Chapter 6

Long-run aspects of fiscal policy and public debt

We consider an economy with a government that provides public goods and services and finances its spending by taxation and borrowing. The term fiscal policy refers to policy that involves decisions about the government’s spending and the financing of this spending, be it by taxes or debt issue. The government’s choice concerning the level and composition of its spending and how to finance it, may aim at:

1 affecting resource allocation (deliver public goods that would otherwise not be supplied in a sufficient amount, correct externalities and other markets failures, prevent monopoly inefficiencies, provide social insurance);

2 affecting income distribution, be it (a) within generations and/or (b) between generations;

3 contribute to macroeconomic stabilization (dampening of business cycle fluctuations through aggregate demand policies).

The design of fiscal policy with regard to the aims 1 and 2 at a disaggregate level is a major theme within public economics. Macroeconomics deals with aim 3 and the big-picture aspects of 1 and 2, like policy to enhance economic growth.

In this chapter we address the issue of fiscal sustainability and long-run implications of debt finance. The is relates to one of the conditions that constrain public financing instruments. To see the issue of fiscal sustainability in a broader context, Section 6.1 provides an overview of conditions and factors that constrain public financing instruments. Section 6.2 introduces
the basics of government budgeting and Section 6.3 defines the concepts of government solvency and fiscal sustainability. In Section 6.4 the analytics of debt dynamics is presented. As an example the Stability and Growth Pact of the EMU (the Economic and Monetary Union of the European Union) is discussed. Section 6.5 looks more closely at the link between government solvency and the government’s No-Ponzi-Game condition and intertemporal budget constraint. This is applied in Section 6.6 to a study of the Ricardian equivalence proposition; applying the Diamond OLG framework we address the question: Is Ricardian equivalence likely to be a good approximation to reality? If not, why?

6.1 An overview of government financing issues

Before entering the more specialized sections, it is useful to have a general idea about circumstances that constrain public financing instruments. These circumstances include:

(i) financing by debt issue is constrained by the need to remain solvent and avoid catastrophic debt dynamics;

(ii) financing by taxes is limited by problems arising from:

(a) distortionary supply-side effects of many kinds of taxes;
(b) tax evasion (cf. the rise of the shadow economy, tax havens used by multinationals, etc.).

(iii) time lags (such as recognition lag, decision lag, implementation lag, and effect lag);

(iv) credibility problems due to time-inconsistency;

(v) limitations imposed by political processes, bureaucratic self-interest, and rent seeking.

Point (i) is discussed in detail in the sections below. The remaining points, (ii) – (v) are not addressed specifically in this chapter. They should always be kept in mind, however, when discussing fiscal policy. Hence a few remarks here. First, the points (ii.a) and (ii.b) give rise to what is known as the Laffer curve (after the American economist Arthur Laffer, 1940-). The Laffer curve refers to a hump-shaped relationship between the income tax rate and the

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tax revenue. For simplicity, suppose the tax revenue equals income times a given average tax rate. A 0% tax rate and most likely also a 100% tax rate generate no tax revenue. As the tax rate increases from a low initial level, a rising tax revenue is obtained. But after a certain point some people may begin to work less (in the legal economy), stop reporting all their income, and stop investing. While Laffer was seemingly wrong about where USA was on the curve (see Fullerton 1982), surely there is a point above which tax revenue depends negatively on the tax rate. This notwithstanding, in practice there is no such thing as the Laffer curve. This is because a lot of contingencies are involved. Income taxes are typically progressive, marginal tax rates differ from average tax rates, it matters how the tax revenue is spent by the government, etc.

That time lags, point (iii), are a constraining factor is especially important for macroeconomic stabilization policy aiming at dampening business cycle fluctuations. If these potential lags are ignored, there is a risk that the government intervention comes too late and ends up amplifying the fluctuations instead of dampening them. In particular the monetarists, lead by the American economist Milton Friedman (1912-2006), warned against this risk.

Point (iv) hints at the fact that when outcomes depend on forward-looking expectations in the private sector, governments sometimes face what is known as the time-inconsistency problem. Time-inconsistency refers to the possible temptation of the government to deviate from its previously announced course of action once the private sector has acted. An example: With the purpose of stimulating private saving, the government announces that it will not tax financial wealth. Nevertheless, when financial wealth has reached a certain level, it constitutes a tempting base for taxation and so a tax on wealth might be levied. To the extent the private sector anticipates this, the attempt to stimulate private saving in the first place fails. We return to this kind of problems in other chapters.

Finally, political processes, bureaucratic self-interest, and rent seeking\footnote{Rent seeking refers to attempts to gain by increasing one’s share of existing wealth, instead of trying to create wealth.} may interfere with fiscal policy. This is a theme in the branch of economics called political economy and is outside the scope of this chapter.

Now to the specifics of government budget accounting and debt financing.

6.2 The government budget

We generally perceive the public sector as consisting of the government (at national as well as local level) and a central bank. In macroeconomics the
term “government” is used in a broad sense, encompassing both legislation and administration concerning spending on public consumption and investment, levying taxes, paying transfers and subsidies, and paying interest on government debt. Within certain limits the government has usually delegated the management of the nation’s currency to the central bank, also called the monetary authority. Until further notice, our accounting treats “government budgeting” as covering the public sector as a whole, that is, the consolidated government and central bank. Government bonds held by the central bank are thus excluded from what we call “government debt”. So the terms government debt and public debt are used synonymously.

The basics of government budget accounting cannot be described without including money, nominal prices, and inflation. Elementary aspects of money and inflation will therefore be included in this section. We shall not, however, consider money and inflation in any systematic way until later chapters.

Table 6.1 lists key variables of government budgeting.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>$Y_t$</td>
<td>real GDP</td>
</tr>
<tr>
<td>$C^g_t$</td>
<td>public consumption</td>
</tr>
<tr>
<td>$I^g_t$</td>
<td>public fixed capital investment</td>
</tr>
<tr>
<td>$G_t$</td>
<td>$C^g_t + I^g_t$ real public spending on goods and services</td>
</tr>
<tr>
<td>$X_t$</td>
<td>real transfer payments</td>
</tr>
<tr>
<td>$\bar{T}_t$</td>
<td>real gross tax revenue</td>
</tr>
<tr>
<td>$T_t \equiv \bar{T}_t - X_t$</td>
<td>real net tax revenue</td>
</tr>
<tr>
<td>$M_t$</td>
<td>the monetary base (currency and bank reserves in the central bank)</td>
</tr>
<tr>
<td>$P_t$</td>
<td>price level (in money) for goods and services (the GDP deflator)</td>
</tr>
<tr>
<td>$D_t$</td>
<td>nominal net public debt</td>
</tr>
<tr>
<td>$B_t \equiv \frac{D_t}{\bar{T}_{t-1}}$</td>
<td>real net public debt</td>
</tr>
<tr>
<td>$b_t \equiv \frac{D_t}{Y_t}$</td>
<td>debt-income ratio</td>
</tr>
<tr>
<td>$i_t$</td>
<td>nominal short-term interest rate</td>
</tr>
<tr>
<td>$\Delta x_t$</td>
<td>$= x_t - x_{t-1}$ (where $x$ is some arbitrary variable)</td>
</tr>
<tr>
<td>$\pi_t \equiv \frac{\Delta P_t}{P_{t-1}}$</td>
<td>inflation rate</td>
</tr>
<tr>
<td>$1 + r_t \equiv \frac{P_{t-1}(1+i_t)}{P_t}$</td>
<td>real short-term interest rate</td>
</tr>
</tbody>
</table>

Note that $Y_t, G_t,$ and $T_t$ are quantities defined per period, or more generally, per time unit, and are thus flow variables. On the other hand, $M_t, D_t,$ and $B_t$ are stock variables, that is, quantities defined at a given point in time, here at the beginning of period $t$. We measure $D_t$ and $B_t$ net of
6.2. The government budget

financial claims held by the government. Almost all countries have positive government debt, but in principle $D_t < 0$ is possible. The monetary base, $M_t$, is currency plus fully liquid deposits in the central bank held by the private sector at the beginning of period $t$; $M_t$ is by definition nonnegative.

Ignoring uncertainty and risk of default, the nominal interest rate on government bonds must be the same as that on other interest-bearing assets in the economy. For ease of exposition we imagine that all government bonds are one-period zero-coupon bonds. That is, each government bond promises a payout equal to one unit of account at the end of the period and then the bond expires. If the number of outstanding bonds (the quantity of bonds) in period $t$ is $q_t$, the government debt has face value (value at maturity) equal to $q_t$. Given the interest rate, $i_t$, the market value at the start of period $t$ of this quantity of bonds will be $D_t = q_t/(1 + i_t)$. The nominal expenditure to be made at the end of the period to redeem the outstanding debt can then be written

$$q_t = D_t(1 + i_t).$$

(6.1)

This is the usual way of writing the expenditure to be made, namely as if the government debt were like a given bank loan of size $D_t$ with a variable rate of interest. We should not forget, however, that given the quantity, $q_t$, of the bonds, the value, $D_t$, of the government debt at the issue date depends negatively on $i_t$.

Anyway, the total nominal government expenditure in period $t$ can be written

$$P_t(G_t + X_t) + D_t(1 + i_t).$$

It is common to refer to this expression as expenditure “in period $t$”. Yet, in a discrete time model (with a period length of a year or a quarter corresponding to typical macroeconomic data) one has to imagine that the payment for goods and services delivered in the period occurs either at the beginning or the end of the period. We follow the latter interpretation and so the nominal price level $P_t$ for period-$t$ goods and services refers to payment occurring at the end of period $t$. As an implication, the real value, $B_t$, of government debt at the beginning of period $t$ (end of period $t-1$) is $D_t/P_{t-1}$. This may look a little awkward but is nevertheless entirely meaningful. Indeed, $D_t$ is a stock of liabilities at the beginning of period $t$ while $P_{t-1}$ is a price referring to a flow paid for at the end of period $t-1$ which is essentially the same point in time as the beginning of period $t$. Anyway, whatever timing convention is chosen, some kind of awkwardness will always arise in discrete time analysis.

