
Beta3'*R1(t)

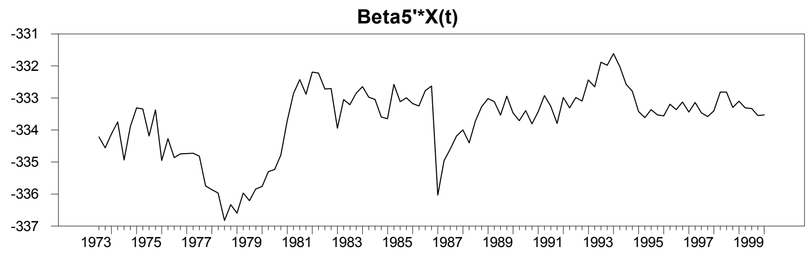


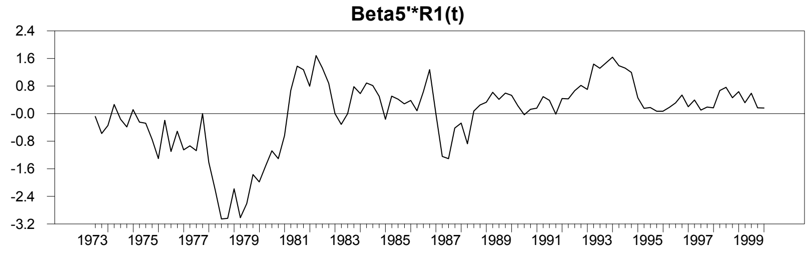
Beta4'*X(t)



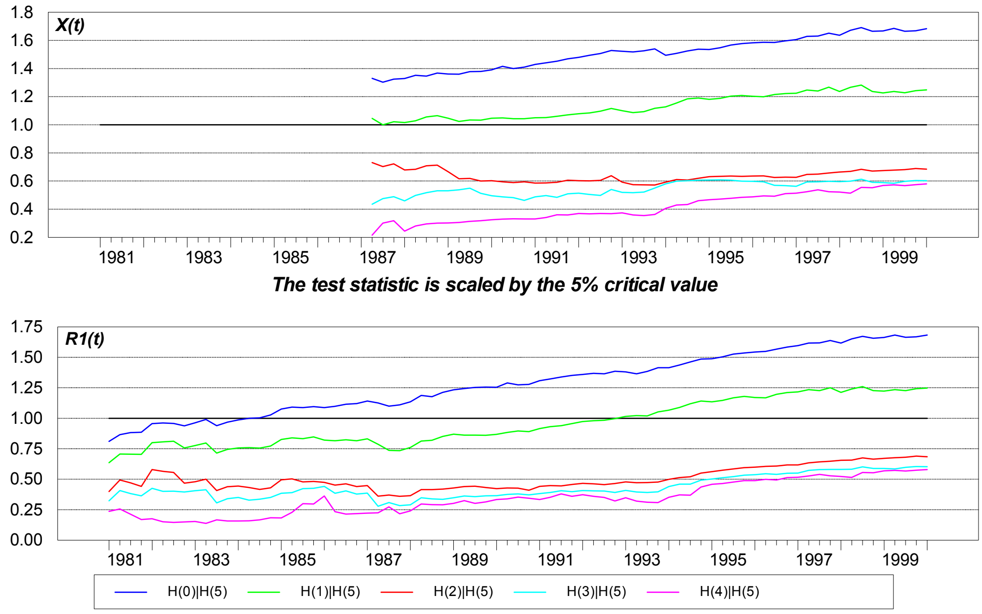
Beta4'*R1(t)







Recursively calculated trace statistics



WARNING: THE CRITICAL VALUES ARE INVALID!

