SUPPLEMENTARY TABLES FOR:

"THE LIKELIHOOD RATIO TEST FOR COINTEGRATION RANKS IN THE I(2) MODEL"

HEINO BOHN NIELSEN* AND ANDERS RAHBEK†

*Department of Economics, University of Copenhagen. heino.bohn.nielsen@econ.ku.dk
†Dept. of Applied Mathematics and Statistics, University of Copenhagen. rahbek@math.ku.dk

ABSTRACT: This note presents estimation and simulation results to compare the likelihood ratio (LR) test and the Two-Step based test for cointegration ranks in the I(2) model. Three empirical examples are considered: Banerjee, Cockerell, and Russell (2001), Rahbek, Kongsted, and Jørgensen (1999), and Nielsen (2002). A final simulation study generates more DGPs to cover a broader range of the parameter space of the I(2) model. Details on the implementation, e.g. the treatment of initial values, are given in the main paper.

1 Banerjee, Cockerell, and Russell (2001)

The original data set includes three price measures, and a number of stationary conditioning variables. Here we exclude the conditioning variables and consider a VAR(2) model with linear trends, $H^*(r,s)$. The LR and Two-Step based tests for the cointegration ranks are reported in Table 1.

r	Two-Step rank test				LR test			
0	203.53	118.91	57.62	47.28	203.53	118.91	57.62	47.28
	[.00]	[.00]	[.02]	[.02]	[.00]	[.00]	[.02]	[.02]
1		78.08	20.37	17.78		47.56	19.21	17.78
		[.00]	[.72]	[.37]		[.06]	[.79]	[.37]
2			11.11	7.05			9.10	7.05
			[.55]	[.35]			[.74]	[.35]
$p-r-s$	3	2	1	0	3	2	1	0

Table 1: *Rank determination for the data in Banerjee, Cockerell, and Russell (2001). Figures in square brackets are asymptotic p−values, see Doornik (1998).*

To analyze the difference further we conduct a Monte Carlo simulation. As the data generating process (DGP) we consider the model $H^*(1,1)$, with parameters set to the maximum likelihood estimates for the data above. More precisely, we use the equation

$$\Delta^2 X_t = \alpha[\rho'\tau^{*\prime}X^*_{t-1} + \psi^{*\prime}\Delta X^*_{t-1}] + \bar{\alpha}_{\perp\Omega}\kappa'\tau^{*\prime}\Delta X^*_{t-1} + \sum_{i=1}^{k-2}\Psi_i\Delta^2 X_{t-i} + \epsilon_t, \qquad (1)$$

with $\epsilon_t \sim N(0,\Omega)$, and parameters and initial values given by

$$\alpha = \begin{pmatrix} -0.0719 \\ 0.0922 \\ -0.1340 \end{pmatrix}, \quad \rho = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \tau^* = \begin{pmatrix} 1 & 0.5466 \\ -0.7551 & 1 \\ -0.2037 & -1.0236 \\ -0.0006 & -0.0111 \end{pmatrix},$$

$$\psi^* = \begin{pmatrix} 10.0879 \\ -3.5693 \\ 0.2735 \\ 1.1028 \end{pmatrix}, \quad \overline{\alpha}_{\perp\Omega}\kappa' = \begin{pmatrix} 0.1745 & -0.0390 \\ 0.7753 & -0.1087 \\ 1.2366 & 0.6834 \end{pmatrix},$$

$$10^3 \cdot \Omega = \begin{pmatrix} 0.0381 & 0.0483 & 0.0642 \\ 0.0483 & 0.3371 & 0.1199 \\ 0.0642 & 0.1199 & 0.9675 \end{pmatrix}, \quad (X_{-1}, X_0) = \begin{pmatrix} 2.9444 & 2.9653 \\ 4.7050 & 4.7230 \\ 3.0106 & 3.0493 \end{pmatrix}.$$

The rejection frequencies of the tests for cointegration ranks are reported in Table 2

T	Models $H^*(r,s)$					
	$H^*(0,0)$	$H^*(0,1)$	$H^*(0,2)$	$H^*(0,3)$	$H^*(1,0)$	$H^*(1,1)$
	Two-Step rank test					
50	100.0	96.2	47.1	52.3	94.4	**20.2**
75	100.0	100.0	81.6	76.7	99.6	**21.8**
100	100.0	100.0	97.1	92.1	100.0	**18.4**
200	100.0	100.0	100.0	100.0	100.0	**11.5**
500	100.0	100.0	100.0	100.0	100.0	**8.1**
1000	100.0	100.0	100.0	100.0	100.0	**6.3**
	LR test					
50	···	···	···	···	32.3	**5.7**
75	···	···	···	···	66.1	**7.5**
100	···	···	···	···	91.4	**7.0**
200	···	···	···	···	100.0	**5.9**
500	···	···	···	···	100.0	**5.7**
1000	···	···	···	···	100.0	**5.4**