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2 If $D_t < 0$, the government has positive net financial claims on the private sector and earns interest on these claims — which is then an additional source of government revenue besides taxation.

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This is because the discrete time approach artificially treats the continuous flow of time as a sequence of discrete points in time.\(^3\)

The government expenditure is financed by a combination of taxes, bonds issue, and change in the monetary base:

\[
P_t \bar{T}_t + D_{t+1} + \Delta M_{t+1} = P_t(G_t + X_t) + D_t(1 + i_t).
\]

(6.2)

By rearranging we have

\[
\Delta D_{t+1} + \Delta M_{t+1} = P_t(G_t + X_t - \bar{T}_t) + i_tD_t.
\]

(6.3)

In standard government budget accounting the nominal \textit{government budget deficit}, \(GBD\), is defined as the excess of total government spending over government revenue, \(P\bar{T}\). That is, according to this definition the right-hand side of (6.3) is the nominal budget deficit in period \(t\), \(GBD_t\). The first term on the right-hand side, \(P_t(G_t + X_t - \bar{T}_t)\), is named the \textit{primary budget deficit} (non-interest spending less taxes). The second term, \(i_tD_t\), is called the \textit{debt service}. Similarly, \(P_t(\bar{T}_t - X_t - G_t)\) is called the \textit{primary budget surplus}. Note that negative values of “deficits” and “surpluses” represent positive values of “surpluses” and “deficits”, respectively.

We immediately see that this budget accounting is different from “regular” budgeting principles. Private companies, for instance, typically have separate capital and operating budgets. In contrast, the budget deficit defined above treats that part of \(G\) which represents government \textit{net investment} parallel to government consumption. Government net investment is attributed as a \textit{cost} in a single year’s account; according to “normal” budgeting principles it is only the \textit{depreciation} on the public capital that should figure as a cost. Likewise the above budget accounting does not consider that a part of \(D\) (or perhaps more than \(D\)) is backed by the value of public physical capital. Similarly, if the government sells a physical asset to the private sector, it will appear as a reduction of the government deficit while in reality it is merely a conversion of an asset from a physical form to a financial form. That is, the cost and asset aspects of government net investment are not properly dealt with in the standard public accounting.\(^4\)

Yet, in our elementary treatment below we will stick to the traditional vocabulary. Where this might create logical problems (as it almost inevitably will do), it helps to imagine that:

\(^3\)This kind of problems is avoided when government budgeting is formulated in continuous time, cf. Chapter 13.

\(^4\)In Chapter 13 we consider a government accounting framework with separate capital and operating budgets.

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6.2. The government budget

(a) all of $G$ is public consumption, i.e., $G_t = C_t^g$ for all $t$.

(b) there is no public physical capital.

An additional anomaly relates to what the accounting usually includes under “public consumption”. A sizeable part of this is in fact investment in an economic sense: expenses on education, research, and health. To avoid confusion we shall treat $C_t^g$ as including only public “consumption” in the “true” sense of the term.

Now, from (6.3) and the definition $\tilde{\mathbb{T}}_t \equiv \tilde{\mathbb{D}}_t - \mathbb{P}_t$ (net tax revenue) follows that real government debt at the beginning of period $t + 1$ is:

$$B_{t+1} \equiv \frac{D_{t+1}}{P_t} = G_t + X_t - \tilde{\mathbb{T}}_t + (1 + i_t) \frac{D_t}{P_t} - \frac{\Delta M_{t+1}}{P_t} = G_t - T_t + \frac{1 + i_t}{1 + \pi_t} B_t - \frac{\Delta M_{t+1}}{P_t} \equiv (1 + r_t) B_t + G_t - T_t - \frac{\Delta M_{t+1}}{P_t}. $$

The last term, $\Delta M_{t+1}/P_t$, in this expression is seigniorage, i.e., public sector revenue obtained by issuing base money.

Suppose real output grows at the constant rate $g_Y$ so that $Y_{t+1} = (1 + g_Y)Y_t$. Then the debt-income ratio can be written

$$b_{t+1} \equiv \frac{B_{t+1}}{Y_{t+1}} = \frac{1 + r_t}{1 + g_Y} b_t + \frac{G_t - T_t}{(1 + g_Y) Y_t} - \frac{\Delta M_{t+1}}{P_t (1 + g_Y) Y_t}. $$

The last term here is the (growth-corrected) seigniorage-income ratio,

$$\frac{\Delta M_{t+1}}{P_t (1 + g_Y) Y_t} = (1 + g_Y)^{-1} \frac{\Delta M_{t+1}}{M_t} \frac{M_t}{P_t Y_t}. $$

If in the long run the base money growth rate, $\Delta M_{t+1}/M_t$, and the nominal interest rate (i.e., the opportunity cost of holding money, cf. Chapter 16) are constant, then the velocity of money and its inverse, the money-income ratio, $M_t/(P_t Y_t)$, are also likely to be more or less constant. So is, therefore, the seigniorage-income ratio. For more developed countries this ratio is generally a small number.\footnote{In Denmark seigniorage was around 0.2 per cent of GDP during the 1990s (Kvar- talsoversigt 4. kvartal 2000, Danmarks Nationalbank).} For some of the emerging economies that have poor institutions for collection of taxes it is considerably higher; and in situations of budget crisis and hyperinflation it may be sizeable (see Montiel, 2003).

Generally budget deficits are financed primarily by debt creation but also to some extent by money creation, as envisioned in the above equations.

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CHAPTER 6. LONG-RUN ASPECTS OF FISCAL POLICY AND PUBLIC DEBT

However, from now on we will in the main ignore the seigniorage term in (6.4).\footnote{The existing fiscal-financial framework for the Eurozone countries officially excludes the European Central Bank. Yet, for the EMU area as a whole, seigniorage is, of course, income, albeit small, for the aggregate consolidated public sector. The seigniorage is every year divided by the national central banks of the Eurozone countries and the central banks then transfer their share to the national treasuries.}

We thus proceed with the simple government accounting equation:

\[ B_{t+1} - B_t = r_t B_t + G_t - T_t, \quad \text{(DGBC)} \]

where the right-hand side is the real budget deficit. This equation is in macroeconomics often called the dynamic government budget constraint (or DGBK for short) although it is in itself just an identity conditional on \( \Delta M = 0 \). It just says that if the real budget deficit is positive and there is no financing by money creation, then the real public debt grows. It comes closer to being a constraint when combined with the requirement that the government stays solvent.

6.3 Government solvency and fiscal sustainability

To be solvent means being able to meet the financial commitments as they fall due. In practice this concept is closely related to the government’s No-Ponzi-Game condition and intertemporal budget constraint (to which we return below), but at the theoretical level it is more general.

We may view the public sector as an infinitely-lived agent in the sense that there is no last date where all public debt has to be repaid. Nevertheless, as we shall see, there tends to be stringent constraints on government debt creation in the long run.

6.3.1 The critical role of the growth-corrected interest rate

Very much depends on whether the real interest rate in the long-run is higher than the growth rate of GDP or not. To see this, suppose the country considered has positive government debt at time 0 and that the government levies taxes equal to its non-interest spending:

\[ \tilde{T}_t = G_t + X_t \quad \text{or} \quad T_t = \tilde{T}_t - X_t = G_t \quad \text{for all} \ t \geq 0. \quad (6.5) \]
This means that taxes cover only the primary expenses while interest payments (and debt repayments when necessary) are financed by issuing new debt. That is, the government attempts a permanent roll-over of the debt including the interest due for payment. This implies that the debt grows at the rate \( r_t \) according to (DGBC).

Assuming, for simplicity, that \( r_t = r \) (a constant), the law of motion for the debt-income ratio is

\[
b_{t+1} = \frac{B_{t+1}}{Y_{t+1}} = \frac{1 + r}{1 + g_Y} \frac{B_t}{Y_t} = \frac{1 + r}{1 + g_Y} b_t, \quad b_0 > 0,
\]

where \( b_0 \) is positive and assumed historically given and we maintained the assumption of a constant output growth rate, \( g_Y \). The solution to this linear difference equation is \( b_t = b_0 \left( \frac{1+r}{1+g_Y} \right)^t \), where we consider both \( r \) and \( g_Y \) as exogenous. We see that the growth-corrected interest rate, \( 1 + \frac{r}{1+g_Y} - 1 \approx r - g_Y \) (for \( g_Y \) and \( r \) “small”) plays a key role. There are two contrasting cases to discuss:

**Case I:** \( r > g_Y \). In this case, \( b_t \to \infty \) for \( t \to \infty \). But this is not a feasible path because, due to compound interest, the debt grows so large in the long run that the government will be unable to find buyers for all the debt. Imagine for example an economy described by the Diamond OLG model. Then the buyers of the debt are the young who place part of their saving in government bonds. But if the stock of these bonds grows at a higher rate than income, the saving of the young cannot in the long run keep track with the fast growing government debt. The private sector sooner or later understands that bankruptcy is threatening and nobody will buy government bonds except at a low price, which means a high interest rate. The high interest rate only aggravates the problem. That is, the fiscal policy (6.5) breaks down. Either the government defaults on the debt or \( \Delta T \) must be increased or \( \Delta T \) decreased (or both) until the growth rate of the debt is no longer higher than \( g_Y \).

If the debt is denominated in the domestic currency, as is usual, and the government directly controls the central bank, then an alternative way out is of course a shift to money financing of the budget deficit, that is, seigniorage. In a situation with full capacity utilization this leads to higher and higher inflation and thus the real value of the debt is eroded. Although the interest payments on the debt are hereby facilitated for a while, this policy has its own costs namely the economic and social costs generated by high inflation, possibly hyperinflation. And this route comes to a dead end if seigniorage reaches the backward-bending part of the “seigniorage Laffer curve”.\(^7\)

\(^7\)See Chapter 18.
Figure 6.1: Real short-term interest rate and annual growth rate of real GDP in Denmark and the US since 1875. The real short-term interest rate is calculated as the money market rate minus the contemporaneous rate of consumer price inflation. Source: Abildgren (2005) and Maddison (2003).