Section 4: Testing

Question 7 and 8

TEST FOR EXCLUSION: LR-test, Chi-Square(r)

r	DGF	ChiSq5	LRC	LRY	LRW	DLPC	RB	DS83	DS871	TREND
1	1	3.8415	0.3229	0.2157	0.3159	19.1177	3.7865	16.2005	5.0182	0.2979
			(0.5698)	(0.6423)	(0.5741)	(0.0000)	(0.0517)	(0.0001)	(0.0251)	(0.5852)
2	2	5.9915	1.0885	15.6641	6.4614	55.1449	15.6011	16.2245	9.3244	4.9560
			(0.5803)	(0.0004)	(0.0395)	(0.0000)	(0.0004)	(0.0003)	(0.0094)	(0.0839)
3	3	7.8147	5.7275	19.9098	9.5802	59.9997	17.0900	17.7731	9.4023	5.0246
			(0.1256)	(0.0002)	(0.0225)	(0.0000)	(0.0007)	(0.0005)	(0.0244)	(0.1700)
4	4	9.4877	6.7955	20.4625	9.6094	61.0645	17.5774	18.6514	9.5130	5.2749
			(0.1471)	(0.0004)	(0.0475)	(0.0000)	(0.0015)	(0.0009)	(0.0495)	(0.2602)

Question 9

```
TEST FOR WEAK EXOGENEITY: LR-Test, Chi-Square(r)
 r  DGF  ChiSq5    LRC       LRY       LRW       DLPC       RB
 1   1   3.8415   7.5033    1.0620    0.0414   16.1524    0.6462
                 (0.0062)  (0.3028)  (0.8387)  (0.0001)  (0.4215)
 2   2   5.9915  13.4829   13.0587   20.4247   37.3937    5.5966
                 (0.0012)  (0.0015)  (0.0000)  (0.0000)  (0.0609)
 3   3   7.8147  15.3682   13.7405   20.4871   39.5280    7.8645
                 (0.0015)  (0.0033)  (0.0001)  (0.0000)  (0.0489)
 4   4   9.4877  16.2033   13.9409   20.6403   40.1767    8.6251
                 (0.0028)  (0.0075)  (0.0004)  (0.0000)  (0.0712)
```

The unrestricted VAR model with an unrestricted constant

```
THE MATRICES BASED ON 3 COINTEGRATING VECTORS:

BETA(transposed)
        LRC     LRY    LRW    DLPC     RB    DS83   DS871
Beta1  0.0191  0.0031 0.0207  1.0000 0.4327 0.0121  0.0068
Beta2 -0.2814 -0.6238 1.0000 -1.2162 6.4191 0.0291  0.0784
Beta3 -0.5520  0.2510 0.3168  0.1786 1.0000 0.0192 -0.0085

ALPHA
       Alpha1    Alpha2    Alpha3
DLRC   -1.0055   -0.1032    0.2916
      (-3.6034) (-2.4093)  (2.0793)
DLRY   -0.4036    0.1719   -0.1521
      (-1.0506)  (2.9155) (-0.7877)
DLRW    0.0104   -0.1472    0.0369
       (0.0570) (-5.2649)  (0.4036)
DDLPC  -0.8269    0.0536   -0.0906
      (-6.9341)  (2.9272) (-1.5119)
DRB     0.0235   -0.0106   -0.0301
       (0.8394) (-2.4779) (-2.1444)

PI
          LRC       LRY       LRW      DLPC        RB       DS83      DS871
DLRC   -0.1511    0.1344   -0.0316   -0.8279   -0.8058   -0.0096   -0.0174
      (-1.9242)  (3.0409) (-0.5103) (-2.9054) (-2.4314) (-2.1309) (-4.3071)
DLRY    0.0279   -0.1467    0.1154   -0.6398    0.7767   -0.0028    0.0120
       (0.2581) (-2.4104)  (1.3517) (-1.6309)  (1.7024) (-0.4526)  (2.1700)
DLRW    0.0212    0.1011   -0.1352    0.1960   -0.9032   -0.0034   -0.0118
       (0.4138)  (3.5050) (-3.3431)  (1.0537) (-4.1760) (-1.1755) (-4.4780)
DDLPC   0.0192   -0.0588    0.0077   -0.9083   -0.1045   -0.0102   -0.0006
       (0.5719) (-3.1108)  (0.2917) (-7.4579) (-0.7378) (-5.3039) (-0.3577)
DRB     0.0201   -0.0009   -0.0197    0.0310   -0.0882   -0.0006   -0.0004
       (2.5510) (-0.1933) (-3.1710)  (1.0864) (-2.6573) (-1.3403) (-1.0370)

Max.LogLikelihood    2652.87240
```

