Figure 19.4: Firm $i$ in a monopolistic competition long-run equilibrium.

plant and equipment of the firm.

It is this last feature that signifies the presence of excess capacity even in the long run equilibrium. As long as the cost curve does not shift (this could happen if the wage level increases), firms are more than willing to accommodate an increased demand (outward shift in the demand curve) at an unchanged price (or even at a lower price) by an increase in supply. This fits well with empirical evidence that productivity and profits are pro-cyclical (varies in the same direction as aggregate output), an issue to which we return in Part VII of this book.

To incorporate better the key role of financial markets, the next section adds an interest-bearing asset to the framework. This takes us to the IS-LM model.

### 19.4 The static IS-LM model

The version of the IS-LM model presented here is essentially the familiar one known from introductory textbooks. It is a static version focusing on mechanisms that are operative within a single short period. This will serve as a point of departure for the more satisfactory dynamic version of the IS-LM model in the next chapter.

We consider a closed economy with a private sector, a government, and a central bank. The produce of the economy consists mainly of manufacturing

C. Groth, Lecture notes in macroeconomics, (mimeo) 2011
CHAPTER 19. THE THEORY OF EFFECTIVE DEMAND

goods and services, supplied under conditions of excess capacity with prices set in advance by firms operating in markets with imperfect competition. Although not very visible in the model, there are private banks that accept deposits and issue loans to households and firms. The money supply in the model stands for money in the $M_1$ or $M_2$ sense and thus includes bank created money in addition to currency in circulation, cf. Chapter 16.

19.4.1 The framework

The IS-LM model addresses the interaction between the output market and the financial markets in the short run.

The output market

Demand Aggregate output demand is given as

$$
Y^d = C(Y^p, Y^e_{+1}, K, r^e) + I(Y^e_{+1}, K, r^e) + G + \varepsilon_D, \tag{19.17}
$$

where the function $C(\cdot)$ represents private consumption, the function $I(\cdot)$ represents private fixed capital investment, $G$ is public spending on goods and services, and $\varepsilon_D$ is a stochastic shift parameter summarizing the role of unspecified exogenous variables that may matter for aggregate output demand. One of these variables could be the general “state of confidence”. As arguments in either the consumption or the investment function, or both, appear $Y^p$ which is current private disposable income, $Y^e_{+1}$, which is expected output the next period (or periods), $K$ which is the installed capital stock, and $r^e$ which is the expected short-term real interest rate. For simplicity the model assumes bank loans and short-term bonds (traded at a centralized auction market) to be perfect substitutes and so there is just one real interest rate, $r$.

The signs of the partial derivatives of the consumption and investment functions in (19.17) are explained as follows. A general tenet from earlier chapters is that consumption depends positively on household wealth. One component of household wealth is financial wealth, here represented by the capital stock $K$. Another component is perceived human wealth (the present value of the expected labor earnings stream), which depends positively on both $Y^p$ and $Y^e_{+1}$. The separate role of disposable income, $Y^p$, reflects the hypothesis that a substantial fraction of households are credit constrained. The role of the interest rate, $r$, reflects the hypothesis that the negative

C. Groth, Lecture notes in macroeconomics, (mimeo) 2011
substitution and wealth effects on current consumption of a rise in the real interest rate dominate the positive income effect.

Firms’ investment depends positively on $Y_{t+1}$, because how much productive capacity firms need next period depends on the expected demand next period. On the other hand, the more capital firms already have, the less they need to invest. Finally, the cost of investing is higher the higher is the real interest rate. These features are consistent with the $q$-theory of investment when considering an economy where production is primarily demand constrained.

Disposable income is given by

$$Y^p \equiv Y - T,$$  \hspace{1cm} (19.18)

where $Y$ is aggregate factor income (= GDP) and $T$ is real net tax revenue (equal to gross tax revenue minus transfers). We assume a quasi-linear net tax revenue function

$$T = \tau + T(Y), \quad 0 \leq T' < 1,$$  \hspace{1cm} (19.19)

where $\tau$ is a constant parameter reflecting “tightness” of discretionary fiscal policy. Fiscal policy is thus described by two variables, $G$ representing government spending and $\tau$ representing the discretionary element in taxation. A balanced primary budget is the special case $\tau + T(Y) = G$. The endogenous part, $T(Y)$, of the tax revenue is determined by the given taxation rules; when $T' > 0$, these rules act as “automatic stabilizers” by softening the effects on disposable income, and thereby on consumption, of changes in output and employment.