Table 2: *Rejection frequencies in a simulation based on Banerjee, Cockerell, and Russell (2001). Results are based on 5000 replications and a nominal 5% level. The tests are not calculated sequentially. Bold indicates rejection frequencies for tests of the correct model (empirical size), while '··· ' indicates that the results are identical for the two tests by construction. The standard error of the rejection frequency corresponding to the true probability π is $s_\pi = \sqrt{\pi(1-\pi)/5000}$, e.g. $s_{0.05} = 0.003$.*

2 Rahbek, Kongsted and Jørgensen (1999)

Data includes nominal money, income, prices and an interest rate. Estimation and simulations are based on a VAR(2) model with linear trends, $H^*(r,s)$. Tests for the cointegration ranks are reported in Table 3.

r	Two-Step rank test					LR test				
0	336.47	231.15	166.18	124.82	115.51	336.47	231.15	166.18	124.82	115.51
	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]
1		173.19	71.83	54.78	45.73		153.12	64.47	53.22	45.73
		[.00]	[.03]	[.04]	[.02]		[.00]	[.12]	[.06]	[.02]
2			36.77	23.14	16.96			29.49	21.57	16.96
			[.44]	[.54]	[.43]			[.82]	[.65]	[.43]
3				12.87	5.25				9.96	5.25
				[.39]	[.57]				[.66]	[.57]
$p-r-s$	4	3	2	1	0	4	3	2	1	0

Table 3: *Rank determination for the data in Rahbek, Kongsted and Jørgensen (1999). See notes to Table 1.*

The DGP for the simulation is given by (1), with $\epsilon_t \sim N(0, \Omega)$ and parameters

$$\alpha = \begin{pmatrix} -0.0937 \\ 0.0107 \\ -0.0105 \\ 0.0207 \end{pmatrix}, \quad \rho = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \tau^* = \begin{pmatrix} 1 & -0.5533 \\ -0.8568 & 1 \\ -0.5181 & 4.4343 \\ 7.4319 & 0.4988 \\ -0.0073 & -0.0379 \end{pmatrix},$$

$$\psi^* = \begin{pmatrix} 13.6978 \\ -6.8955 \\ 1.0642 \\ 3.9482 \\ -5.3117 \end{pmatrix}, \quad \overline{\alpha}_{\perp\Omega}\kappa' = \begin{pmatrix} 0.0266 & 0.0433 \\ -0.0079 & 0.0145 \\ 0.0386 & -0.2686 \\ -0.1283 & 0.0179 \end{pmatrix},$$

$$10^3 \cdot \Omega = \begin{pmatrix} 0.1948 & -0.0053 & -0.0153 & -0.0796 \\ -0.0053 & 0.0666 & -0.0027 & 0.0397 \\ -0.0153 & -0.0027 & 0.1685 & 0.0240 \\ -0.0796 & 0.0397 & 0.0240 & 0.1895 \end{pmatrix},$$

$$(X_{-1}, X_0) = \begin{pmatrix} 9.0098 & 9.0158 \\ -1.9347 & -1.9211 \\ 11.1169 & 11.1307 \\ 0.0490 & 0.0500 \end{pmatrix}.$$

The rejection frequencies of the tests for cointegration ranks are reported in Table 4

T	Models $H^*(r,s)$						
	$H^*(0,0)$	$H^*(0,1)$	$H^*(0,2)$	$H^*(0,3)$	$H^*(0,4)$	$H^*(1,0)$	$H^*(1,1)$
	Two-Step rank test						
50	100	99	94	88	98	97	**29**
75	100	100	100	99	100	100	**21**
100	100	100	100	100	100	100	**15**
200	100	100	100	100	100	100	**11**
500	100	100	100	100	100	100	**6**
1000	100	100	100	100	100	100	**6**
	LR test						
50	...	...	...	...	...	82	**13**
75	...	...	...	...	...	100	**9**
100	...	...	...	...	...	100	**9**
200	...	...	...	...	...	100	**8**
500	...	...	...	...	...	100	**5**
1000	...	...	...	...	...	100	**5**