Question 10

Estimation of relation (1)

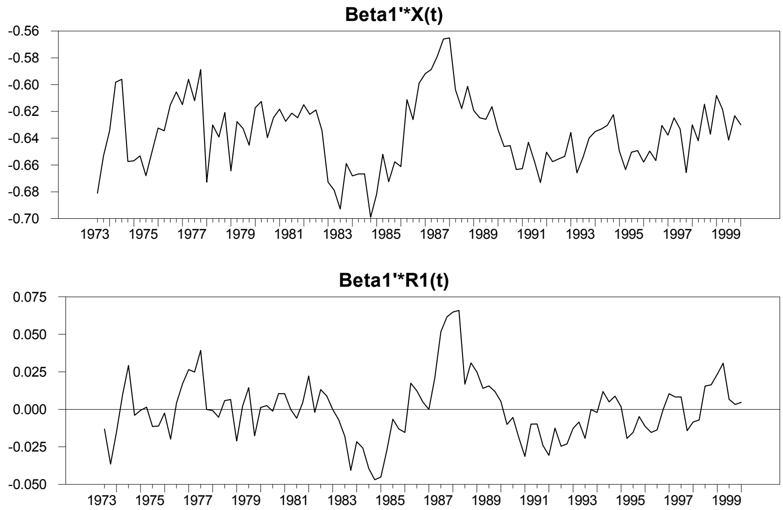
```
THE MATRICES BASED ON 3 COINTEGRATING VECTORS:

BETA(transposed)
        LRC     LRY     LRW    DLPC      RB      DS83   DS871
Beta1  1.0000 -0.6386 -0.3395  0.0000  0.0000 -0.0215 0.0407
Beta2  1.0104  1.0000  1.0904 74.7252 27.8961  0.8885 0.4262
Beta3 -0.1324 -0.7344  1.0000 -0.5103  6.9568  0.0360 0.0920

ALPHA
       Alpha1    Alpha2    Alpha3
DLRC   -0.1487   -0.0116   -0.0695
      (-1.9627) (-3.0398) (-1.5398)
DLRY    0.0544   -0.0076    0.1421
       (0.5217) (-1.4508)  (2.2868)
DLRW    0.0014    0.0017   -0.1366
```

```
       (0.0285)  (0.6811) (-4.6377)
DDLPC   0.0356   -0.0119    0.0328
       (1.0998) (-7.3444)  (1.7015)
DRB     0.0179    0.0003   -0.0140
       (2.3590)  (0.8395) (-3.0882)
```

Graphs of the estimated just-identified relation (1)



Question 11

Estimation of relation (2)

```
LR-TEST FOR RESTRICTED MODEL: CHISQR(0.0516,1) =  0.8203
*** No Bartlett Correction for this test

THE MATRICES BASED ON 3 COINTEGRATING VECTORS:

BETA(transposed)
        LRC      LRY      LRW     DLPC      RB      DS83   DS871
Beta1  1.0000 -0.6686 -0.3314  0.0000  0.0000 -0.0210 0.0449
Beta2  1.0097  1.0000  1.0305 73.2324 27.1379  0.8694 0.4145
Beta3 -0.1536 -0.7204  1.0000 -0.4765  6.9028  0.0368 0.0915

ALPHA
       Alpha1    Alpha2    Alpha3
DLRC   -0.1419   -0.0118   -0.0692
      (-1.9210) (-3.0349) (-1.5059)
DLRY    0.0590   -0.0078    0.1412
       (0.5809) (-1.4659)  (2.2356)
DLRW   -0.0027    0.0018   -0.1368
      (-0.0571)  (0.7080) (-4.5708)
DDLPC   0.0343   -0.0122    0.0323
       (1.0886) (-7.3444)  (1.6486)
DRB     0.0177    0.0003   -0.0144
       (2.4006)  (0.8482) (-3.1321)
```