Finally, we assume that expected output next period, $Y_{t+1}^e$, is an increasing function of current output,

$$Y_{t+1}^e = \varphi(Y), \quad 0 < \varphi' \leq 1.$$  \hspace{1cm} (19.20)

We may write aggregate private output demand in a more compact way. First, since we only consider a single period, we can treat the amount of installed capital as a given constant, $\bar{K}$. For convenience, we suppress the explicit reference to $\bar{K}$ in the consumption and investment functions. Second, inserting (19.18) and 19.20) into (19.17), we can express aggregate private demand by the function $D(Y, r^e, \tau) \equiv C(Y - \tau - T(Y), \varphi(Y), r^e) + I(\varphi(Y), r^e)$. Thus, total demand is

\[
Y^d = D(Y, r^e, \tau) + G + \varepsilon_D, \quad \text{where} \quad \varepsilon_D \\
0 < D_Y = C_{Y^p}(1 - T') + (C_{Y_{t+1}^*} + I_{Y_{t+1}^*})\varphi'(Y) < 1, \quad \text{and} \quad D_{r^e} = C_{r^e} + I_{r^e} < 0, \quad \text{and} \quad D_{\tau} = C_{Y^p} \cdot (-1) \in (-1, 0).
\]

C. Groth, Lecture notes in macroeconomics, (mimeo) 2011
Production Prices on goods and services have been set in advance by firms operating in markets with monopolistic competition. Owing to menu costs, in response to shifts in demand firms prefer to change production rather than price. There is scope for maintaining profitability this way, because prices are above marginal costs. Behind the scene there is an aggregate production function, \( Y = F(K, N) \), where \( N \) is the level of employment and technological change is ignored because the focus is on the short run. The conception is that “under normal circumstances” there is excess capacity in the sense that \( \bar{K} \) is large enough so that for some \( N < \bar{N} \), where \( \bar{N} \) is the labor force, output demand can be satisfied, i.e., \( F(\bar{K}, N) = Y^d \). The total unemployment, \( \bar{N} - N \), is traditionally decomposed into two components. One component is the so-called natural (or structural) level of unemployment, \( \bar{N} - N_n \), which is that level of unemployment which generates neither upward or downward pressure on the inflation rate (according to some theories there is rather a range of unemployment rates with this property). The remainder, \( N_n - N \), is named cyclical unemployment; \( N_n \) is called the natural level of employment and the corresponding output level \( Y_n = F(\bar{K}, N_n) \) the natural level of output (some authors call it full-employment output).

The financial markets

For simplicity it is assumed that essentially only two financial assets exist, money and interest-bearing short-term bonds. Although not directly visible in the model, there are commercial banks that accept deposits and provide bank loans to households and firms. The money supply, \( M \), in the model thus stands for “broad money”, in the \( M_1 \) or \( M_2 \) sense, say, thereby including bank created money in addition to currency in circulation, cf. Chapter 16. The banks offer bank loans with an interest rate equal to the bond rate.\(^{10}\) All interest-bearing assets are considered perfect substitutes from the point of view of the lender as well as the borrower.\(^{11}\) At the start of the period agents are able to allocate their portfolio between money and the interest-bearing asset.

The demand for money is given by

\[
M^d = P \cdot (L(Y, i) + \varepsilon_L), \quad L_Y > 0, \; L_i < 0, \tag{19.23}
\]

where \( P \) is the output price level (the GDP deflator) and \( i \) is the short-term nominal interest rate. Ignoring for a moment the term \( \varepsilon_L \), real money demand

\(^{10}\) Or at least with a one-to-one relationship to this rate.

\(^{11}\) To simplify, the model assumes that none of the components in the money supply earn interest. In practice even checkable deposits in the banks often earn a small nominal interest, but this is ignored.

C. Groth, Lecture notes in macroeconomics, (mimeo) 2011
19.4. The static IS-LM model

is given by the function \( L(Y, i) \), sometimes called the \textit{liquidity preference function}. The positive sign of the first partial derivative appearing in (19.23) reflects that the output level, \( Y \), is an approximate statistic (a “proxy”) for the flow of transactions for which money is needed. The negative sign of the second partial derivative in (19.23) reflects that the interest rate, \( i \), is the opportunity cost of holding money instead of interest-bearing bonds. The term \( \varepsilon_L \) in (19.23) is a stochastic shift parameter summarizing the role of unspecified exogenous variables that may matter for real money demand.

The part of financial wealth not held in the form of money is held in the form of interest-bearing one-period bonds. Such a bond gives a payoff equal to 1 unit of account at the end of the period. Let the market price of the bond at the beginning of the period be \( Q \) units of account. The implicit nominal interest rate, \( i \), is then the solution to

\[
(1 + i)^{-1} = Q \quad \text{i.e.,} \quad i = (1 - Q)/Q. \tag{19.24}
\]

There is a definitional link between the nominal interest rate and the expected short-term real interest rate, \( r^e \). In a \textit{continuous}-time setting we would have \( r^e = i - \pi^e_+ \) with \( i \) as the instantaneous nominal interest rate (with continuous compounding) and \( \pi^e_+ \) (\( \equiv \hat{P}/P \)) as the (forward-looking) instantaneous inflation rate, the superscript \( e \) indicating expected value.