Table 4: *Rejection frequencies in a simulation based on Rahbek, Kongsted and Jørgensen (1999). Based on 1000 replications. The standard error in this case is* $s_{0.05} = 0.007$. *See notes to Table 2.*

3 Nielsen (2002)

Data includes real exports, a measure of the export market, the export price, the competing price, domestic production costs, and the exchange rate. Based on a VAR(3). The original model and the statistics in Table 5 are based on $H^*(r,s)$ augmented with an intervention dummy, and a level shift in all directions. The dummies are excluded for in the simulation. Tests for the cointegration ranks are reported in Table 5.

r	Two-Step rank test							LR test						
0	427.53	357.30	303.50	252.68	210.69	179.54	170.90	427.53	357.30	303.50	252.68	210.69	179.54	170.90
	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]
1		305.40	236.79	188.85	148.89	115.65	111.92		285.79	222.04	174.87	142.53	114.76	111.92
		[.00]	[.00]	[.00]	[.01]	[.08]	[.01]		[.00]	[.00]	[.01]	[.03]	[.09]	[.01]
2			222.98	162.66	121.33	84.44	76.85			162.62	127.41	99.47	80.54	76.85
			[.00]	[.00]	[.00]	[.11]	[.04]			[.03]	[.09]	[.17]	[.18]	[.04]
3				132.64	86.22	49.72	44.09				92.57	68.60	47.42	44.09
				[.00]	[.02]	[.43]	[.21]				[.18]	[.28]	[.53]	[.21]
4					67.89	27.39	20.35					42.01	24.86	20.35
					[.00]	[.60]	[.53]					[.51]	[.74]	[.53]
5						21.37	9.00						13.92	9.00
						[.12]	[.41]						[.58]	[.41]
$p-r-s$	6	5	4	3	2	1	0	6	5	4	3	2	1	0

Table 5: *Rank determination for the data in Nielsen (2002). The model allows for a break in the levels for the German reunification. The p-values are based on simulations in Nielsen (2002).*

The DGP for the simulation is given by (1), with $\epsilon_t \sim N(0, \Omega)$ and parameters

$$\alpha = \begin{pmatrix} -0.2480 & 0.0316 & 0.1677 \\ -0.1670 & -0.0594 & 0.0051 \\ -0.0477 & 0.0304 & -0.0641 \\ 0.0170 & 0.0027 & -0.0499 \\ 0.1242 & 0.0032 & -0.0195 \\ -0.0822 & -0.0051 & -0.0828 \end{pmatrix}, \quad \rho = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\tau^* = \begin{pmatrix} 1 & -1.3268 & -1.4496 & 3.8087 & 0.6755 \\ 0.1091 & 1 & 0.6184 & 4.5120 & 0.9832 \\ -0.2082 & -8.6090 & 1 & -0.0002 & 0.3012 \\ -1.0441 & 1.1250 & 0.7662 & 1 & -0.8620 \\ -1.3485 & 4.3912 & 0.5109 & 1.1631 & 1 \\ 1.8365 & 6.2262 & -1.4198 & -0.9290 & -0.1493 \\ -0.0236 & 0.0125 & 0.0113 & -0.1148 & -0.0171 \\ 0.0352 & 0.2156 & 0.1219 & 0.4324 & 0.0645 \end{pmatrix},$$

$$\psi^* = \begin{pmatrix} 0.4591 & -4.3842 & -1.1460 \\ 0.5046 & 9.8296 & -0.5898 \\ 4.9906 & -15.8242 & 3.2639 \\ -0.2633 & 8.4162 & -0.7078 \\ -3.1966 & -0.4893 & -0.9255 \\ -1.2417 & 4.6155 & 10.0901 \\ 0.4864 & -0.1370 & -0.4984 \\ -0.0353 & 0.1932 & -0.0537 \end{pmatrix},$$

$$\overline{\alpha}_{\perp\Omega}\kappa' = \begin{pmatrix} 0.3171 & -0.0152 & 0.3840 & -0.0156 & -0.3276 \\ 0.0483 & 0.0563 & -0.1009 & -0.0002 & -0.3252 \\ -0.0156 & 0.0086 & -0.0133 & 0.0201 & -0.1314 \\ 0.2817 & -0.0112 & 0.1145 & -0.1695 & 0.6051 \\ -0.0242 & 0.0121 & 0.0906 & 0.1074 & -0.6424 \\ 0.0326 & -0.0530 & 0.2379 & 0.0380 & -0.0344 \end{pmatrix},$$