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6.3. Government solvency and fiscal sustainability

Case II: \( r \leq g_Y \). If \( r = g_Y \), we get \( b_t = b_0 \) for all \( t \geq 0 \). Since the debt, rising at the rate \( r \), does not in the long run increase faster than national income, the government has no problem finding buyers of its newly issued debt. Thereby the government is able to finance its interest payments — it stays solvent. The growing debt is passed on to ever new generations with higher income and saving and the debt roll-over implied by (6.5) can continue forever. If \( r < g_Y \), we get \( b_t \to 0 \) for \( t \to \infty \), and the same conclusion holds a fortiori.

The government can thus pursue a permanent debt roll-over policy as implied by (6.5) and still remain solvent if in the long run the interest rate is not higher than the growth rate of the economy. But in the opposite case, permanent debt roll-over is not possible and sooner or later at least part of the interest payments must be tax financed.

Which of the two cases is relevant for the real world? The answer is not obvious if by the “interest rate” is meant the risk-free interest rate. Fig. 6.1 shows for Denmark (upper panel) and the US (lower panel) the time paths of the real short-term interest rate and the GDP growth rate, both on an annual basis. Overall the levels of the two are more or less the same, although on average the interest rate is in Denmark slightly higher but in the US somewhat lower than the growth rate.

Nevertheless, it is generally believed that there is good reason for paying attention to the case \( r > g_Y \), also for a country like the US. This is because we live in a world of uncertainty and imperfect credit markets (lack of trust), an aspect the above line of reasoning has not incorporated. Indeed, the prudent debt policy needed in the case \( r > g_Y \) can be shown to apply for a larger range of circumstances when uncertainty is present (see Abel et al. 1989, Ball et al. 1998, Blanchard and Weil 2001). As a rough indication we may say that a prudent debt policy is needed when \( r > g_Y - \varepsilon \) for some “small” but positive \( \varepsilon \). On the other hand there is a different feature which draws the matter somewhat in the opposite direction. This is the possibility that a tax, \( \tau \in (0,1) \), on interest income is in force so that the net interest rate on the government debt is \((1 - \tau)r\) rather than \( r \).

6.3.2 Sustainable fiscal policy

The concept of sustainable fiscal policy is closely related to the concept of government solvency. As already noted, to be solvent means being able to meet the financial commitments as they fall due. A given fiscal policy is called sustainable if by applying its spending and tax rules forever, the government stays solvent.

To be specific, suppose \( G_t \) and \( T_t \) are determined by fiscal policy rules
CHAPTER 6. LONG-RUN ASPECTS OF FISCAL POLICY
AND PUBLIC DEBT

represented by the functions

\[ G_t = G(x_{1t}, \ldots, x_{nt}, t), \quad \text{and} \quad T_t = T(x_{1t}, \ldots, x_{nt}, t), \]

where \( t = 0, 1, 2, \ldots \), and \( x_{1t}, \ldots, x_{nt} \) are key macroeconomic and demographic variables (like net national income, old-age dependency ratio, rate of unemployment, extraction of natural resources, say oil in the ground, etc.). In this way a given fiscal policy is characterized by the rules \( G(\cdot) \) and \( T(\cdot) \). Suppose further that we have an economic model, \( M \), of how the economy functions.

**DEFINITION 1** Let the current period be period 0 and let the public debt at the beginning of period 0 be given. Then, given a forecast of the evolution of the demographic and foreign economic environment in the future and given the economic model \( M \), the fiscal policy \( (G(\cdot), T(\cdot)) \) is **sustainable** relative to this model if the forecast calculated on the basis of \( M \) is that the government stays solvent under this policy. The fiscal policy \( (G(\cdot), T(\cdot)) \) is called **unsustainable**, if it is not sustainable.

A given fiscal policy is thus sustainable relative to a given model only if the fiscal policy rules imply a stable debt-income ratio according to the model. The terms “sustainable”/“unsustainable” convey the intuitive meaning. It is all about the question: can the current tax and spending rules continue forever?

This definition of fiscal sustainability is silent about the presence of uncertainty. Without going into detail about the difficult uncertainty issue, we may elaborate a little on the definition by letting “stays solvent” be reformulated as “stays solvent with 100-\( \varepsilon \) percent probability”, where \( \varepsilon \) is a “small” positive number.

Owing to the increasing pressure on public finances caused by reduced birth rates, increased life expectancy, and a fast-growing demand for medical care, many industrialized countries have for a long time been assessed to be in a situation where their fiscal policy is not sustainable (Elmendorf and Mankiw 1999). The implication is that sooner or later one or more expenditure rules and/or tax rules (in a broad sense) will have to be changed.

Two major kinds of strategies have been suggested. One kind of strategy is the **pre-funding strategy**. The idea is to prevent sharp future tax increases by ensuring a fiscal consolidation prior to the expected future demographic changes. Another strategy (alternative or complementary to the former) is to attempt a gradual increase in the labor force by letting the age limits for retirement and pension increase along with expected lifetime — this is the **indexed retirement strategy**. The first strategy implies that current generations bear a large part of the adjustment cost. In the second strategy the costs are shared by current and future generations in a way more similar to

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the way the benefits in the form of increasing life expectancy are shared. We shall not here go into detail about these matters, but refer the reader to the growing literature about securing fiscal sustainability in the ageing society, see Literature notes.

6.4 Debt arithmetic

A key tool for evaluating fiscal sustainability is debt arithmetic, i.e., the analytics of debt dynamics. The previous section described the important role of the growth-corrected interest rate. Here we will consider the minimum primary budget surplus required for fiscal sustainability in different situations.

6.4.1 The required primary budget surplus

Let $\gamma$ denote the spending-income ratio, $G/Y$, and $\tau$ the net tax-income ratio, $T/Y$. Then from (6.4) with $\Delta M = 0$ follows that the debt-income ratio $b_t \equiv B_t/Y_t$ changes over time according to

$$b_{t+1} = \frac{B_{t+1}}{Y_{t+1}} = \frac{1 + r}{1 + g_Y} b_t + \frac{\gamma - \tau}{1 + g_Y}. \quad (6.6)$$

Suppose, until further notice, that $g_Y$, $r$, $\gamma$, and $\tau$ are constant. There are three cases to consider. Case 1: $r > g_Y$. As emphasized above this case is generally considered the one of most practical relevance. And it is in this case that latent debt instability is present and the government has to pay attention to the danger of runaway debt dynamics. To see this, note that the solution of the linear difference equation (6.6) is

$$b_t = (b_0 - b^*) \left( \frac{1 + r}{1 + g_Y} \right)^t + b^*, \quad \text{where} \quad b^* = \frac{\tau - \gamma}{r - g_Y}. \quad (6.7)$$

Here $b_0$ is historically given. But the steady-state debt-income ratio, $b^*$, depends on fiscal policy. The important feature is that the growth-corrected interest factor is in this case higher than 1 and has the exponent $t$. Therefore, if fiscal policy is such that $b^* < b_0$, the debt-income ratio explodes. The solid curve in the uppermost panel in Fig. 6.2 shows a case where fiscal policy is such that $\tau - \gamma < (r - g_Y)b_0$ whereby we get $b^* < b_0$ when $r > g_Y$, so that the debt-income ratio, $b_t$, grows without bound.

With reference to the net asset position and fiscal stance of the US government, economist and Nobel laureate George Akerlof remarked:

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“It takes some time after running off the cliff before you begin to fall. But the law of gravity works, and that fall is a certainty” (Akerlof 2004, p. 6).

Somewhat surprisingly, perhaps, when \( \rho > \gamma \), the rekindled explosion in the long run even if \( \nu > \theta \), namely if \( \tau - \gamma < (r - g_Y) b_0 \). Debt explosion can even arise if \( b_0 < 0 \), namely if \( \tau - \gamma < (r - g_Y) b_0 < 0 \).

The only way to avoid the snowball effects of compound interest when the growth-corrected interest rate is positive is to ensure a primary budget surplus as a share of GDP, \( \hat{\sigma} \), high enough such that \( \hat{\beta} \geq \beta_0 \). That is, the minimum required primary surplus as a share of GDP, \( \hat{\sigma} \), is

\[
\hat{\sigma} = \frac{T - G}{Y} = (r - g_Y) b_0. 
\]

(6.8)

If by adjusting \( \tau \) and/or \( \gamma \), the government obtains \( \tau - \gamma = \hat{s} \), then \( b^* = b_0 \) whereby \( b_t = b_0 \) for all \( t \geq 0 \) according to (6.7).

Note that \( \hat{s} \) will be larger:

- the higher is the initial level of debt, \( b_0 \); and,
- when \( b_0 > 0 \), the higher is the growth-corrected interest rate, \( r - g_Y \).

For fixed spending-income ratio \( \gamma \), the minimum needed net tax rate is \( \hat{\tau} = \gamma + (r - g_Y) b_0 \). The difference, \( \hat{\tau} - \tau \), indicates the size of the needed adjustment, were it to take place at time 0. A more involved indicator of the sustainability gap, taking the room for manoeuvre into account, is \( (\hat{\tau} - \tau)/(1 - \tau) \).\(^8\) Delaying the adjustment increases the size of the needed policy action, since the debt-income ratio has become higher in the meantime.

Note also that if for some reason (it be economic or political) an indebted government with fiscal problems can not raise the primary surplus as a share of GDP above some threshold value \( \bar{\sigma} \). Then, replacing \( \tau - \gamma \) in (6.8) by \( \bar{\sigma} \), the obtained equation can be interpreted as giving the maximum interest rate consistent with absence of runaway debt dynamics:

\[
\bar{\rho} = \frac{\hat{s}}{b_0} + g_Y. 
\]

(6.9)

Thus, the larger is \( b_0 \) the lower is this critical interest rate, \( \bar{\rho} \). The situation may lead to financial investors becoming worried about default, because they

\(^8\)With a Laffer curve in mind, in this formula one should in principle replace the number 1 by a tax rate estimated to maximize the tax revenue in the country considered. This is easier said than done, however, because as noted in the first section of this chapter there are many uncertainties and contingencies involved in the construction of a Laffer curve.
Figure 6.2: Evolution of the debt-income ratio in the cases $r > g_Y$ (the three upper panels) and $r < g_Y$ (the two lower panels), respectively.