In a \textit{discrete}-time \textit{monetary} short-run model, as considered here, the appropriate way of defining \( r^e \) is more involved. The holding of money is motivated by the need (or at least convenience) of ready liquidity to carry out expected as well as unexpected spending in the near future. To perform this role, money must be held in advance, that is, at the beginning of the (short) period in which the purchases are to be made (“cash in advance”). If the price of a good is \( P \) euro to be paid at the end of the period and you have to hold this money already from the beginning of the period, you effectively pay \( P + iP \) for the good, namely the purchase price, \( P \), plus the opportunity cost, \( iP \). Postponing the purchase one period thus gives a saving equal to \( P + iP \). The price of the good next period is \( P_{+1} \) which, with cash in advance, must be held already from the beginning of that period. So the effective real gross rate of return obtained by postponing the purchase one period is

\[
1 + r = (1 + i)P \frac{1}{P_{+1}} = \frac{1 + i}{1 + \pi_{+1}},
\]

where \( \pi_{+1} \equiv (P_{+1} - P)/P \) is the inflation rate from the current to the next period. As seen from the current period, \( P_{+1} \) and \( \pi_{+1} \) are generally not

\[2\text{In continuous time with compound interest, } Q = e^{-i}.\]

C. Groth, Lecture notes in macroeconomics, (mimeo) 2011
known. Hence we write

\[ r^e = \frac{1 + i}{1 + \pi^e_{t+1}} - 1 \approx i - \pi^e_{t+1}, \]  

where the approximation is valid for “small” \( i \) and \( \pi^e_{t+1} \).

**Keynesian equilibrium**

The static IS-LM model assumes clearing in both the output and the money market:

\[
\begin{align*}
Y &= D(Y, i - \pi^e_{t+1}, \tau) + G + \epsilon_D, \quad \text{(IS)} \\
\frac{M}{P} &= L(Y, i) + \epsilon_L, \quad \text{(LM)}
\end{align*}
\]

where, for simplicity, we have used the approximation in (19.25), and where \( M \) is the available money stock at the beginning of the period. In reality the central bank has direct control only over the monetary base. Yet the traditional understanding of the model is that through this, the central bank has also full control over \( M \). Thus, with \( M \) given by monetary policy, the interpretation of the equations (IS) and (LM) is that output (a quantity) and the nominal interest rate quickly adjust so as to clear the output and money markets.

Should we not also consider clearing in the market for bonds? We do not have to, because the balance sheet constraint guarantees that clearing in the money market implies clearing in the bond market — and vice versa. To see this, let \( W \) denote the nominal financial wealth of the non-bank public and let \( x \) denote the number of one-period bonds (government bonds and corporate bonds) available to the non-bank public after subtraction of its bank loans. Then \( M + Qx = W \). With \( x^d \) denoting the demanded quantity of bonds, we have \( M^d + Qx^d = W \). This is called the balance sheet constraint. Subtracting the first from the second of these two equations gives

\[ M^d - M + Q(x^d - x) = 0. \]  

(19.26)

Given \( Q > 0 \), it follows that if and only if \( M^d = M \), then \( x^d = x \). That is, clearing in one of the asset markets implies clearing in the other. So it is enough to consider one of these two markets explicitly. Usually the money market is considered.

The IS equation postulates clearing in a flow market: so much output per time unit matches the demand per time unit for this output. In contrast, the LM equation postulates clearing in a stock market: so much liquidity
demand matches the available money stock at a given point in time, here
the start of the period. At the theoretical level the period length is thus
understood to be short. This is somewhat at odds with the period length of
data for aggregate output, consumption, and investment, usually a year or
at least a quarter of a year. So in econometric analyses, instead of letting \( M \)
and \( i \), for which we have data on a daily and even shorter basis, sometimes
the average money stock and interest rate over a year or a quarter of a year
is used.

The equations (IS) and (LM) constitute the traditional IS-LM model. The variables \( P, \pi^e_{t+1}, \tau, G, \varepsilon_D, \varepsilon_L \), and, in the traditional interpretation, \( M \),
are exogenous. Although the current price level, \( P \), can rightly be seen as
predetermined and maintained through the period, the price level \( P_{t+1} \) set
for the next period would presumably to some extent respond to current
events (as a Phillips curve would tell). So expected inflation, \( \pi^e_{t+1} \) ought
to be endogenous. It is therefore a deficiency of the model that \( \pi^e_{t+1} \) is treated
as exogenous. Yet this may give an acceptable approximation as long as the
sensitivity of expected inflation to current events is small.

Below we analyze the functioning of the described economy under three
alternative monetary policies. We first consider the traditional case where
the central bank is assumed to maintain the money supply at a given target
level. Next we consider the case where the central bank, through open market
operations, maintains \( i \) at a certain target level. The third monetary policy
to be considered is an interest rate rule where both \( i \) and \( M \) are endogenous.
We assume throughout that the choice of monetary policy has to be decided
before the shocks, \( \varepsilon_D \) and \( \varepsilon_L \), are known.

19.4.2 Alternative monetary policies

Money stock as target

Here the central bank maintains the money supply at a certain target level,
\( M \). In addition to the output level, the interest rate is endogenous. The determination of the endogenous variables, \( Y \) and \( i \), is conveniently illustrated by
an IS-LM diagram as in Fig. 19.5. The IS curve is the locus of combinations
of \( Y \) and \( i \) that are consistent with clearing in the output market, i.e., consist-
sent with the equation (IS). Since \( D_r < 0, D(\cdot) \) is a monotonous function
of \( i \). Consequently, (IS) determines \( i \) as an implicit function of \( Y, \pi^e_{t+1}, \tau, G, \varepsilon_D \),
and \( \varepsilon_D \):

\[
i = IS(Y, \pi^e_{t+1}, \tau, G, \varepsilon_D).
\]

C. Groth, Lecture notes in macroeconomics, (mimeo) 2011
CHAPTER 19. THE THEORY OF EFFECTIVE DEMAND

Figure 19.5: The IS-LM cross when $M$ is exogenous (a case with equilibrium output below the natural level, $Y_n$).