$$10^3 \cdot \Omega = \begin{pmatrix} 0.7328 & 0.2283 & 0.0246 & 0.0202 & 0.1658 & 0.0401 \\ 0.2283 & 0.2255 & 0.0116 & 0.0032 & 0.0631 & -0.0027 \\ 0.0246 & 0.0116 & 0.0391 & 0.0083 & 0.0210 & 0.0258 \\ 0.0202 & 0.0032 & 0.0083 & 0.0837 & -0.0079 & 0.0145 \\ 0.1658 & 0.0631 & 0.0210 & -0.0079 & 0.1800 & 0.0124 \\ 0.0401 & -0.0027 & 0.0258 & 0.0145 & 0.0124 & 0.0979 \end{pmatrix},$$

$$(X_{-2}, X_{-1}, X_0) = \begin{pmatrix} -0.2740 & -0.2941 & -0.2553 \\ -0.3003 & -0.2280 & -0.1954 \\ -0.3321 & -0.3231 & -0.3058 \\ -0.3310 & -0.3318 & -0.3153 \\ -0.0860 & -0.0750 & -0.1035 \\ -0.3757 & -0.3554 & -0.3663 \end{pmatrix}.$$

The rejection frequencies of the tests for cointegration ranks are reported in Table 6.

T	Models $H^*(r,s)$																				
	$(0,0)$	$(0,1)$	$(0,2)$	$(0,3)$	$(0,4)$	$(0,5)$	$(0,6)$	$(1,0)$	$(1,1)$	$(1,2)$	$(1,3)$	$(1,4)$	$(1,5)$	$(2,0)$	$(2,1)$	$(2,2)$	$(2,3)$	$(2,4)$	$(3,0)$	$(3,1)$	$(3,2)$
	Two-Step rank test																				
50	100	100	99	98	98	99	100	100	97	88	80	84	94	98	81	55	50	71	92	48	**22**
75	100	100	100	100	100	100	100	100	100	100	98	94	97	100	99	81	59	76	100	73	**25**
100	100	100	100	100	100	100	100	100	100	100	100	99	99	100	100	98	76	89	100	91	**21**
200	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	**12**
500	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	**9**
1000	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	**7**
	LR test																				
50	···	···	···	···	···	···	···	96	87	74	73	81	···	58	35	31	42	···	14	7	**11**
75	···	···	···	···	···	···	···	100	100	98	95	93	···	97	76	54	51	···	39	14	**13**
100	···	···	···	···	···	···	···	100	100	100	100	99	···	100	99	84	69	···	78	28	**12**
200	···	···	···	···	···	···	···	100	100	100	100	100	···	100	100	100	100	···	100	95	**8**
500	···	···	···	···	···	···	···	100	100	100	100	100	···	100	100	100	100	···	100	100	**7**
1000	···	···	···	···	···	···	···	100	100	100	100	100	···	100	100	100	100	···	100	100	**7**

Table 6: *Rejection frequencies in a simulation based on Nielsen (2002). DGP is* $H^*(3,2)$*. Based on 1000 replications. See notes to Table 4.*

4 More DGPs

The main drawback of the simulations based on estimated models is the specificity, i.e. that the results are specific to the chosen Monte Carlo setup. Here we consider a series of simulations where the DGP itself is drawn randomly from a class of relevant models.

We consider a $p = 3$ dimensional VAR(2). Instead of the intractable task of systematically covering the entire parameter space of the model we randomly draw coefficients for the DGPs. We then check that the eigenvalues of the companion matrix suggest a behavior similar to I(2) models seen in empirical applications. To be specific, a DGP is accepted if the following three requirements are met: (1) The model is non-explosive. (2) The largest unrestricted eigenvalue has a positive real part. (3) The largest unrestricted eigenvalue has a larger real than complex component. The requiments in $(2)-(3)$ exclude DGPs with close to seasonal unit roots. Below we present results for 150 different DGPs and 1000 replications per DGP.

First we consider 150 random DGPs in $H^{*}(1,0)$. Graph (A) illustrates the rejection frequencies when testing the correct model, $H^{*}(1,0)$. The empirical sizes of the LR test vary between 5% and 20% with an average of 9%. Based on the Two-Step test the rejection frequencies vary between 7% and 89%, and with an average of 21%. This illustrates that if $s = 0$ there is a big potential distortion in the Two-Step test.