Estimation of relation (3)

```
LR-TEST FOR RESTRICTED MODEL: CHISQR(0.0000,1) =  0.9963
*** No Bartlett Correction for this test

THE MATRICES BASED ON 3 COINTEGRATING VECTORS:

BETA(transposed)
        LRC      LRY      LRW     DLPC      RB      DS83    DS871
Beta1  1.0000 -1.0000  0.0000 -2.0415  2.0415 -0.0309  0.0694
Beta2 -1.1408  1.0000 -0.3019 -8.0072 -6.9551 -0.0948 -0.1431
Beta3 -1.9518  1.0150  1.0000  1.3595  2.8153  0.0713 -0.0401
```

```
ALPHA
       Alpha1    Alpha2    Alpha3
DLRC   -0.0234    0.1096    0.0014
      (-0.3167)  (3.9651)  (0.0233)
DLRY    0.2966    0.0252    0.1230
       (2.9200)  (0.6609)  (1.4548)
DLRW   -0.2248    0.0104   -0.1321
      (-4.6680)  (0.5771) (-3.2965)
DDLPC   0.1663    0.0763    0.0308
       (5.2719)  (6.4560)  (1.1719)
DRB    -0.0215   -0.0018   -0.0202
      (-2.9103) (-0.6589) (-3.2894)
```

Question 12

Estimation of relation (4) with zero restrictions on the shift dummies

```
LR-TEST FOR RESTRICTED MODEL: CHISQR(2.9643,1) =  0.0851
*** No Bartlett Correction for this test

THE MATRICES BASED ON 3 COINTEGRATING VECTORS:

BETA(transposed)
        LRC     LRY     LRW    DLPC    RB      DS83    DS871
Beta1 -5.0173  2.0279  2.4829 1.0000  0.0000  0.0000  0.0000
Beta2  1.0000 -0.3175 -0.3120 6.4359  2.7413  0.0795  0.0425
Beta3  0.0691  1.0000 -1.2576 1.4415 -9.2402 -0.0467 -0.1103

ALPHA
       Alpha1    Alpha2    Alpha3
DLRC    0.0108   -0.1391    0.0623
       (0.5654) (-3.2861)  (2.0238)
DLRY   -0.0146   -0.0702   -0.1174
      (-0.5525) (-1.1951) (-2.7492)
DLRW   -0.0099    0.0060    0.1057
      (-0.7904)  (0.2153)  (5.2303)
DDLPC  -0.0316   -0.1302   -0.0326
      (-3.8686) (-7.1644) (-2.4649)
DRB    -0.0011    0.0036    0.0080
      (-0.5829)  (0.8296)  (2.5409)
```

Question 13

Estimation of a just-identified system

```
TESTING RESTRICTIONS
Zero degrees of freedom -> No test

THE MATRICES BASED ON 3 COINTEGRATING VECTORS:

BETA(transposed)
        LRC      LRY      LRW      DLPC      RB      DS83     DS871
Beta1  1.0000  -0.6386  -0.3395   0.0000   0.0000  -0.0215   0.0407
      (0.0000) (0.0470) (0.0903) (0.0000) (0.0000) (0.0184) (0.0174)
Beta2  1.0000  -1.0000  -0.0005  -2.0396   2.0396  -0.0309   0.0694
      (0.0000) (0.0000) (0.1020) (0.1755) (0.1755) (0.0230) (0.0196)
Beta3  0.0017   0.0631  -0.0318   1.0000   0.0000   0.0101   0.0000
      (0.0199) (0.0155) (0.0203) (0.0000) (0.0000) (0.0019) (0.0000)

ALPHA
       Alpha1    Alpha2    Alpha3
DLRC    0.2467   -0.3951   -1.6337
       (1.1492) (-2.4314) (-3.3572)
DLRY   -0.3532    0.3808    0.1369
      (-1.1946)  (1.7024)  (0.2044)
DLRW    0.4653   -0.4428   -0.7072
       (3.3200) (-4.1760) (-2.2270)
DDLPC   0.0722   -0.0512   -1.0128
       (0.7864) (-0.7378) (-4.8697)
DRB     0.0634   -0.0432   -0.0572
```

```
        (2.9484)  (-2.6573)  (-1.1735)
```