The partial derivatives of this function can be found by taking the differential on both sides of (IS):

$$dY = D_Y dY + D_{\tau^e} (di - d\pi^e_{t+1}) + D_{\pi^e} d\pi + dG + d\varepsilon_D.$$  \hspace{1cm} (19.27)

We find $\partial i / \partial Y_{IS}$ by setting $d\pi^e_{t+1} = dG = d\tau = d\varepsilon_D = 0$ and reordering:

$$\frac{\partial i}{\partial Y_{IS}} = \frac{1 - D_Y}{D_{\tau^e}} < 0,$$  \hspace{1cm} (19.28)

where the sign comes from (19.22).

Similarly, the $LM$ curve is the locus of combinations of $Y$ and $i$ that are consistent with clearing in the money market, i.e., consistent with the equation (LM). Since $L_i < 0$, $L(\cdot)$ is a monotonous function of $i$. Therefore, (LM) determines $i$ as an implicit function of $M/P, Y$, and $\varepsilon_L$:

$$i = i_{LM}(Y, \frac{M}{P}, \varepsilon_L).$$

The partial derivatives can be found by taking the differential on both sides of (LM):

$$d \frac{M}{P} = L_Y dY + L_i di + d\varepsilon_L.$$  \hspace{1cm} (19.29)

We find $\partial i / \partial Y_{LM}$ by setting $d(M/P) = d\varepsilon_L = 0$ and reordering:

$$\frac{\partial i}{\partial Y_{LM}} = \frac{-L_Y}{L_i} > 0.$$  \hspace{1cm} (19.30)

C. Groth, Lecture notes in macroeconomics, (mimeo) 2011
19.4. The static IS-LM model

Fig. 19.5 shows the downward sloping IS curve and the upward sloping LM curve. A solution \((Y, i)\) to the model is unique and we can write \(Y\) and \(i\) as implicit functions of all the exogenous variables:

\[
Y = f\left(\frac{M}{P}, \pi_+^e, \tau, G, \varepsilon_D, \varepsilon_L\right), \quad (19.31)
\]

\[
i = g\left(\frac{M}{P}, \pi_+^e, \tau, G, \varepsilon_D, \varepsilon_L\right). \quad (19.32)
\]

The figure

Comparative statics How do \(Y\) and \(i\) depend on the exogenous variables? A qualitative answer can easily be derived by taking into account in what direction the IS curve or the LM curve in Fig. 19.5 shift in response to each exogenous variable. A quantitative answer is obtained by finding the partial derivatives of the implicit functions \(f(\cdot)\) and \(g(\cdot)\). A convenient method is the following. There are given two equations, (19.27) and (19.29), and two new endogenous variables, the changes \(\Delta Y\) and \(\Delta i\). The changes, \(d\pi_+^e, dG, d\tau, d\varepsilon_D, d(M/P),\) and \(d\varepsilon_L\), in the exogenous variables are our new exogenous variables. The system is simultaneous (not recursive). We first reorder (19.27) and (19.29) so that \(\Delta Y\) and \(\Delta i\) appear on the left-hand side and the differentials of the exogenous variables on the right-hand side of each equation:

\[
(1 - D_Y)\Delta Y - D_r \Delta i = -D_{\pi_+^e} d\pi_+^e + D_\tau d\tau + dG + d\varepsilon_D,
\]

\[
L_Y \Delta Y + L_i \Delta i = d\frac{M}{P} - d\varepsilon_L.
\]

From this linear system we find \(\Delta Y\) and \(\Delta i\) by Cramer’s rule:

\[
dY = \frac{| -D_r d\pi_+^e + D_\tau d\tau + dG + d\varepsilon_D \quad -D_r^e \n\begin{array}{c}
d\frac{M}{P} - d\varepsilon_L \\ L_i
\end{array} |}{\Delta} = L_i (-D_{\pi_+^e} d\pi_+^e + D_\tau d\tau + dG + d\varepsilon_D) + D_r^e (d\frac{M}{P} - d\varepsilon_L), \quad (19.33)
\]

and

\[
di = \frac{| 1 - D_Y \quad -D_r d\pi_+^e + D_\tau d\tau + dG + d\varepsilon_D \n\begin{array}{c}
L_Y \\ d\frac{M}{P} - d\varepsilon_L
\end{array} |}{\Delta} = (1 - D_Y)(d\frac{M}{P} - d\varepsilon_L) - L_Y (-D_{\pi_+^e} d\pi_+^e + D_\tau d\tau + dG + d\varepsilon_D),
\]

C. Groth, Lecture notes in macroeconomics, (mimeo) 2011
where the determinant $\Delta$ is defined by

$$
\Delta = \begin{vmatrix}
1 - D_Y & -D_r \\
L_Y & L_i
\end{vmatrix} = (1 - D_Y)L_i + D_r L_Y < 0. \quad (19.34)
$$

