

Introduction to Time Series

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Outline

- (1) **What is a time series?**
- (2) **Important characteristics of time series data.**
- (3) **Data sampling** and stochastic processes.
Cross-sectional data vs. time series data.
- (4) **Stationarity.**
- (5) **Measuring time dependence.**
- (6) Non-stationarity and **transformations to stationarity.**

Time Series Data

- A time series is a set of observations

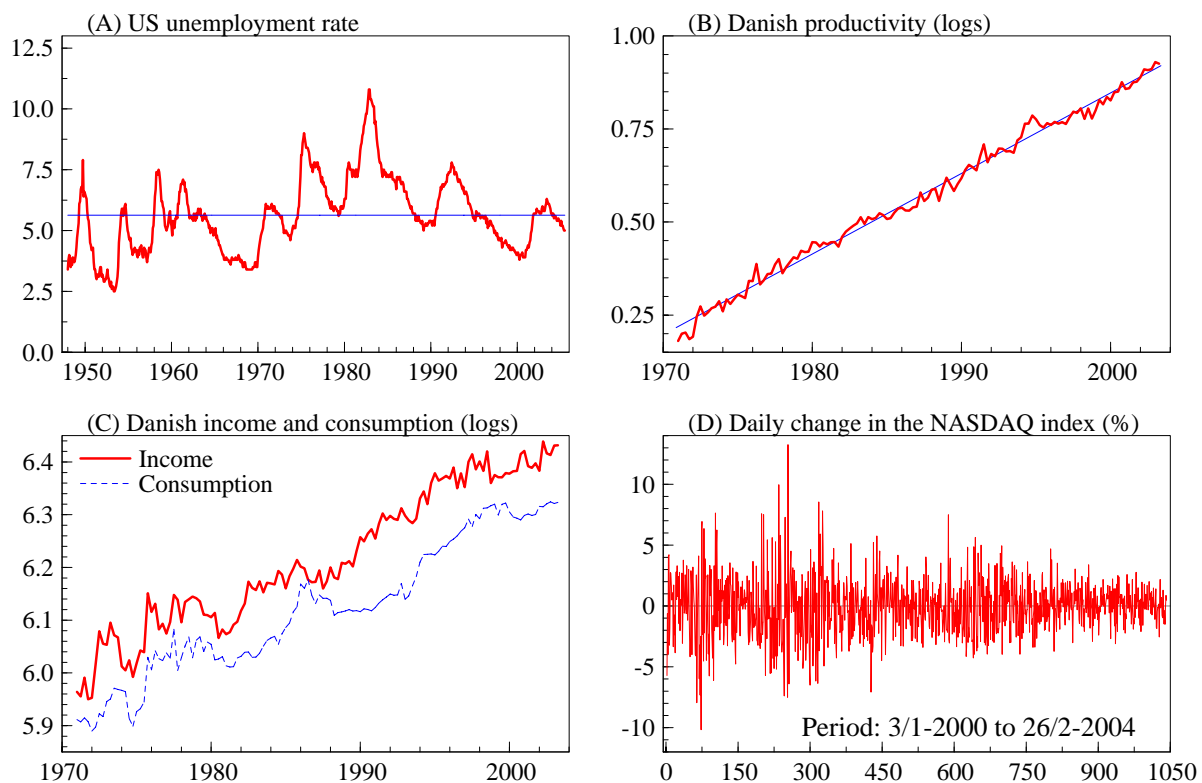
$$y_1, y_2, \dots, y_t, \dots, y_T,$$

where t is the time index. **Natural temporal ordering.**

- It holds that y_{t-1} is observed when y_t is determined.
Focus on conditional models $y_t \mid y_{t-1}, y_{t-2}, \dots$
- Most data in **macro-economics** and **finance** come in this form.
- Very different characteristics.
The tools should match the characteristics of the data.

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Characteristics of Time Series Data



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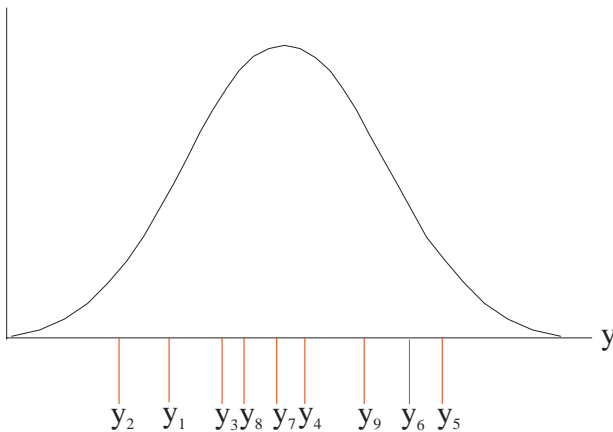
Random Sampling (Cross-Sectional Data)

- The observations x_1, \dots, x_N are randomly drawn from a fixed population.
 N Observations from the same distribution. No ordering.