Estimation of an over-identified system

```
LR-TEST FOR RESTRICTED MODEL: CHISQR(5.2091,2) =  0.0739
*** No Bartlett Correction for this test

THE MATRICES BASED ON 3 COINTEGRATING VECTORS:
```

BETA(transposed)

	LRC	LRY	LRW	DLPC	RB	DS83	DS871
Beta1	1.0000	-0.3423	-0.5601	0.0000	0.0000	-0.0132	0.0008
	(0.0000)	(0.0819)	(0.0881)	(0.0000)	(0.0000)	(0.0028)	(0.0027)
Beta2	1.0000	-1.0000	0.0000	-3.9295	3.9295	-0.0239	0.0575
	(0.0000)	(0.0000)	(0.0000)	(0.3506)	(0.3506)	(0.0167)	(0.0161)
Beta3	0.7103	-0.1815	-0.4194	1.0000	0.0000	0.0000	0.0000
	(0.0198)	(0.0684)	(0.0743)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

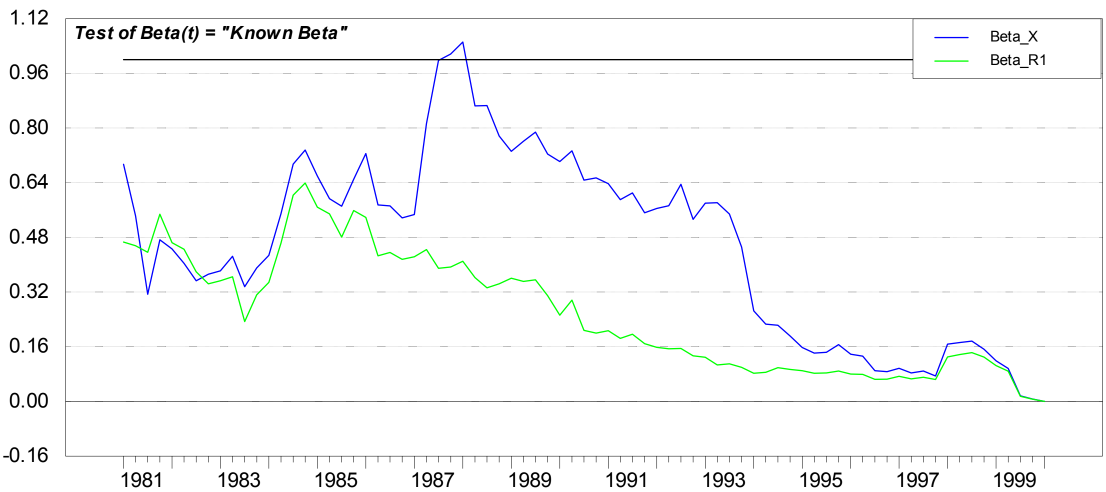
ALPHA

	Alpha1	Alpha2	Alpha3
DLRC	1.3430	-0.2555	-1.7779
	(3.1817)	(-3.3870)	(-3.7998)
DLRY	-0.3759	0.2183	0.1997
	(-0.6456)	(2.0981)	(0.3094)
DLRW	0.7331	-0.2243	-0.6852
	(2.6481)	(-4.5342)	(-2.2329)
DDLPC	0.7764	-0.0243	-1.0147
	(4.2919)	(-0.7515)	(-5.0601)
DRB	0.0530	-0.0152	-0.0323
	(1.2335)	(-1.9801)	(-0.6782)

Own calculated t-values (in brackets) of BETA(transposed)