Finally, we get the partial derivatives of $f(\cdot)$ and $g(\cdot)$, respectively, w.r.t. the real money supply, $M/P$, by setting $d\pi_{i+1} = d\tau = dG = d\varepsilon_D = d\varepsilon_L = 0$ and reordering. We get:

$$
\frac{\partial Y}{\partial (\frac{M}{P})} = f_{M/P} = \frac{D_r}{(1 - D_Y)L_i + D_r L_Y} > 0,
$$

$$
\frac{\partial i}{\partial (\frac{M}{P})} = g_{M/P} = \frac{1 - D_Y}{(1 - D_Y)L_i + D_r L_Y} < 0,
$$

in view of (19.22) and (19.34). Such partial derivatives of the endogenous variables of a short-run model w.r.t. an exogenous variable are called short-run multipliers. The short-run effect on $Y$ of a small increase in $M/P$ can be calculated as $dY = (\partial Y/\partial (\frac{M}{P}))d(M/P)$. Thus, the partial derivative acts as a multiplier on the increase, $d(M/P)$, in the exogenous variable.\(^{13}\)

The intuitive interpretation of the signs of these multipliers is the following. The central bank increases the money supply by an open market purchase of bonds held by the private sector. Immediately after this, the supply of money is higher than before and the supply of bonds available to the public is lower. At the initial interest rate there is now excess supply of money and excess demand for bonds. But the attempt of agents to get rid of their excess cash in exchange for more bonds can not succeed in the aggregate because the supplies of bonds and money are given. Instead, what happens is that the price of bonds goes up, that is, the interest rate goes down, cf. (19.24), until the available supplies of money and bonds are willingly held by the agents.

Shifts in the values of the parameters $\varepsilon_D$ and $\varepsilon_L$ may be seen as “disturbances” or “shocks” to the system, coming from unspecified sources outside the system. To see how such “demand shocks” and “liquidity preference shocks”, respectively, affect output under the given monetary policy, we first

\(^{13}\)Instead of using Cramer’s rule, in the present case we could just substitute $di$, as determined from (19.29), into (19.27) and then find $dY$ from this equation. In the next step, the found solution for $dY$ can be inserted into (19.29), which then gives the solution for $di$. However, if $L_i$ were a function that could become nil, this procedure might invite a temptation to rule this out by assumption. That would imply an unnecessary reduction of the domain of $f(\cdot)$ and $g(\cdot)$. The only truly necessary assumption is that $\Delta \neq 0$ and that is automatically satisfied in the present problem.
set $d\pi_{e+1} = d\tau = dG = d\frac{M}{P} = 0$ in the $dY$ equation (19.33). Then, setting $d\varepsilon_L = 0$ (or $d\varepsilon_D = 0$), we find the partial derivative of $Y$ w.r.t. $\varepsilon_D$ (or $\varepsilon_L$):

$$\frac{\partial Y}{\partial \varepsilon_D} = f_{\varepsilon_D} = \frac{L_i}{(1 - D_Y)L_i + D_r L_Y}$$

(19.35)

$$\frac{\partial Y}{\partial \varepsilon} = f_{\varepsilon} = \frac{-D_r}{(1 - D_Y)L_i + D_r L_Y} < 0.$$  (19.36)

As expected, a positive demand shock is expansionary, while a positive liquidity preference shock is contractionary because it raises the interest rate. Note that $\partial Y/\partial G = \partial Y/\partial \varepsilon_D$ in view of the way $\varepsilon_D$ enters the equation (IS).

How does a higher expected inflation affect $Y$, $i$, and $r^e$? We find

$$\frac{\partial Y}{\partial \pi^e_{+1}} = f_{\pi^e_{+1}} = \frac{-D_r L_i}{(1 - D_Y)L_i + D_r L_Y} > 0,$$

$$\frac{\partial i}{\partial \pi^e_{+1}} = g_{\pi^e_{+1}} = \frac{D_r L_Y}{(1 - D_Y)L_i + D_r L_Y} \in (0, 1),$$

$$\frac{\partial r^e}{\partial \pi^e_{+1}} = \frac{\partial (i - \pi^e_{+1})}{\partial \pi^e_{+1}} = g_{\pi^e_{+1}} - 1 = \frac{- (1 - D_Y)L_i}{(1 - D_Y)L_i + D_r L_Y} \in (-1, 0).$$

(19.37)

A higher expected inflation rate thus leads to a less than one-to-one increase in the nominal interest rate and thereby a smaller expected real interest rate. Only if money demand were independent of the nominal interest rate ($L_i = 0$), as the quantity theory of money claims, would the expected real interest rate not be affected.

The nominal interest rate as target

Here the central bank maintains the nominal interest rate at a certain target level, $i > 0$. This version of the model corresponds better to how central banks operate in practice. The central bank announces a target level of the nominal interest rate and through open-market operations adjusts the monetary base until $M$ is such that the target rate is realized.