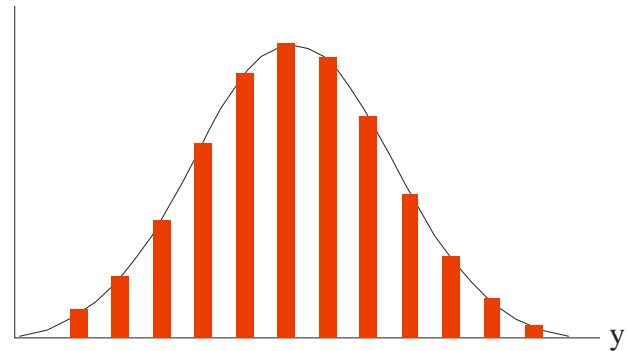
- When N is large we can characterize the distribution.

Law of large numbers: $N^{-1} \sum_{i=1}^N x_i \rightarrow E[x_i]$.

Random sampling from population



Sample distribution approximates the population



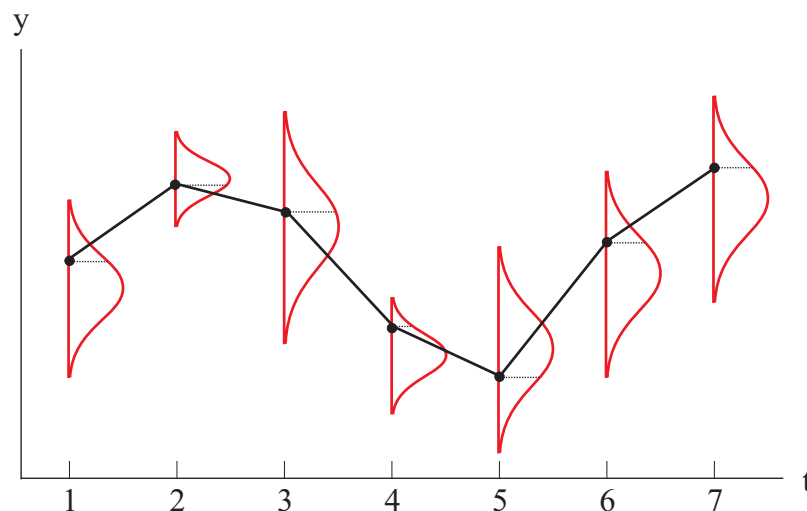
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Stochastic Processes (Time Series Data)

- Observation y_t is a realization of a random variable y_t .

Only one observation per random variable!

- The sequence of random variables y_1, \dots, y_T is denoted a **stochastic process**.



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- If we could rerun history M times:

Stochastic process: $y_1, y_2, \dots, y_t, \dots, y_T$

Realization 1 :	$y_1^{(1)}$	$y_2^{(1)}$	\dots	$y_t^{(1)}$	\dots	$y_T^{(1)}$
	\vdots	\vdots		\vdots		\vdots
Realization m :	$y_1^{(m)}$	$y_2^{(m)}$	\dots	$y_t^{(m)}$	\dots	$y_T^{(m)}$
	\vdots	\vdots		\vdots		\vdots
Realization M :	$y_1^{(M)}$	$y_2^{(M)}$	\dots	$y_t^{(M)}$	\dots	$y_T^{(M)}$

- Consider the **ensemble mean**, $E[y_t]$, estimated with the average

$$\hat{E}[y_t] = \frac{1}{M} \sum_{m=1}^M y_t^{(m)}.$$

- Fundamentally different from the **time average** of a realized sample path

$$\bar{y} = \frac{1}{T} \sum_{t=1}^T y_t^{(1)}.$$

Stationarity

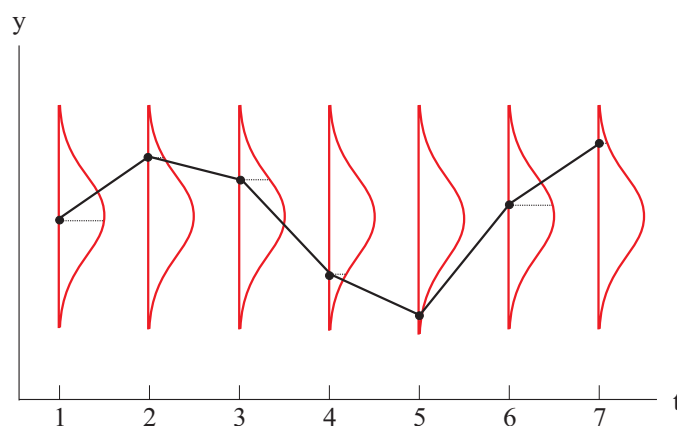
- A time series, $y_1, y_2, \dots, y_t, \dots, y_T$, is called **strictly stationary** if the distributions

$$(y_{t_1}, y_{t_2}, \dots, y_{t_n}) \quad \text{and} \quad (y_{t_1+h}, y_{t_2+h}, \dots, y_{t_n+h})$$

are the same for all h . The distribution of y_t does not depend on t .