	LRC	LRY	LRW	DLPC	RB	DS83	DS871
Beta1	1.0000	-0.3423	-0.5601	0.0000	0.0000	-0.0132	0.0008
	(0.0000)	(-4.1795)	(-6.3575)	(0.0000)	(0.0000)	(-4.7143)	(0.2963)
Beta2	1.0000	-1.0000	0.0000	-3.9295	3.9295	-0.0239	0.0575
	(0.0000)	(0.0000)	(0.0000)	(-11.2080)	(11.2079)	(-1.4311)	(3.5714)
Beta3	0.7103	-0.1815	-0.4194	1.0000	0.0000	0.0000	0.0000
	(35.8737)	(-2.6535)	(-5.6447)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Recursive graph for Beta constancy



The test statistic is scaled by the 5% critical value

Section 5: The moving average representation

Question 14

```
THE MA-REPRESENTATION AND DECOMPOSITION OF THE TREND

ALPHA Orthogonal
  LRC   LRY   LRW  DLPC   RB
  0.10  0.03 -0.15 -0.11 0.98
 -0.14 -0.68 -0.52  0.49 0.01

BETA Orthogonal(transposed)

LRC -0.70  1.02 -3.99 -0.19  0.65
LRY -0.62 -0.70 -0.51  0.03 -0.01

The Impact Matrix, C
  LRC   LRY   LRW  DLPC   RB
  0.01  0.40  0.43 -0.23 -0.70
  0.20  0.51  0.22 -0.46  0.99
 -0.33  0.23  0.85  0.19 -3.90
 -0.02 -0.03  0.01  0.04 -0.19
  0.07  0.03 -0.09 -0.08  0.64

T-values for C
  LRC   LRY   LRW  DLPC   RB
  0.08  4.33  2.56 -0.87 -0.50
  0.99  4.36  1.04 -1.39  0.56
 -1.71  2.00  4.16  0.61 -2.26
 -1.83 -3.48  0.95  1.71 -1.66
  2.27  1.55 -2.90 -1.60  2.44

The Long-Run covariance matrix, C*Sigma*C
  LRC   LRY   LRW  DLPC   RB
  0.00
  0.00  0.00
  0.00  0.00  0.00
 -0.00 -0.00  0.00  0.00
  0.00  0.00 -0.00 -0.00 0.00

The Linear Trends in the Levels
 LRC  LRY  LRW  DLPC   RB
 0.00 0.00 0.00 -0.00 -0.00
```

Question 16

```
THE SHORT-RUN MATRICES:

LAGGED DIFFERENCES:
GAMMA(1)
       DLRC{1}   DLRY{1}   DLRW{1}  DDLPC{1}   DRB{1}
DLRC   -0.2678   -0.1189    0.0410    0.4930   -0.8824
      (-2.5767) (-1.8100)  (0.2913)  (2.5624) (-1.0804)
DLRY    0.3222   -0.2387   -0.0117    0.3102   -0.8294
       (2.2517) (-2.6390) (-0.0606)  (1.1710) (-0.7377)
DLRW   -0.2176   -0.0364   -0.1162   -0.2397   -2.4657
      (-3.2071) (-0.8495) (-1.2662) (-1.9088) (-4.6260)
DDLPC   0.0564   -0.0066   -0.0379   -0.0329    0.0961
       (1.2700) (-0.2345) (-0.6301) (-0.3996)  (0.2754)
DRB     0.0278   -0.0013   -0.0083    0.0234    0.2796
       (2.6675) (-0.1987) (-0.5889)  (1.2120)  (3.4186)

SHIFT DUMMIES:
        DDS83    DDS871
DLRC   -0.0031   -0.0111
      (-0.2042) (-0.7321)
DLRY    0.0030   -0.0150
       (0.1408) (-0.7181)
DLRW    0.0152   -0.0291
       (1.5133) (-2.9338)
```

```
DDLPC   0.0056   -0.0082
       (0.8456) (-1.2624)
DRB    -0.0076    0.0019
      (-4.9096)  (1.2636)

DUMMY VARIABLES:
         D75      D774
DLRC    0.0490   -0.0599
       (4.0385) (-3.9094)
DLRY    0.0571   -0.0014
       (3.4158) (-0.0643)
DLRW    0.0120    0.0082
       (1.5198)  (0.8244)
DDLPC  -0.0325    0.0146
      (-6.2624)  (2.2218)
DRB    -0.0003   -0.0015
      (-0.2597) (-0.9791)

CONSTANT

DLRC    0.4087
       (2.0834)
DLRY   -0.2289
      (-0.8477)
DLRW    0.3970
       (3.1007)
DDLPC   0.2061
       (2.4587)
DRB     0.0477
       (2.4292)
```