Now $i$ is exogenous, while $M$ is endogenous along with $Y$. Instead of the upward-sloping LM curve we get a horizontal line, the $IR$ line (“IR” for interest rate) in Fig. 19.6. The model is now recursive. Since $M$ does not enter the IS equation, $Y$ is given by this equation independently of the LM equation. Indeed, the equation (IS) determines $Y$ as an implicit function

$$Y = h(i - \pi^e_{+1}, \tau, G, \varepsilon_D).$$

(19.38)
Comparative statics By considering the equation (IS) we shall find the partial derivatives of the $h$ function. For example, the partial derivatives w.r.t. the interest rate, public spending, a demand shock, and a liquidity preference shock, respectively, are

$$\frac{\partial Y}{\partial i} = \frac{D_\varphi}{1 - D_Y} < 0,$$

$$\frac{\partial Y}{\partial G} = \frac{\partial Y}{\partial \varepsilon_D} = \frac{1}{1 - D_Y} > 1,$$

$$\frac{\partial Y}{\partial \varepsilon_L} = 0.$$

The last derivative shows that a liquidity preference shock does not decrease output. The shock is immediately counteracted by a change in the money supply in the same direction, so that the interest rate remains unchanged. Thus, the liquidity preference shock is “cushioned” by the monetary policy. On the other hand, a shock to output demand has a larger effect on output than in the case of money stock targeting (compare (19.39) to (19.35)). This is because under money stock targeting a dampening rise in the interest rate was allowed to take place, the so-called financial crowding-out effect. From this kind of analysis, Poole (1970) concluded that:

- a money stock targeting rule is preferable (in the sense of implying less volatility) if most shocks are output demand shocks, while
19.4. The static IS-LM model

Figure 19.7: Given a fixed interest rate, a 45° Keynes diagram displays the equilibrium output level \( r = i - \pi^e_{+1}, \tau, G, \) and \( \varepsilon_D \) given.

- an interest rate targeting rule is preferable if most shocks are liquidity preference shocks.

Since there is no financial crowding-out under interest rate targeting, the production outcome can also be illustrated by a standard 45° Keynes diagram as in Fig. 19.7.

By inserting the solution for \( Y \) from (19.38) into (LM), we find the money supply required to obtain the target interest rate, \( i \), to be

\[
M = P \cdot (L(Y, i) + \varepsilon_L) = P \cdot (L(h(i - \pi^e_{+1}, \tau, G, \varepsilon_D), i) + \varepsilon_L).
\]

By taking the differential on both sides of (LM) we get

\[
dM = P(L_Y dY + L_i di + d\varepsilon_L) + \frac{M}{P} dP. \tag{19.40}
\]
This implies, e.g.,
\[
\frac{\partial M}{\partial h} = P(L_Y \frac{\partial Y}{\partial h} + L_i) = P(L_Y \frac{D_Y}{1 - D_Y} + L_i) < 0,
\]
\[
\frac{\partial M}{\partial \varepsilon_D} = PL_Y \frac{1}{1 - D_Y} > 0,
\]
\[
\frac{\partial M}{\partial \varepsilon_L} = P > 0,
\]
\[
\frac{\partial M}{\partial P} = \frac{M}{P} > 0.
\]

**The balanced budget multiplier** Under the interest rate targeting rule the effect on \( Y \) of a one-unit increase in \( G \) is given by the multiplier \( \frac{\partial Y}{\partial G} = 1/(1 - D_Y) > 1 \), from (19.39). Since the tax parameter \( \tau \) is unchanged, it is understood that the increase in \( G \) is financed by allowing the budget deficit to increase.

What will be the effect on \( Y \), if we instead assume a balanced budget policy? Then the increase in \( G \) is financed by a corresponding increase in \( T \), brought about by the required change of \( \tau \). That is, \( d\tau \) is determined by the requirement
\[
d\tau = d\tau + T'(Y)dY = dG.
\]
We have
\[
dY = \frac{\partial Y}{\partial G}dG + \frac{\partial Y}{\partial \tau}d\tau = \frac{\partial Y}{\partial G}dG + \frac{\partial Y}{\partial \tau}(dG - T'(Y)dY).
\]
By ordering,
\[
dY = \frac{\frac{\partial Y}{\partial \tau} + \frac{\partial Y}{\partial \tau}d\tau}{1 + \frac{\partial Y}{\partial \tau}d\tau}dG.
\]
From (19.27) we find \( \frac{\partial Y}{\partial \tau} = D_Y/(1 - D_Y) < 0 \). Thus, the total effect of a unit increase in \( G \) under a balanced budget policy is
\[
\frac{dY}{dG} = \frac{\frac{\partial Y}{\partial \tau} + \frac{\partial Y}{\partial \tau}d\tau}{1 + \frac{\partial Y}{\partial \tau}d\tau}dG = \frac{1 + D_Y}{1 - D_Y + D\tau T'} = \frac{1 + D\tau}{1 - D_Y + D\tau T'}
\]
\[
= \frac{1 - C_{Y\tau}}{(C_{Y+i} + I_{Y+i})\varphi(Y)} \geq 1,
\]
where we have used (19.22). This is called the balanced budget multiplier. In case \( C_{Y+i} + I_{Y+i} = 0 \), it equals exactly 1. If the interest rate were allowed to increase, as in the case with \( M \) exogenous, then the balanced budget multiplier would be smaller. But with the interest rate targeting rule this financial crowding-out effect is eliminated.