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fear a rise in the actual interest rate, \( r \), whereby \( r - \bar{r} > 0 \). The worrying scenario is that

\[
r - \bar{r} > 0 \implies \hat{s} > \bar{s} \implies b^* < b_0 \implies b_t \text{ takes off.}
\]

That is, if the actual interest rate should rise above the critical interest rate, \( \bar{r} \), the minimum required primary surplus as a share of GDP, \( \hat{s} \), given in (6.8), will exceed \( \bar{s} \), thus leading to \( b^* < b_0 \) and thereby runaway debt dynamics, i.e., default. Moreover, the risk that \( r - \bar{r} \) becomes positive is larger the larger is \( b_0 \), cf. (6.9) The fear that it may happen may be enough to trigger a fall in the market price of government bonds which means a rise in the actual interest rate, \( r \). So financial investors’ fear can be a self-fulfilling expectation.9

As an alternative scenario, suppose that the debt build-up can be – and is – prevented already from time 0 by ensuring that the primary surplus as a share of income, \( \tau - \gamma \), at least equals \( \hat{s} \) so that \( b^* \geq b_0 \). The solid curve in the second-from-the-top panel in Fig. 6.2 illustrates the resulting evolution of the debt-income ratio if \( b^* \) is at the level corresponding to the hatched horizontal line. Presumably, the government would in such a state of affairs relax its fiscal policy after a while in order not to accumulate large government net wealth. Yet, the pre-funding strategy vis-a-vis the fiscal challenge of population ageing (referred to above) is in fact based on accumulating some positive public financial net wealth as a buffer before the substantial effects of population ageing set in. In this context, the higher the growth-corrected interest rate, the shorter the time needed to reach a given positive net wealth position.

Case 2: \( r = g_Y \). In this knife-edge there is still a danger of instability, but less explosive. The formula (6.7) is no longer valid. Instead the solution of (6.6) is \( b_t = b_0 + [(\gamma - \tau)/(1 + g_Y)] t \). Here, a non-negative primary surplus is sufficient to avoid \( b_t \to \infty \) for \( t \to \infty \).

Case 3: \( r < g_Y \). This is the case of stable debt dynamics. The formula (6.7) is again valid, but now implying a non-exploding debt-income ratio even if there is a negative primary budget surplus and initial debt is large. The curve in the second from bottom panel in Fig. 6.2 illustrates such a situation. In spite of \( b^* < b_0 \), with unchanged fiscal policy, \( b \) will converge to \( b^* \). This is so whether \( b^* \) is positive (as in the figure) or negative. In fact, if the growth-corrected interest rate remains negative, permanent debt roll-

---

9Several observers see the events in the South European part of the Eurozone in 2010-2012 as a manifestation of such a process (De Grauwe and Ji, 2013). The process came to a halt when the European Central Bank in September 2012 effectively declared its willingness to act as a “lender of last resort” on a conditional basis.

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over can handle the financing and taxes need never be levied.\textsuperscript{10} Finally, the bottom panel in Fig. 6.2 shows the case where with a large primary deficit ($\gamma - \tau > 0$), the excess of output growth over the interest rate still implies convergence towards a constant debt-income ratio, albeit a high one.

In the above analysis we have simplified by assuming that several variables, including $\gamma$, $\tau$, and $r$, are given constants. The ongoing rise in the dependency ratio, due to a decreased birth rate and rising life expectancy, together with a rising request for medical care is likely to generate upward pressure on $\gamma$ and downward pressure on $\tau$. Thereby a high initial debt-income ratio becomes more challenging.\textsuperscript{11}

On the other hand, as $rB_t$, is income to the private sector, it can, along with the factor income, $Y_t$, be taxed at the average tax rate $\tau$. Then the critical inequality is no longer $r > g_Y$ but $(1 - \tau)r > g_Y$, which is less likely to hold (cf. Exercise 6.?).

### 6.4.2 Debt arithmetic and the Stability and Growth Pact of the EMU

The Maastrict criteria, after the Treaty of Maastrict 1992, for joining the Economic and Monetary Union (EMU) of the EU specified both a government deficit rule and a government debt rule. The first is the rule saying that the nominal budget deficit must not be above 3 percent of nominal GDP. The debt rule says that the debt should not be above 60 percent of GDP. The deficit rule and the debt rule were upheld in the Stability and Growth Pact (SGP) which was implemented in 1997 as one of the “great pillars” of the EMU institutional framework.\textsuperscript{12}

\textsuperscript{10}When the GDP growth rate exceeds the interest rate on government debt, a large debt-income ratio can be brought down quite fast, as witnessed by the evolution of both UK and US government debt in the first three decades after the second world war.

\textsuperscript{11}A sustained government budget deficit may also endogenously raise the interest rate in the economy, a topic to which we return in Chapter 13.

\textsuperscript{12}The other “great pillar” is the European Central Bank (ECB). Notice that some of the EMU member states (Greece, Italy, and Belgium) have had debt-income ratios above 100 percent since the early 1990s. Yet they became full members of the EMU. The 60 percent debt rule in the SGP is to be understood as a long-run ceiling and, by the stock nature of debt, cannot be a here-and-now requirement.

Moreover, the measure of government debt, called the EMU debt, used in the SGP criterion is based on the book value of the financial liabilities rather than the market value. In addition, the EMU debt is more of a gross nature than the standard government net debt measure, corresponding to our $D$. The EMU debt measure allows fewer of the government financial assets to be subtracted from the government financial liabilities in order to reach a net debt measure. In our discussion we ignore these complications.
The two rules (with associated detailed arrangements and contingencies) are meant as discipline devices aiming at “sound budgetary policy”. The declared goal is protection of the European Central Bank (ECB) against political demands on the ECB to loosen monetary policy and “bail out” a member country “too big to fail”. An extreme fiscal crisis in one or more of the euro-zone countries could set in and entail a state of affairs approaching default on government debt. This is in fact what we have seen in southern Europe in the wake of the Great Recession triggered by the financial crisis in 2008. Such a situation is likely to generate open or concealed political pressure on the ECB to curb soaring interest rates and spreading of financial problems through buying government bonds from the country in trouble. The lid on deficit spending was meant as a means to avoid such a state of affairs which might interfere with the ECB’s one and only concern with price stability.

The pact has an exemption clause referring to “exceptional” circumstances. These circumstances are defined as “severe economic recession” the interpretation of which was, by the reform of the SGP in March 2005, changed from an annual fall of real GDP of at least 1-2% to simply “negative growth”. Thus, owing to the international economic crisis that broke out in 2008, the deficit rule was suspended temporarily for many of the EMU countries.

The link between the deficit and the debt rule

Whatever the virtues or vices of the SGP and whatever its future status, let us ask the plain question: what is the logical relationship, if any, between the 3 percent and 60 percent tenets?

To answer this, consider a deficit rule saying that the (total) nominal budget deficit must never be above $\alpha \cdot 100$ percent of nominal GDP. By (6.3) with $\Delta M_{t+1} = 0$ this is equivalent to the requirement

$$D_{t+1} - D_t = GBD_t = i_t D_t + P_t(G_t - T_t) \leq \alpha P_t Y_t.$$  (6.10)

In the Maastrict Treaty as well as the SGP, $\alpha = 0.03$. Here we consider the general case: $\alpha > 0$. To see the implication for the debt-income ratio in the long run, let us first imagine a situation where the deficit ceiling, $\alpha$, is always binding for the economy we look at. Then $D_{t+1} = D_t + \alpha P_t Y_t$ and so

$$b_{t+1} \equiv \frac{B_{t+1}}{Y_{t+1}} = \frac{D_{t+1}}{P_t Y_{t+1}} = \frac{D_t}{(1+\pi)P_{t-1}(1+g_Y)Y_t} + \frac{\alpha}{1+g_Y},$$

assuming constant output growth rate, $g_Y$, and inflation rate $\pi$. This reduces
6.4. Debt arithmetic

to

\[ b_{t+1} = \frac{1}{(1 + \pi)(1 + g_Y)} b_t + \frac{\alpha}{1 + g_Y}. \] (6.11)

Assuming (realistically) that \((1 + \pi)(1 + g_Y) > 1\), this linear difference equation has the stable solution

\[ b_t = (b_0 - b^*) \left( \frac{1}{(1 + \pi)(1 + g_Y)} \right)^t + b^* \to b^* \text{ for } t \to \infty, \] (6.12)

where

\[ b^* = \frac{(1 + \pi)\alpha}{(1 + \pi)(1 + g_Y) - 1}. \]

Consequently, if the deficit rule (6.10) is always binding, the debt-income ratio tends in the long run to be proportional to the deficit bound \(\alpha\); the factor of proportionality is a decreasing function of the long-run growth rate of real GDP and the inflation rate. This result confirms the general tenet that if there is economic growth, perpetual budget deficits need not lead to fiscal problems.

If on the other hand the deficit rule is \textit{not} always binding, then the budget deficit is on average smaller than above so that the debt-income ratio will in the long run be \textit{smaller} than \(b^*\).

The conclusion is the following. With one year as the time unit, suppose the deficit rule is \(\alpha = 0.03\) and that \(g_Y = 0.03\) and \(\pi = 0.02\) (the upper end of the inflation interval aimed at by the ECB). Further, suppose the deficit rule is never violated. Then in the long run the debt-income ratio will be \textit{at most} \(b^* = 1.02 \times 0.03/(1.02 \times 1.03 - 1) \approx 0.60\). This is in agreement with the debt rule of the Maastricht Treaty and the SGP according to which the maximum value allowed for the debt-income ratio is 60%.

Although there is nothing sacred about either of the numbers 0.60 or 0.03, they are at least mutually consistent as long as 0.60 is considered an upper bound for the debt-income ratio. On the other hand, if a debt-income ratio at 60 percent is considered \textit{acceptable} for a given country, then it is the \textit{average} deficit over the business cycle that should be 3 percent, not the deficit ceiling.