Section 6: The identification of the short run structure

Construction of the ECM-variables

```
CI1=LrC-0.3423*Lry-0.5601*LrW-0.0132*Ds83+0.0008*Ds871;
CI2=LrC-Lry-3.9295*DLPc+3.9295*Rb-0.0239*Ds83+0.0575*Ds871;
CI3=0.7103*LrC-0.1815*Lry-0.4194*LrW+Rb;

Means, standard deviations and correlations (using DKconsumption 1973_1 2000_1.xls)
The sample is 1973 (1) - 2000 (1)
Means
          CI1          CI2          CI3
     -0.63420   -0.0079259     -0.20986

ecm1=CI1+0.63420;
ecm2=CI2+0.0079259;
ecm3=CI3+0.20986;
```

Tests for significance of each variable

```
F-test on regressors except unrestricted: F(60,425) = 5.64845 [0.0000] **
F-tests on retained regressors, F(5,90) =
      DLrC_1       4.70101 [0.001]**      DLrW_1      0.641075 [0.669]
     DDLPc_1       3.31547 [0.009]**       DRb_1       4.17757 [0.002]**
      DLry_1       1.31059 [0.267]        ecm1_1       2.39351 [0.044]*
      ecm2_1       10.2748 [0.000]**      ecm3_1       1.54477 [0.184]
       DDs83       4.51669 [0.001]**      DDs871       3.29080 [0.009]**
         D75       7.70996 [0.000]**        D774       3.37303 [0.008]**
    Constant U     4.43211 [0.001]**

correlation of URF residuals (standard deviations on diagonal)
               DLrC         DLrW        DDLPc          DRb         DLry
DLrC       0.016147      0.30730     -0.18004     0.015245      0.24580
DLrW        0.30730     0.010193     -0.23002     -0.29017     0.055823
DDLPc      -0.18004     -0.23002    0.0071891    -0.061786    -0.062230
DRb        0.015245     -0.29017    -0.061786    0.0015178     -0.12911
DLry        0.24580     0.055823    -0.062230     -0.12911     0.020927
correlation between actual and fitted
         DLrC         DLrW        DDLPc           DRb          DLry
```

```
      0.63871      0.63646      0.78386      0.69869      0.55907
```