C. Groth, Lecture notes in macroeconomics, (mimeo) 2011
The paradox of thrift  \hspace{1em} An instructive special case of (19.17) is:

\[
\begin{align*}
\delta & = \chi + \theta + \gamma + \Delta \\
\text{with} \quad & \chi > 0, \quad 0 < c_1 < 1, \quad c_2 > 0, \quad 0 < \tau_1 < 1.
\end{align*}
\]

In addition to linearizing the consumption and tax revenue functions we have, for simplicity, assumed a negligible influence of expected future output on consumption and investment. Under the interest rate targeting rule, the equilibrium condition, \( Y = Y^d \), now yields

\[
Y = \frac{c_0 - c_1\tau - c_2(i - \pi_{+1}^{e}) + \bar{I}(i - \pi_{+1}^{e}) + G + \varepsilon_D}{1 - c_1(1 - \tau_1)}.
\]  \hspace{1em} (19.41)

Consider a negative demand shock \((\varepsilon_D < 0)\). This leads to lower production and higher unemployment. Suppose people respond by prudence and attempt to save more, say by reducing \(c_0\) and/or \(c_1\). This worsens the incipient recession and the attempt at increasing saving is defeated. The reason is that the decreased propensity to consume lowers aggregate demand, thereby reducing production. The resulting lower income brings aggregate consumption further down and so on through the renowned Kahn-Keynes “multiplier process”\(^{14}\). Whereas consumption is reduced, aggregate private saving is not increased, in spite of the endeavours at the individual level to increase saving. Indeed, aggregate private saving is

\[
S^p = Y - T - C = Y^d - C - T = I + G + \varepsilon_D - T
\]  \hspace{1em} (19.42)

and remains unchanged because \(I = \bar{I}(i - \pi_{+1}^{e})\) is unchanged for fixed \(i\). What happens is only that income is reduced exactly as much as consumption. This is Keynes’ famous paradox of thrift. It is an example of a fallacy of composition, a term used by philosophers to denote the error of concluding from what is locally valid to what is globally valid. Such inference overlooks the possibility that when many agents act at the same time, the conditions framing each agent’s actions are affected.

Note that by allowing consumption and investment to depend on \(Y_{+1}^{e}\) as before, the paradox of thrift comes through in an even stronger form. In response to a decrease in \(c_0\) and/or \(c_1\), aggregate private saving will go down, because \(I\) on the right-hand side of (19.42) will go down.

\(^{14}\)Keynes (1936, pp. 113 ff.)
A liquidity trap  In case of a large adverse demand shock, the IS curve may be moved so much leftward in the IS-LM diagram that whatever the money supply, output will end up smaller than the natural level, \( Y_n \). Then the economy is in a liquidity trap: conventional monetary policy can not accomplish “full employment”. The phenomenon is illustrated in Fig. 19.8.

The background is that the nominal interest rate has a lower bound, 0. An increase in \( M \) can not bring \( i \) below 0. Agents would prefer holding cash at zero interest rather than short-term bonds at negative interest. That is, equilibrium in the asset markets is then consistent with the “=” in the LM equilibrium being replaced by “\( \geq \)”.

Fiscal policy will be effective, however, by moving the IS curve rightward. Other policy options are considered in the next chapter.

A counter-cyclical interest rate rule

Suppose the central bank conducts stabilization policy by using the counter-cyclical interest rate rule

\[
i = i_0 + i_1 Y, \quad i_1 > 0,
\]

where \( i_0 \) and \( i_1 \) are policy parameters and \( i_0 \), if negative, is not so small that the zero bound for \( i \) becomes topical under “normal circumstances”. This policy rule is “counter-cyclical” in the Keynesian meaning of a policy intended to dampen the deviations of \( Y \) from “normal”. The policy is
counter-cyclical although the interest rate behaves “pro-cyclically” in the terminology of business cycle econometrics. In this terminology a variable is called pro- or counter-cyclical depending on whether its correlation with output is positive or negative, respectively.

If the LM curve in Fig. 19.5 is made linear, that diagram covers the interest rate rule (19.43). The interpretation of the LM curve is now different, however, hence we will rather name it the IRR curve (IRR for interest rate rule). Both $\pi$ and $\pi^c$ are here endogenous. The fixed interest rate target from above is a limiting case of this interest rate rule, namely the case $i_1 = 0$. But by having $i_1 > 0$, the counter-cyclical interest rate rule yields qualitative effects more in line with those of a money targeting rule. If $i_1 > \delta i / \delta Y_{LM}$ from (19.30), the stabilizing response of $i$ to a decrease in $Y$ is even stronger than under the money targeting rule.

**Comparative statics** Inserting (19.43) into (IS) gives

$$Y = D(Y, i_0 + i_1 Y - \pi^c_{+1}, \tau) + G + \varepsilon_D.$$  

By taking the differential on both sides we find

$$\frac{\partial Y}{\partial \varepsilon_D} = \frac{\partial Y}{\partial \varepsilon_L} = \frac{1}{1 - D_Y - D_{r\varepsilon} i_1} \in (0, 1),$$

$$\frac{\partial Y}{\partial i_1} = \frac{D_{r\varepsilon} Y}{1 - D_Y - D_{r\varepsilon} i_1} < 0,$$

$$\frac{\partial Y}{\partial \pi^c_{+1}} = -\frac{D_{r\varepsilon} D_{\varepsilon^c}}{1 - D_Y - D_{r\varepsilon} i_1} > 0,$$

$$\frac{\partial Y}{\partial \varepsilon_L} = 0.$$

We see that all multipliers become $\approx 0$, if the reaction coefficient $i_1$ is large enough. In particular, undesired fluctuations due to demand shocks are damped this way.