Anyway, by (6.12) we see that if \(\alpha\) and \(g_Y\) are given, then the maximum long-run debt-income ratio depends negatively on the rate of inflation. In this way a deficit rule like (6.10) is more restrictive regarding the implied maximum long-run debt income ratio the higher is inflation. The reason inflation has this role is that the growth factor, \(\beta = [(1 + \pi)(1 + g_Y)]^{-1}\), for \(b_t\) in (6.11) depends negatively on the inflation rate so that the debt-income ratio settles down at a lower level, \(\alpha(1 + g_Y)^{-1}(1 - \beta)^{-1}\), the higher is \(\pi\).
The strictness regarding the implied maximum long-run debt income ratio will for given inflation rate, \( \pi \), be independent of both the nominal and real interest rate, however (this follows from the formula for \( \beta \) and the fact that \( (1 + i)(1 + r)^{-1} = 1 + \pi \)).

As to the strictness of the deficit rule (6.10) with regard to the current budget deficit from (6.10) together with (6.3) with \( \Delta M_{t+1} = 0 \), we have

\[
\Delta D_{t+1} \equiv D_{t+1} - D_t = iD_t + P_t(G_t - T_t) = [i(1 + \pi)^{-1}b_t + \gamma - \tau] P_t Y_t \leq \alpha P_t Y_t.
\]

where we have used that \( D_t/(P_t Y_t) \equiv P_{t-1}D_t/(P_t(P_{t-1} Y_t)) \equiv (1 + \pi)^{-1}b_t \). Let \( b_t > 0 \) be given. Then for given \( i \), \( i(1 + \pi)^{-1}b_t \) is lower and the maximum primary deficit, \( \gamma - \tau \), compatible with the deficit rule thereby higher the higher is the inflation rate. On the other hand \( i(1 + \pi)^{-1} = [(1 + r)(1 + \pi) - 1] (1 + \pi)^{-1} = 1 + r - (1 + \pi)^{-1} \) is for given \( r \) higher and the maximum primary deficit, \( \gamma - \tau \), compatible with the deficit rule thereby lower the higher is the inflation rate. Finally, for given inflation rate, the maximum primary deficit, \( \gamma - \tau \), compatible with the deficit rule is lower the higher is the interest rate, whether nominal or real.

The debate about the design of the SGP

An intricate feature of the EMU is that the member countries, although quite different in political and economic structure, are subject to the same one-size-fits-all monetary policy. Facing asymmetric shocks (definéir), individual member countries in need of aggregate demand stimulation in a recession have by joining the EMU renounced on both interest rate policy and currency depreciation and so the only left policy tool for stabilization is fiscal policy.

After the founding of the SGP in 1997, critics have maintained that when considering the need for fiscal stimuli even in a mild recession, a ceiling at 0.03 may be too low, giving too little scope for counter-cyclical fiscal policy (including the free working of the “automatic fiscal stabilizers”). As an economy moves towards recession, the deficit rule may, bizarrely, force the government to tighten fiscal policy although the situation calls for a stimulation of aggregate demand. The pact has therefore sometimes been called the Instability and Depression Pact.

Besides it has been objected that there is no reason to have a flow rule (a deficit rule) on top of the stock rule (the debt rule), since it is the long-run stock that matters for whether the fiscal accounts are worrying. To put it differently, a criterion for a deficit rule should relate the deficit to the trend nominal GDP, which we may denote \((PY)^*\). Such a criterion would imply

\[
\Delta D \leq \alpha (PY)^*.
\]

(6.14)

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Then
\[
\frac{\Delta D}{PY} \leq \frac{\alpha (PY)^*}{PY}
\]

In recessions the ratio \((PY)^*/(PY)\) is high, in booms it is low. This has the advantage of allowing more room for budget deficits when they are needed – without necessarily interfering with the long-run aim of stabilizing government debt below some specified ceiling.

A further step in this direction is a rule directly in terms of the structural or cyclically adjusted budget deficit rather than the actual year-by-year deficit. The cyclically adjusted budget deficit in a given year is defined as the value the deficit would take in case actual output were equal to trend output in that year. Denoting the cyclically adjusted budget deficit \(\Delta D^*\), the rule would be
\[
\frac{\Delta D^*}{(PY)^*} \leq \alpha.
\]

In fact, in its original version the SGP contained an additional rule like that, but in the very strict form of \(\alpha \approx 0\). This requirement was implicit in the directive that the cyclically adjusted budget “should be close to balance or in surplus”. By this bound it is imposed that the debt-income ratio should be close to zero in the long run. Many EMU countries certainly had — and have — larger cyclically adjusted deficits. Taking steps to comply with such a low structural deficit bound could be costly.\(^{13}\) The minor reform of the SGP endorsed in March 2005 allowed more contingencies, also concerning this structural bound. And by the recent reform, the Fiscal Pact of 2012, the lid on the cyclically adjusted deficit was raised to 0.5%, still a quite small number.

At a more general level critics have contended that bureaucratic budget rules imposed on sovereign nations will never be able to do their job unless they encompass incentive compatible elements. Regarding the composition of government expenditure, critics have argued that the SGP pact entails a problematic disincentive to public investment. In this context Blanchard and Giavazzi (2004) maintain that a fiscal rule should be based on a proper accounting of public investment instead of simply ignoring the composition of government expenditure. This issue is taken up in Chapter 13.

A general counterargument raised against the criticisms of the SGP has been that the potential benefits of the proposed alternative rules are more than offset by the costs in terms of reduced simplicity, measurability, and transparency (Schuknecht, 2005).

\(^{13}\)Moreover, to insist on exactly a zero debt-income ratio in the long run has no foundation in public economics or general welfare theory.

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Sovereign debt crisis

We close this section by some remarks on the 2010-12 European sovereign debt crisis. In the first months of 2010 anxiety broke out about the Greek government debt crisis to spill over to Spain, Portugal, Italy, and Ireland, thus widening bond yield spreads in these countries vis-a-vis Germany in the midst of a serious economic recession. Moreover, the solvency of big German banks that were among the prime creditors of Greece was endangered. The major Eurozone governments and the International Monetary Fund (IMF) reached an agreement to help Greece (and indirectly its creditors) with loans and guarantees for loans, conditional on the government of Greece imposing yet another round of fiscal austerity measures. In the wake of these events the European Commission took steps to intensify the monitoring of the fiscal policies of the EMU members.

The “fiscal consolidation” initiatives required by the EU Commission to be carried out in most EU countries in 2011-2013 have by many observers, including the IMF, been judged as having been self-defeating. By constraining aggregate demand the policy has increased unemployment. Thereby, instead of decreasing budget deficits, the policy has decreased the numerator in the debt-income ratio, \( D/(PY) \). Fiscal multipliers are judged to be large “in the 0.9 to 1.7 range since the Great Recession” (IMF, World Economic Outlook, Oct. 2012) because of high unemployment (idle resources), monetary policy maintaining low interest rates, and negative spillover effects through trade linkages when fiscal consolidation is synchronized across countries. “It is ironic that, given that the EU was set up in part to avoid coordination failures in economic policy, it should deliver the exact opposite” (Holland and Portes, 2012).

The whole crisis has pointed to the basic difficulties faced by the Eurozone. In spite of the member countries being economically very different, they are subordinate to the same one-size-fits-all monetary policy. Bureaucratic policy rules and surveillance procedures for the fiscal policy of sovereign nations can hardly replace self-regulation; enforcement mechanisms are bound to be weak. Moreover, abiding by the fiscal rules of the SGP was certainly no assurance of not ending up in a fiscal crisis in the wake of the crisis in the financial sector 2008-2009, as witnessed by Ireland and Spain.
6.5 Solvency and the intertemporal government budget constraint

Up to now we have considered the issue of government solvency in the perspective of the dynamics of the debt-income ratio. It is sometimes useful to view government solvency from another angle, that of the intertemporal budget constraint (GIBC). Under a certain condition given below, the intertemporal budget constraint – and the associated No-Ponzi-Game (NPG) condition – are just as relevant for a government as they are for private agents. We first consider the relationship between solvency and the NPG condition. We concentrate on the government debt measured in real terms.

6.5.1 When is the NPG condition necessary for solvency?

Consider a situation with a constant interest rate \( r > 0 \). The relevant NPG condition is that the present discounted value of the public debt in the far future is not positive, i.e.,

\[
\lim_{t \to \infty} B_t (1 + r)^{-t} \leq 0. \tag{NPG}
\]

This condition says that government debt is not allowed to grow in the long run at a rate above (or just equal to) the interest rate.\(^{14}\) That is, a fiscal policy satisfying the NPG condition rules out a permanent debt roll-over. The designation No-Ponzi-Game condition refers to a guy from Boston, Charles Ponzi, who in the 1920s made a fortune out of an investment scam based on the chain letter principle. The principle is to pay off old investors with money from new investors. Ponzi was sentenced to many years in prison for his transactions; he died poor — and without friends.

Note, however, that since there is no final date, the NPG condition does not say that all debt ultimately has to be repaid or even that the debt has ultimately to be non-increasing. It “only” says that the debtor, here the government, is not allowed to let the debt grow forever at a rate as high as (or higher than) the interest rate. And for instance the U.K. as well as the U.S. governments have had positive debt for centuries.

Let \( \lim_{t \to \infty} Y_{t+1}/Y_t = 1 + g_Y \). We have:

PROPOSITION 1 Assume an initial debt \( B_0 > 0 \). Then:

\(^{14}\)If there is taxation of interest income at the rate \( \tau_r \in (0, 1) \), then the after-tax interest rate, \( (1 - \tau_r)\), is the relevant discount rate.

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(i) if \( r > g_Y \), solvency requires (NPG) to be satisfied;

(ii) if \( r \leq g_Y \), the government can remain solvent without satisfying (NPG).