Next we consider 150 random DGPs with ranks $H^{*}(1,1)$. Graph (B) illustrates the rejection frequencies when the model $H^{*}(1,0)$ is tested, i.e. empirical power. The rejection frequencies for the LR test vary between 8% and 100% with an average of 87%. For the Two-Step test the rejection frequencies vary between 16% and 100%, and with an average of 96%. Graph (C) depicts the rejection frequencies when the true model $H^{*}(1,1)$ is tested. In this case, $s = 1$, the average size distortion is much smaller - apart from a limited number of cases.

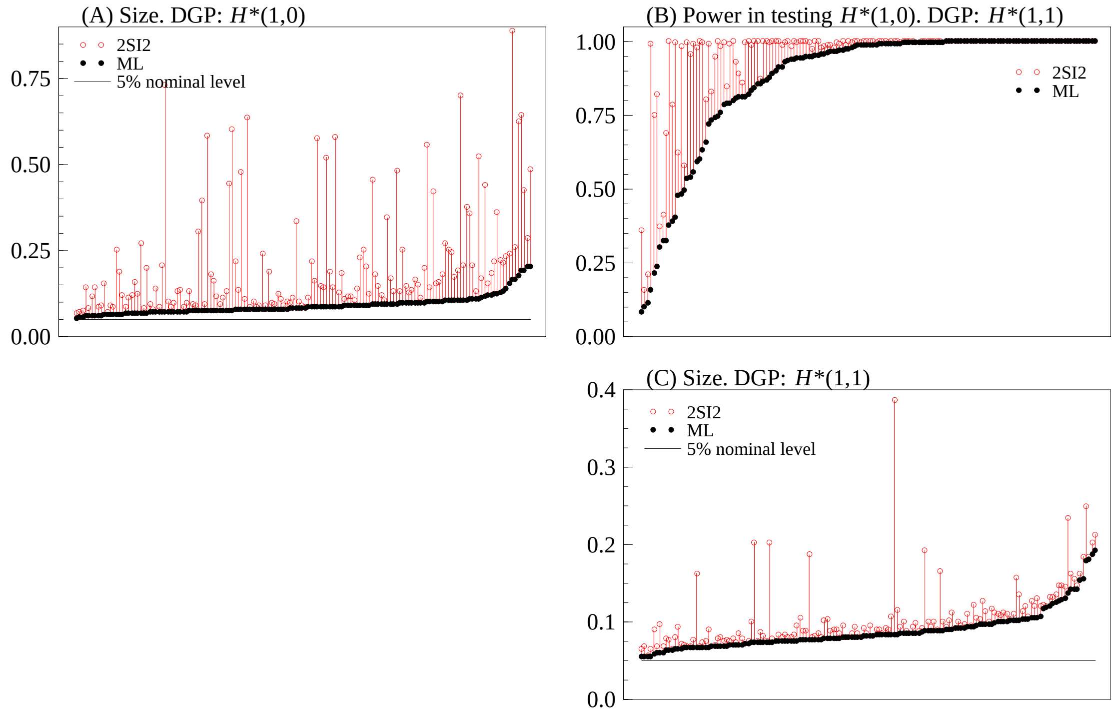

Figure 1: *(A) Rejection frequencies for test of the model $H^*(1,0)$ when it is also the DGP (size). (B) Rejection frequencies for test of the model $H^*(1,0)$ when $H^*(1,1)$ is the DGP (power). (C) Rejection frequencies for test of the model $H^*(1,1)$ when it is also the DGP (size). All tests are calculated for T=75. Results are sorted by the LR test and are based on 150 randomly drawn DGPs and 1000 replications for each DGP.*

References

Banerjee, A., L. Cockerell, and B. Russell (2001): "An I(2) Analysis of Inflation and the Markup," *Journal of Applied Econometrics*, 16, 221–240.

Doornik, J. A. (1998): "Approximations to the Asymptotic Distribution of Cointegration Tests," *Journal of Economic Surveys*, 12(5), 573–593.

Nielsen, H. B. (2002): "An I(2) Cointegration Analysis of Price and Quantity Formation in Danish Manufactured Exports," *Oxford Bulletin of Economics and Statistics*, 64(5), 449–472.

Nielsen, H. B., and A. Rahbek (2003): "Likelihood Ratio Testing for Cointegration Ranks in the I(2) Model," *Preprint 2003:11, Department of Applied Mathematics and Statistics, University of Copenhagen.*

Rahbek, A., H. C. Kongsted, and C. Jørgensen (1999): "Trend-Stationarity in the I(2) Cointegration Model," *Journal of Econometrics*, 90, 265–289.