The corresponding changes in $i$ are given as $\partial i / \partial x = i_1 \partial Y / \partial x$ for $x = G, \varepsilon_D, i_1, \pi^c_{+1}$, and $\varepsilon_L$, respectively. From (19.40) we find the corresponding changes in $M$ as $\partial M / \partial x = P(L_Y + i_1 L_i) \partial Y / \partial x$ for $x = G, \varepsilon_D, i_1$, and $\pi^c_{+1}$; finally, from (19.40) we have again $\partial M / \partial \varepsilon_L = P > 0$.

### 19.5 Conclusion

The IS-LM framework is based on the presumption that for short-run analysis of effects of demand shocks it is acceptable, as a first approximation, to treat

C. Groth, Lecture notes in macroeconomics, (mimeo) 2011
the nominal price level as an exogenous constant. The menu cost theory is one of the microfoundations provided for this presumption. The idea is that there are fixed costs associated with changing prices. The main theoretical insight of the theory is that even small menu costs can be enough to prevent firms from changing their price. This is because the opportunity cost of not changing price is only of second order, i.e., “small”; this is a reflection of the envelope theorem. So, nominal prices are sticky in the short run. But owing to imperfect competition (price > MC), the effect on aggregate output, employment, and welfare of not changing prices is of first order, i.e., “large”.

Traditionally, the IS-LM model has been seen as only one building block of the more elaborate aggregate supply-aggregate demand (AS-AD) framework of many macroeconomic textbooks. In that framework the IS-LM model describes just the demand side of a larger model where the level of nominal wages is an exogenous constant in the short run, but the price level is endogenous.

In this chapter we have interpreted the IS-LM model in another way, namely as an independent model in its own right, based on the approximation that both nominal wages and prices are fixed in the short run. Here “fixed” means: set in advance by agents operating in imperfectly competitive markets and being hesitant with regard to frequent or large price changes. Given the pre-set wages and prices, output and employment become fully demand-determined. Not prices, but quantities are the equilibrating factors. This is a complete upside-down compared with the long-run theory of the previous chapters where output and employment were supply-driven — with absolute and relative prices as the equilibrating factors.

When aggregate production is demand determined, the level of production and employment is significantly sensitive to fiscal and monetary policy.

19.6 Bibliographic notes

The IS-LM model was constructed by Hicks (1937) in an attempt to summarize the analytical content of Keynes’ General Theory of Employment, Interest and Money (1936). Notwithstanding the questioning of the IS-LM model’s achievements as interpretation of “what Keynes really meant” (see, e.g., Leijonhufvud 1968), the model has remained a cornerstone of mainstream short-run macroeconomics. The demand side of the large macroeconometric models which governments, financial institutions, and trade unions use to predict macroeconomic evolution in the near future is built on the IS-LM model. Empirically the IS-LM model does a quite good job (see Gali 1992 and Rudebusch and Svensson 1998). At the theoretical level...
the IS-LM model has been criticized for being ad hoc, not derived from the “primitives” (the behavior of firms and households, given technology, preferences, budget constraints, and market structure). In recent years, however, micro-foundations of more elaborate versions of the IS-LM model have been provided (McCallum and Nelson 1999, Sims 2000, Dubey and Geanakoplos 2003, Walsh 2003, and Woodford 2003). Some of these “modernizations” are considered in later chapters.

To be added: comparison between Keynes (1936) and Keynes (1939).

Balanced budget multiplier: Haavelmo (1945).

19.7 Appendix

ENVELOPE THEOREM FOR AN UNCONSTRAINED MAXIMUM Let \( \phi = \phi(\alpha, \xi) \) be a continuously differentiable function of two variables, of which one, \( \alpha \), is conceived as a parameter and the other, \( \xi \), as a control variable. Let \( \xi(\alpha) \) be a value of \( \xi \) at which \( \frac{\partial \phi}{\partial \xi}(\alpha, \xi(\alpha)) = 0 \), i.e., \( \frac{\partial \phi}{\partial \xi}(\alpha, \xi(\alpha)) = 0 \). Let \( F(\alpha) \equiv f(\alpha, g(\alpha)) \). Provided \( F(\alpha) \) is differentiable,

\[
F'(\alpha) = \frac{\partial f}{\partial \alpha}(\alpha, g(\alpha)),
\]

where \( \partial f/\partial \alpha \) denotes the partial derivative of \( f(\cdot) \) w.r.t. the first argument.

Proof. \( F'(\alpha) = \frac{\partial L}{\partial \alpha}(\alpha, g(\alpha)) + \frac{\partial L}{\partial \xi}(\alpha, g(\alpha))g'(\alpha) = \frac{\partial L}{\partial \alpha}(\alpha, g(\alpha)) = 0 \) by definition of \( g(\alpha) \). \( \square \)

That is, when calculating the total derivative of a function w.r.t. a parameter and evaluating this derivative at an interior maximum w.r.t. a control variable, the envelope theorem allows us to ignore the terms that arise from the chain rule. This is also the case if we calculate the total derivative at an interior minimum.\(^{15}\)

19.8 Exercises

\(^{15}\)For extensions and more rigorous framing of the envelope theorem, see for example Sydsæter et al. (2006).