A parsimonious short run adjustment structure

```
MOD( 5) Estimating the model by FIML (using DKconsumption 1973_1 2000_1.xls)
        The estimation sample is: 1973 (3) to 2000 (1)

Equation for: DLrC
                  Coefficient  Std.Error  t-value  t-prob
DLrC_1              -0.160253    0.07897    -2.03   0.045
ecm3_1              -0.429135    0.09292    -4.62   0.000
D75                 0.0353470    0.01253     2.82   0.006
D774               -0.0571562    0.01533    -3.73   0.000
Constant       U   0.00452926   0.001569     2.89   0.005

sigma = 0.0159694

Equation for: DLrW
                  Coefficient  Std.Error  t-value  t-prob
DLrC_1              -0.148461    0.05615    -2.64   0.010
DDLPc_1             -0.204934     0.1032    -1.99   0.050
DRb_1                -1.89163     0.4788    -3.95   0.000
ecm2_1             -0.0851468    0.02331    -3.65   0.000
DDs871             -0.0343791   0.009144    -3.76   0.000
Constant       U   0.00453656   0.001019     4.45   0.000

sigma = 0.0102906

Equation for: DDLPc
                  Coefficient  Std.Error  t-value  t-prob
DLrC_1               0.157547    0.03982     3.96   0.000
DDLPc_1             -0.225597    0.07984    -2.83   0.006
ecm2_1              0.0765304    0.01686     4.54   0.000
D75                -0.0291238   0.005797    -5.02   0.000
Constant       U -0.000879992  0.0007225    -1.22   0.226

sigma = 0.00734164

Equation for: DRb
                  Coefficient  Std.Error  t-value  t-prob
DLrC_1              0.0200039   0.008152     2.45   0.016
DRb_1                0.278020    0.08001     3.47   0.001
ecm1_1              0.0831756    0.02126     3.91   0.000
ecm2_1             -0.0134184   0.003661    -3.66   0.000
ecm3_1             -0.0666083    0.02650    -2.51   0.014
DDs83             -0.00732492   0.001460    -5.02   0.000
Constant       U-8.58616e-005  0.0001472   -0.583   0.561

sigma = 0.00148853

Equation for: DLry
                  Coefficient  Std.Error  t-value  t-prob
ecm2_1               0.213748    0.04952     4.32   0.000
ecm3_1              -0.262590     0.1503    -1.75   0.084
D75                 0.0554740    0.01761     3.15   0.002
Constant       U   0.00274064   0.002062     1.33   0.187

sigma = 0.0213276

log-likelihood     1842.33341  -T/2log|Omega|     2601.46553
no. of observations       107  no. of parameters          27

LR test of over-identifying restrictions: Chi^2(28)=   31.999 [0.2745]
BFGS using analytical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence

correlation of structural residuals (standard deviations on diagonal)
               DLrC         DLrW        DDLPc           DRb         DLry
DLrC       0.015969      0.31208     -0.18851     0.0019702      0.27898
DLrW        0.31208     0.010291     -0.26109      -0.30112     0.069906
```

```
DDLPc      -0.18851     -0.26109    0.0073416    -0.060360    -0.080067
DRb      0.0019702     -0.30112    -0.060360    0.0014885     -0.11504
DLry       0.27898     0.069906    -0.080067     -0.11504     0.021328
```

Question 18

Partial system conditional on the bond rate and real wealth

```
MOD(18) Estimating the model by FIML (using DKconsumption 1973_1 2000_1.xls)
        The estimation sample is: 1973 (3) to 2000 (1)

Equation for: DLrC
                  Coefficient  Std.Error  t-value  t-prob
ecm3_1              -0.267741    0.09459    -2.83   0.006
D774               -0.0578160    0.01528    -3.78   0.000
DLrW                 0.325400     0.1329     2.45   0.016
DDLPc               -0.577043     0.2140    -2.70   0.008
Constant       U   0.00254936   0.001588     1.61   0.112

sigma = 0.0155257

Equation for: DDLPc
                  Coefficient  Std.Error  t-value  t-prob
DLrC_1               0.142298    0.03866     3.68   0.000
DDLPc_1             -0.279855    0.07940    -3.52   0.001
ecm2_1              0.0570806    0.01768     3.23   0.002
D75                -0.0313621   0.005920    -5.30   0.000
DLrW                -0.128683    0.06176    -2.08   0.040
Constant       U -0.000310421  0.0007511   -0.413   0.680

sigma = 0.00715779

Equation for: DLry
                  Coefficient  Std.Error  t-value  t-prob
ecm2_1               0.144481    0.04150     3.48   0.001
D75                 0.0550138    0.01705     3.23   0.002
Constant       U   0.00275405   0.002075     1.33   0.188

sigma = 0.0214601

log-likelihood     941.872035  -T/2log|Omega|      1397.3513
no. of observations       107  no. of parameters          14

LR test of over-identifying restrictions: Chi^2(25)=   35.138 [0.0857]
BFGS using analytical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence

correlation of structural residuals (standard deviations on diagonal)
               DLrC        DDLPc         DLry
DLrC       0.015526      0.18593      0.26279
DDLPc       0.18593    0.0071578    -0.017666
DLry        0.26279    -0.017666     0.021460
```