**Proof.** For the debt-income ratio we have \( b_{t+1} \equiv B_{t+1}/Y_{t+1} \) so that

\[
\lim_{t \to \infty} \frac{b_{t+1}}{b_t} = \lim_{t \to \infty} \frac{B_{t+1}/Y_{t+1}}{B_t/Y_t} = \lim_{t \to \infty} \frac{B_{t+1}/B_t}{Y_{t+1}/Y_t} = \lim_{t \to \infty} \frac{B_{t+1}/B_t}{1+g_Y}.
\]  

(6.15)

Case (i): \( r > g_Y \). If \( \lim_{t \to \infty} B_t \leq 0 \), then (NPG) is trivially satisfied. Assume \( \lim_{t \to \infty} B_t > 0 \). For this situation we prove the statement by contradiction. Suppose (NPG) is not satisfied. Then, \( \lim_{t \to \infty} B_t (1+r)^{-t} > 0 \), implying that \( \lim_{t \to \infty} B_{t+1}/B_t \geq 1 + r \). In view of (6.15) this implies that \( \lim_{t \to \infty} b_{t+1}/b_t \geq (1+r)/(1+g_Y) > 1 \). Thus, \( b_t \to \infty \), which violates solvency. By contradiction, this proves that solvency implies (NPG) in this case.

Case (ii): \( r \leq g_Y \). Consider the permanent debt roll-over policy \( T_t = G_t \) for all \( t \geq 0 \). By (DGBP) of Section 6.2 this policy yields \( B_{t+1}/B_t = 1 + r \); hence, in view of (6.15), \( \lim_{t \to \infty} b_{t+1}/b_t = (1+r)/(1+g_Y) \leq 1 \). The policy consequently implies solvency. On the other hand the solution of the difference equation \( B_{t+1} = (1+r)B_t = B_0(1+r)^t \) is \( B_t = B_0(1+r)^t \). Thus \( B_t(1+r)^{-t} = B_0 > 0 \) for all \( t \), thus violating (NPG). \( \square \)

Hence imposition of the NPG condition on the government relies on the interest rate being in the long run higher than the growth rate of GDP. If instead \( r \leq g_Y \), the government can cut taxes, run a budget deficit, and postpone the tax burden indefinitely. So in that case the government can run a Ponzi Game and still stay solvent. But as alluded to earlier, if uncertainty is added to the picture, matters become more complicated and qualifications to Proposition 1 are needed (Blanchard and Weil, 2001). The prevalent view among macroeconomists is that imposition of the NPG condition on the government is generally warranted.

While in the case \( r > g_Y \), the NPG condition is necessary for solvency, it is not sufficient. Indeed, we could have

\[
1 + g_Y < \lim_{t \to \infty} B_{t+1}/B_t < 1 + r.
\]  

(6.16)

Here (NPG) is satisfied, yet, by (6.15), \( \lim_{t \to \infty} b_{t+1}/b_t > 1 \) so that the debt-income ratio explodes. (Note that we are here dealing with a partial equilibrium analysis, which takes the interest rate as given.)

A way to obtain solvency is for example to impose that the primary budget surplus as a share of GDP should be constant during the debt stabilization. Thus, ignoring short-run differences between \( Y_{t+1}/Y_t \) and \( 1+g_Y \) and between...
6.5. Solvency and the intertemporal government budget constraint

$r_t$ and its long-run value, $r$, the minimum primary surplus as a share of GDP, $\hat{s}$, required to obtain $b_{t+1}/b_t \leq 1$ for all $t \geq 0$ is $\hat{s} = (r - g_Y)b_0$ as in (6.8). This $\hat{s}$ is a measure of the burden that the government debt imposes on the economy. If the fiscal policy steps needed to realize $\hat{s}$ are not taken, the debt-income ratio will grow further, thus worsening the fiscal stance in the future by increasing $\hat{s}$.

6.5.2 Equivalence of NPG and IBC

The condition under which the NPG condition is relevant for solvency is also the condition under which the government’s intertemporal budget constraint is relevant. To show this we let $t$ denote the current period and $t + i$ denote a period in the future. From Section 6.2, debt accumulation is described by

$$B_{t+1} = (1 + r)B_t + G_t + X_t - \bar{T}_t,$$

where $B_t$ is given. (6.17)

The government intertemporal budget constraint (GIBC), as seen from the beginning of period $t$, is the requirement

$$\sum_{i=0}^{\infty} (G_{t+i} + X_{t+i})(1 + r)^{-(i+1)} \leq \sum_{i=0}^{\infty} \bar{T}_{t+i}(1 + r)^{-(i+1)} - B_t.$$ (GIBC)

This says that the present value of current and expected future government spending cannot exceed the government’s net wealth (the present value of current and expected future tax revenue minus existing government debt). By the symbol $\sum_{i=0}^{\infty} x_i$ we mean $\lim_{T \to \infty} \sum_{i=0}^{T} x_i$, assuming this limit exists in the improper interval $[-\infty, +\infty]$.

To see the connection between the dynamic accounting relationship (6.17) and the intertemporal budget constraint, note that by rearranging (6.17) and using forward substitution we get

$$B_t = (1 + r)^{-1}(\bar{T}_t - G_t - X_t) + (1 + r)^{-1}B_{t+1}$$

$$= \sum_{i=0}^{j} (1 + r)^{-(i+1)}(\bar{T}_{t+i} - G_{t+i} - X_{t+i}) + (1 + r)^{-(j+1)}B_{t+j+1}$$

$$\leq \sum_{i=0}^{\infty} (1 + r)^{-(i+1)}(\bar{T}_{t+i} - G_{t+i} - X_{t+i}),$$

if and only if government debt ultimately grows at a rate which is less than $r$, so that

$$\lim_{j \to \infty} (1 + r)^{-(j+1)}B_{t+j+1} \leq 0.$$ (6.19)

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This latter condition is exactly the NPG condition above. Reordering (6.18) gives (GIBC). We have thus shown:

**PROPOSITION 2** Given the book-keeping relation (6.17), then:

(i) (NPG) is satisfied if and only if (GIBC) is satisfied;

(ii) there is strict equality in (NPG) if and only if there is strict equality in (GIBC).

We know from Proposition 1 that in the “normal case” where \( r > g_Y \), (NPG) is needed for government solvency. The message of (i) of Proposition 2 is that then also (GIBC) need be satisfied. So, when \( r > g_Y \), to remain solvent a government has to realistically plan taxation and spending profiles such that the present value of current and future primary budget surpluses matches the current debt. Otherwise debt default threatens.\(^{15}\) In view of the remarks around the inequalities in (6.16), however, satisfying the GIBC is only a necessary condition (if \( r > g_Y \), not in itself a sufficient condition for solvency. A simple condition under which the GIBC is sufficient for solvency is that both \( G_t \) and \( T_t \) are proportional to \( Y_t \), cf. Example 1 below.

On the other hand, if \( r \leq g_Y \), it follows from propositions 1 and 2 together that the government can remain solvent *without* satisfying its intertemporal budget constraint.

Returning to the “normal case” where \( r > g_Y \), it is certainly *not* required that the budget is balanced all the time. The point is “only” that for a given planned expenditure path, a government should plan realistically a stream of future tax revenues the PV of which equals the PV of planned expenditure plus the current debt. If during an economic recession for example, a budget deficit is run so that the public debt rises, then higher taxes than otherwise must be raised in the future.

**EXAMPLE 1** Consider a small open economy facing an exogenous constant real interest rate \( r \). Suppose that at time \( t \) government debt is \( B_t > 0 \). GDP, \( Y_t \), grows at the constant rate \( g_Y \), and \( r > g_Y \). Assume \( G_t = \gamma Y_t \) and \( T_t \equiv \bar{T}_t - X_t = \tau Y_t \), where \( \gamma \) and \( \tau \) are positive constants. What is the minimum size of the primary budget surplus as a share of GDP required for satisfying the government’s intertemporal budget constraint as seen from time \( t \)? Inserting into the formula (6.18), with strict equality, yields \( \sum_{i=0}^{\infty} (1+r)^{-(i+1)} (\tau - \gamma) Y_{t+i} \)

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\(^{15}\)Government debt defaults have their own economic as well as political costs. Yet, they occur now and then. Recent examples include Russia in 1998 and Argentina in 2001-2002. Since 2010, Greece has been on the brink of debt default.
6.6 Ricardian equivalence?

We now turn to the question how budget policy affects resource allocation and intergenerational distribution. The role of budget policy for economic activity within a time horizon corresponding to the business cycle is not our topic here. Our question is about the longer run: does it matter for aggregate consumption and aggregate saving in an economy without cyclical unemployment whether the government finances its current spending by taxes or borrowing?

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6.6.1 Two differing views

There are two opposite answers in the literature to this question. Some macroeconomists answer the question in the negative. This is the debt neutrality view, also called the Ricardian equivalence view. The influential American economist Robert Barro is in this camp. Other macroeconomists answer the question in the positive. This is the debt non-neutrality view or absence of Ricardian equivalence view. The influential French-American economist Olivier Blanchard is in this camp.

The two different views rest on two different models of the economic reality. To spell this out, let us first display the common point of departure for the two different models. The point of departure is a situation where \( r > g_Y \), regarded as the “normal” situation. Then, to remain solvent, the government has to satisfy its no-Ponzi-game condition or, equivalently, its intertemporal budget constraint. This follows from (i) of Proposition 1 and Proposition 2. The debate concentrates on the “ordinary” case where the government does not tax so heavily that public net debt turns to public net financial wealth \((B < 0)\) and as such accumulates in the long run at a speed equal to the interest rate or faster. Excluding this kind of “capitalist government”, strict equality instead of weak inequality will rule in both the no-Ponzi-game condition, \((6.19)\), and the intertemporal budget constraint of the government, \((\text{GIBC})\). Then there is equality also in \((6.18)\), which may thus, with period 0 as the initial period and \(T_t \equiv \bar{T}_t - X_t\), be written

\[
\sum_{t=0}^{\infty} T_t (1 + r)^{-(t+1)} = \sum_{t=0}^{\infty} G_t (1 + r)^{-(t+1)} + B_0. \tag{6.20}
\]

The initial debt, \(B_0\), is historically given. For a given planned time path of \(G_t\), equation (6.20) shows that a tax cut in any period has to be met by an increase in future taxes of the same present discounted value as the tax cut.

Ricardian equivalence

The Ricardian equivalence view is the conception that government debt is neutral in the sense that for a given time path of government spending, aggregate private consumption is unaffected by a temporary tax cut. The temporary tax cut do not make the households feel richer because they know that the ensuing rise in government debt leads to higher taxes in the future. The essential claim is that the timing of taxes does not matter. The name Ricardian equivalence comes from a — seemingly false — association of this view with the early nineteenth-century British economist David Ricardo. It

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is true that Ricardo articulated the possible logic behind debt neutrality. But he suggested several reasons that debt neutrality would not hold in practice and in fact he warned against high public debt levels (Ricardo, 1969, pp. 161-164). Therefore, to cite Elmendorf and Mankiw (1999, p. 1643), it is doubtful whether Ricardo was a Ricardian.

Debt neutrality was rejuvenated, however, by Robert Barro in a paper entitled “Are government bonds net wealth [of the private sector]?”, a question which Barro answered in the negative (Barro 1974). Barro’s debt neutrality view rests on a representative agent model, that is, a model where the household sector is described as consisting of a fixed number of infinitely-lived forward-looking “dynasties”. A change in the timing of (lump-sum) taxes does not change the present value of the infinite stream of taxes imposed on the individual dynasty. A cut in current taxes is offset by the expected higher future taxes. Though current government saving \((T - G - rB)\) goes down, private saving and bequests left to the members of the next generation go up equally much.

More precisely, the logic of the debt neutrality view is as follows. Suppose, for simplicity, that the government waits only 1 period to increase taxes. Then, for each unit of account current taxes are reduced, taxes next period are increased by \((1 + r)\) units of account. The present value as seen from the end of the current period of this future tax increase is \((1 + r)/(1 + r) = 1\). As \(-1 + 1 = 0\), the change in the time profile of taxation will neither make the dynasty feel richer nor poorer. Consequently, its current and planned future consumption will be unaffected. That is, its current saving goes up just as much as its current taxation is reduced. In this way the altruistic parents make sure that the next generation is fully compensated for the higher future taxes. Current private consumption in society is thus unaffected and aggregate saving stays the same.\(^{16}\)

**Absence of Ricardian equivalence**

Olivier Blanchard and others dissociate themselves from such representative agent models because of their coarse demography. Instead attention is drawn to overlapping generations models which lead to a refutation of Ricardian equivalence. The essential point is that those persons who benefit from lower taxes today will at most be a fraction of those who bear the higher tax burden in the future.

Taxes levied at different times are levied at partly different sets of agents, the timing of taxes generally matters. The current tax cut makes current tax

---

\(^{16}\)The complete Barro model is presented in Chapter 7.
payers feel wealthier and this tends to increase their consumption and lead to a decrease in aggregate saving. The present generations benefit and future tax payers (partly future generations) bear the cost in the form of access to less national wealth than otherwise.

The next subsection provides an example showing in detail how a change in the timing of taxes affects aggregate private consumption and saving in an overlapping generations (OLG) framework.

### 6.6.2 A small open OLG economy with a budget deficit

We consider a Diamond-style OLG model of a small open economy (henceforth SOE) with a government sector. As earlier we let $L_t$ denote the size of the young generation and $L_t = L_0(1 + n)^t$, $n \geq 0$.

**Some national accounting for the open economy**

We start with a little national accounting for our SOE. In the notation from Chapter 3 gross national saving is

$$S_t = Y_t - C_t - G_t = Y_t - (c_{1t}L_t + c_{2t}L_{t-1}) - G_t,$$

where $G_t$ is the amount of the public good in period $t$. It is assumed that the production of $G_t$ uses the same technology, and therefore involves the same unit production costs, as the other components of GDP. Since our focus here is not on the distortionary effects of taxation, taxes are assumed to be lump sum, i.e., levied on individuals irrespective of their economic behavior.

With $A_t$ denoting private financial (net) wealth and $B_t$ government (net) debt, *national wealth, $V_t$*, of SOE at the beginning of period $t$ is by definition the sum of private financial wealth and government financial wealth. The latter equals minus the government debt if the government has no physical assets which we assume. Thus,

$$V_t = A_t + (-B_t).$$

(6.22)

With $NFD_t$ denoting (net) foreign debt (also called external debt) at the beginning of period $t$, we also have

$$V_t = K_t - NFD_t,$$

saying, if $NFD_t > 0$, that some of the capital stock is directly or indirectly owned by foreigners. On the other hand, if $NFD_t < 0$, SOE has positive net claims on resources in the rest of the world.
When the young save, they accumulate private financial wealth. The private financial wealth at the start of period $t + 1$ must in our Diamond framework equal the saving by the young in the previous period, i.e.,

$$A_{t+1} = s_t L_t = S_{1t}. \quad (6.23)$$

So, by (6.22) forwarded one period, we have

$$V_{t+1} = A_{t+1} + (-B_{t+1}) = s_t L_t - B_{t+1}. \quad (6.24)$$

*Remark.* The reader might alternatively want to reach this conclusion from the perspective of *national saving*. By definition the increase in *national* wealth equals net *national* saving, $S_t - \delta K_t$, which in turn equals net private saving, $s_t L_t + (-A_t)$, minus net public dissaving. The latter equals the size of the government budget deficit, $GBD$. That is,

$$V_{t+1} - V_t = S_t - \delta K_t = S_{1t} S_{2t} - GBD_t = S_{1t} + (-A_t) - GBD_t = A_{t+1} - A_t - (B_{t+1} - B_t),$$

where the last equality comes from (6.23) and the maintained assumption that budget deficits are fully financed by debt issue. □

Regarding the relations of SOE to international markets we re-introduce the three standard assumptions (from Chapter 5):

(a) There is perfect mobility of goods and financial capital across borders.

(b) There is no uncertainty and domestic and foreign financial claims are perfect substitutes.

(c) There is no labor mobility across borders.

The assumptions (a) and (b) imply real interest rate equality. That is, the real interest rate in our SOE must equal the real interest rate in the world financial market, $r$. And by saying that the SOE is “small” we mean it is small enough to not affect the world market interest rate as well as other world market factors. We imagine that all countries trade one and the same homogeneous good. International trade will then only be *intertemporal* trade, i.e., international borrowing and lending of this good.

We assume that $r$ is constant over time and that $r > n$. 

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CHAPTER 6. LONG-RUN ASPECTS OF FISCAL POLICY
AND PUBLIC DEBT

Technology and preferences

GDP is produced by an aggregate neoclassical production function with CRS:

\[ Y_t = F(K_t, L_t), \]

where \( K_t \) and \( L_t \) are input of capital and labor, respectively. Technological change is ignored. Imposing perfect competition in all markets, markets clear so that \( L_t \) can be interpreted as both employment and labor supply (exogenous). Profit maximization leads to \( f'(k_t) = r + \delta \), where \( k_t \equiv K_t / L_t \), \( f(k_t) \equiv F(k_t, 1) \), and \( \delta \) is a constant capital depreciation rate, \( 0 \leq \delta \leq 1 \). When \( f \) satisfies the condition \( \lim_{k \to 0} f'(k) > r + \delta > \lim_{k \to \infty} f'(k) \), there is always a solution in \( k \) to this equation and it is unique (since \( f'' < 0 \)) and constant over time (as long as \( r \) and \( \delta \) are constant). Thus,

\[ k_t = f'^{-1}(r + \delta) \equiv k, \text{ for all } t. \quad (6.25) \]

The stock of capital, \( K_t \), is determined as \( K_t = kL_t \).

In view of firms’ profit maximization, the equilibrium real wage before tax is

\[ w_t = \frac{\partial Y_t}{\partial L_t} = f(k) - f'(k)k \equiv w, \quad (6.26) \]

a constant. GDP will evolve according to

\[ Y_t = f(k)L_t = f(k)L_0(1 + n)^t = Y_0(1 + n)^t = Y_0(1 + n)^t \]

and thus grow at the same rate as the labor force, i.e., \( g_Y = n \).

Suppose the role of the government sector is to deliver a service, a “public good”. Let \( G_t \) be the size of this service in period \( t \). Think of a non-rival good like “rule of law”, TV-transmitted theatre, or another public service free of charge etc. Suppose

\[ G_t = G_0(1 + n)^t, \]

where \( 0 < G_0 < F(K_0, L_0) \). Without consequences for the qualitative results, we assume a CRRA period utility function \( u(c) = (c^{1-\theta} - 1)/(1 - \theta) \), where \( \theta > 0 \). To keep things simple, the utility of the public good enters additively in lifetime utility so that it does not affect marginal utilities of private consumption. In addition we assume that the public good does not affect productivity in the private sector. There is a lump-sum tax on the young as well as the old in period \( t \), \( \tau_{1t} \) and \( \tau_{2t} \), respectively. Possibly, \( \tau_{1t} \) or \( \tau_{2t} \) is negative, in which case there is a transfer to either the young or the old.
The consumption-saving decision of the young will be the solution to the following problem:

\[
\max \left( \frac{c_{1t}^{1-\theta} - 1}{1-\theta} + v(G_t) + (1 + \rho)^{-1} \left[ \frac{c_{2t+1}^{1-\theta} - 1}{1-\theta} + v(G_{t+1}) \right] \right) \\
\text{s.t.} \\
c_{1t} + s_t = w - \tau_{1t}, \\
c_{2t+1} = (1 + r)s_t - \tau_{2t+1}, \\
c_{1t} \geq 0, c_{1t+1} \geq 0,
\]

where the function \( v(\cdot) \) represents the utility contribution of the public good. The implied Euler equation can be written

\[
\frac{c_{2t+1}}{c_{1t}} = \left( \frac{1 + r}{1 + \rho} \right)^{1/\theta}
\]

Inserting the two budget constraints and ordering gives

\[
s_t = \frac{w - \tau_{1t} + \tau_{2t+1} \left( \frac{1+\rho}{1+r} \right)^{1/\theta}}{1 + (1 + \rho)^{1/\theta}(1 + r)^{(\theta-1)/\theta}}.
\]

Consumption in the first and second period then is

\[
c_{1t} = w - \tau_1 - s_t = z_1 h_t \tag{6.27}
\]

and

\[
c_{2t+1} = z_2 h_t, \tag{6.28}
\]

respectively, where

\[
z_1 = \frac{(1 + \rho)^{1/\theta}(1 + r)^{(\theta-1)/\theta}}{1 + (1 + \rho)^{1/\theta}(1 + r)^{(\theta-1)/\theta}} \in (0, 1),
\]

\[
z_2 = \left( \frac{1 + r}{1 + \rho} \right)^{1/\theta} z_1 = \frac{1 + r}{1 + (1 + \rho)^{1/\theta}(1 + r)^{(\theta-1)/\theta}},
\]

and

\[
h_t \equiv w - \tau_{1t} - \frac{\tau_{2t+1}}{1 + r} \tag{6.29}
\]

represents the “human wealth” of the young, evaluated at the end of period \( t \). This last equation says that \( h_t \) is the after-tax human wealth, that is, the present value of disposable lifetime income; we assume \( \tau_{1t} \) and \( \tau_{2t} \) in every period are such that \( h_t > 0 \). The consumption in both the first and the second period of life is thus proportional to individual human wealth.\(^{17}\)

\(^{17}\)This was to be expected in view of the homothetic life time utility function and the constant interest rate.

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CHAPTER 6. LONG-RUN ASPECTS OF FISCAL POLICY
AND PUBLIC DEBT

The tax revenue in period $t$ is $T_t = \tau_{1t}L_t + \tau_{2t}L_{t-1}$. Suppose $B_0 = 0$ and that in the reference path the budget is and remains balanced for all $t$, i.e., $T_t = G_t$. For simplicity, let the lump-sum taxes be constants, $\tau_1$ and $\tau_2$. Then the tax scheme is some pair $(\tau_1, \tau_2)$ satisfying

$$
\left(\tau_1 + \frac{\tau_2}{1+n} \right) L_0 = G_0.
$$

In view of the tax scheme $(\tau_1, \tau_2)$ being time independent, so are individual human wealth and the individual consumption levels, which we may hence denote $c_1$ and $c_2$. In the reference path aggregate private consumption now grows at the same constant rate as GDP and public consumption, the rate $n$. Indeed,

$$
C_t = c_1L_1 + \frac{c_2}{1+n}L_t = (c_1 + \frac{c_2}{1+n})L_0(1+n)^t = C_0(1+n)^t.
$$

A one-time tax cut

What are the consequences of a one-time cut in taxation by $x > 0$ for every individual, whether young or old? Letting the tax cut occur in period 0, it amounts to creating a budget deficit in period 0 equal to $(L_0 + L_{-1})x$. At the start of period 1 there is thus a debt $B_1 = (L_0 + L_{-1})x$. Since we assumed $r > n = \gamma$, government solvency now requires a rise in the present value of future taxes (as seen from the beginning of period 1) equal to $(L_0 + L_{-1})x$. This may be accomplished by for instance raising the tax on all individuals from period 1 onward by $m$. The required value of $m$ will satisfy

$$
\sum_{t=1}^{\infty} (L_0 + L_{-1})(1+n)^t m(1+r)^{-t} = (L_0 + L_{-1})x.
$$

This gives

$$
m \sum_{t=1}^{\infty} \left(\frac{1+n}{1+r}\right)^t = x, \quad (r > n)
$$

so that

$$
m = \frac{r - n}{1+n}x \equiv \bar{m}.
$$

The higher the growth-corrected interest rate, $r - n$, the higher the needed rise in future taxes.

Let the value of the variables in this alternative fiscal regime be marked with a prime. In period 0 the tax cut unambiguously benefit the old whose increase in consumption equals the saved taxes:

$$
c_{20}' - c_{20} = x > 0.
$$

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The young in period 0 enjoy an increase in after-tax human wealth equal to

\[ h'_0 - h_0 = w - \tau_1 x - \frac{\tau_2 + \bar{m}}{1 + r} - \frac{w - \tau_1}{1 + r} \]

\[ = \left( 1 - \frac{r - n}{(1 + r)(1 + n)} \right) x \quad \text{(by (6.30))} \]

\[ = \frac{1 + (2 + r)n}{(1 + r)(1 + n)} x > 0. \]

But all future generations are worse off, since they do not benefit from the tax relief in period 0, but instead have to pay for this by a reduction in individual after-tax human wealth. Indeed, for \( t \geq 1 \),

\[ h'_t - h_t = w - \tau_1 - \bar{m} - \frac{\tau_2 + \bar{m}}{1 + r} - \frac{w - \tau_1}{1 + r} \]

\[ = - \left( \bar{m} + \frac{\bar{m}}{1 + r} \right) < 0. \]

All things considered, since both the young and the old in period 0 increase their consumption, aggregate consumption rises and aggregate saving (\( \equiv \) national saving) therefore falls, implying less national wealth and less aggregate consumption in the future. Thus Ricardian equivalence fails. Indeed, according to this analysis, budget deficits crowd out private saving and reduce future national wealth.

The lower aggregate saving in period 0 results in higher net foreign debt at the beginning of period 1 than otherwise. Essentially, the reason that the generations 1, 2,..., are worse off than otherwise is the emergence of this debt, which requires interest payments. In a closed economy these future generations would also be worse off, but that would be because the national capital stock at the beginning of period 1 would be smaller than otherwise, in view of the smaller aggregate saving in period 0.

The fundamental point underlined by OLG models is that there is a difference between the public sector’s future tax base, including the resources of individuals yet to be born, and the future tax base emanating from individuals alive today. This may be called the \textit{composition-of-tax-base argument} for a tendency to non-neutrality of shifting the timing of (lump-sum) taxation.

The above analysis is based on the standard assumption in long-run analysis that production occurs under full capacity utilization. What if the economy is in an economic depression with high unemployment due to insufficient aggregate demand? Also in this situation does Barro maintain that a cut in (lump-sum) taxes to stimulate aggregate demand is futile because it
has no real effect. The argument is again that foreseeing the higher taxes needed in the future, people will save more to prepare themselves (or their descendants through higher bequests) for paying the higher taxes in the future.\footnote{See Barro ().} The opposite view is, first, that the composition-of-tax-base argument speaks against this as usual. Second, there is in a depression an additional and quantitatively important factor, namely that not only is consumption affected positively by the tax cut, but thereby aggregate demand and aggregate activity are stimulated and thereby consumption is further stimulated.

Further perspectives

Although the above composition-of-tax-base argument against Ricardian equivalence seems logically convincing, another question is how large quantitative deviations from Ricardian equivalence it can deliver. Taking into account the sizeable life expectancy of the average citizen, it can be shown that the composition-of-tax-base argument alone delivers only modest deviations (.... and Summers, 198?). Additional sources of deviation have been put forward in the literature, a theme to which we return in chapters 13 and 19.

In the real world taxes are not lump sum, but usually distortionary. This fact should, of course, always be kept in mind when discussing practical issues of fiscal policy. Yet, it is not an argument against the possible theoretical validity of the Ricardian equivalence view. This is so because Ricardian equivalence (in the proper sense of the word) claims absence of allocational effects of deficit financing when taxes are lump sum. There is hardly disagreement that the timing of distortionary taxes generally matters.

In the end, whether Ricardian equivalence is a good or bad approximation to reality is an empirical question. Sometimes an observed positive correlation between the government budget deficit and private financial saving is taken as an argument for Ricardian equivalence. That is a mistake, however. A sector’s financial saving is defined as the sector’s income minus its spending on goods and services (in contrast to spending on purely financial assets, “paper assets”). This is the same as the sector’s total saving minus its capital investment. For a closed economy a positive correlation between the government budget deficit and private financial saving is a trivial implication of accounting. Indeed, the closed-economy identity $I = S$ implies $I_p + I_g = S_p + S_g$, where the subindices $p$ and $g$ stand for “private” and “government”, respectively. Thus,

\[(S_p - I_p) + (S_g - I_g) = 0.\]

\footnote{See Barro ()}
6.6. Ricardian equivalence?

This says that private financial saving plus government financial saving must necessarily sum to zero and therefore be negatively correlated. Moreover, the saving of the government equals government income, $T - rB$, minus government consumption, $C_g$, so that the financial saving of the government is

$$S_g - I_g = T - rB - C_g - I_g = T - rB - G = -GBD = GBS,$$

where $GBD$ is the real government budget deficit and $GBS$ the real government budget surplus. So, merely by accounting do we have

$$S_p - I_p = -(S_g - I_g) = GBD.$$

Consequently, there will automatically be a positive correlation between the government budget deficit and private financial saving whether or not Ricardian equivalence holds.

For an open economy we have that the current account surplus, $CAS$, can be written

$$CAS = S - I = S_p + S_g - (I_p + I_g) = (S_p - I_p) + (S_g - I_g).$$

This says that unless the current account surplus moves one-to-one (or more) with government financial saving, there will by mere accounting (i.e., independently of whether Ricardian equivalence holds or not) be a negative relationship between government and private financial saving.

The Ricardian equivalence hypothesis is however the statement that even for an open economy neither aggregate saving nor aggregate investment depends on government saving. In view of the accounting relation

$$CAS = S - I,$$

there should thus, according to the Ricardian equivalence hypothesis, be no relationship between budget deficits and current account deficits. Yet, the simple cross-country regression analysis for 19 OECD countries over the 1981-86 period reported in Obstfeld and Rogoff (1996, p. 144) indicates a positive relationship between budget and current account surpluses. Warning about the omitted variable problem, the authors also consider two historical episodes of drastic shifts in tax policy, namely the U.S. tax cuts in the early 1980s and the large debt-financed transfer program from the western part to the eastern part of unified Germany starting in 1990. These episodes, which mirror a Ricardian experiment of a ceteris paribus tax cut, support the debt non-neutrality view that private saving does not rise one-to-one in response to government dissaving.\(^{19}\) On the other hand, there certainly exists evidence

\(^{19}\)Obstfeld and Rogoff (1996, p. 144-45).

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that consumers sometimes worry about the future fiscal stance of the country; and government spending multipliers tend to be *smaller* in highly indebted countries (see Ilzetzki et al., 2009).

### 6.7 Literature notes

(incomplete)

A reader wanting to go more into detail with the debate about the EMU and the Stability and Growth Pact is referred to the discussions in for example Buiter (2003), Buiter and Grafe (2004), Fogel and Saxena (2004), and Wyplosz (2005).

De Grauwe’s book.