- The time series is called **weakly stationary** if

$$\begin{aligned} E[y_t] &= \mu \\ V[y_t] &= E[(y_t - \mu)^2] = \gamma_0 \\ Cov[y_t, y_{t-k}] &= E[(y_t - \mu)(y_{t-k} - \mu)] = \gamma_k \quad \text{for } k = 1, 2, \dots \end{aligned}$$



Main Result

Stationarity implies that y_t ($t = 1, 2, \dots, T$) contain information on the same distribution. And y_t fluctuates around a constant level: **equilibrium correcting**.

- We make the additional assumption that y_t and y_{t-k} becomes approximately independent for $k \rightarrow \infty$. This technical assumption called **weak dependence**, ensures that a law of a large numbers (LLN) hold.

Then \bar{y} is a consistent estimator of $E[y_t]$.

- Given **stationarity** and **weak dependence** of y_t and x_t , most properties of OLS in the IID case carry over to the time series regression

$$y_t = x_t' \beta + \epsilon_t.$$

We return to this issue later.

The most important distinction in time series econometrics is whether the time series of interest are stationary or not.

Measuring Time Dependence

A characteristic feature of time series is a clear dependence over time.

We can measure the dependence by the correlation

$$\text{Corr}(y_t, y_{t-k}) = \frac{\text{Cov}(y_t, y_{t-k})}{\sqrt{V(y_t) \cdot V(y_{t-k})}}, \quad k = \dots, -2, -1, 0, 1, 2, \dots$$

Under stationarity the formula simplifies to the so-called **autocorrelation function** (ACF)

$$\rho_k = \frac{\text{Cov}(y_t, y_{t-k})}{V(y_t)}, \quad k = \dots, -2, -1, 0, 1, 2, \dots$$

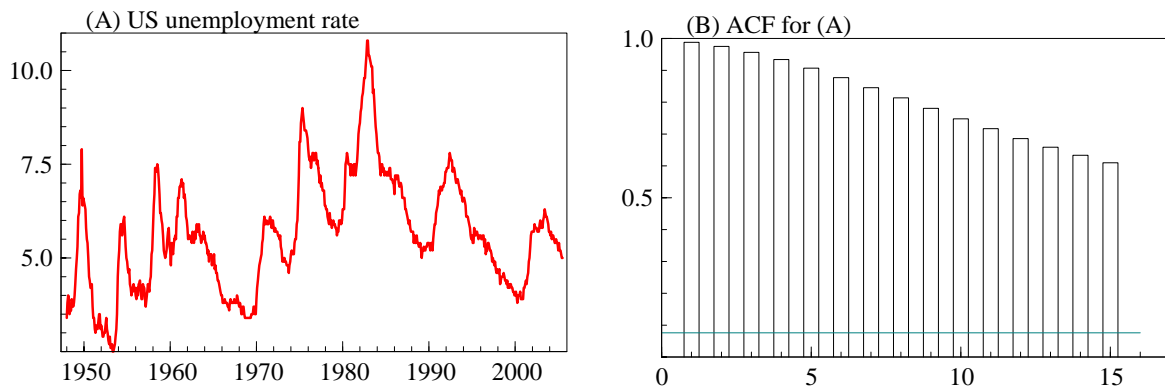
which can be estimated by e.g.

$$\hat{\rho}_k = \frac{\frac{1}{T-k} \sum_{t=k+1}^T (y_t - \bar{y})(y_{t-k} - \bar{y})}{\frac{1}{T-k} \sum_{t=k+1}^T (y_{t-k} - \bar{y})^2}.$$

Empirical Example

Look at the US unemployment rate.

- It is not clear whether u_t equilibrium corrects.
- It fluctuates within bounds, but deviations are very persistent and equilibrium correction is very slow. u_t and u_{t-k} are highly correlated for large values of k .
- We return to formal testing later.



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Transformation to Stationarity

Many economic time series are not stationary.

Sometimes, a non-stationary time series can be transformed to stationarity.

Three important cases:

(A) Remove a deterministic trend.

Trend stationary.

(B) Take first differences.

Difference stationary or integrated or first order.

(C) Combine several variables.

Cointegration.

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(A) If the non-stationary of y_t is due to a deterministic trend, then the de-trended variable

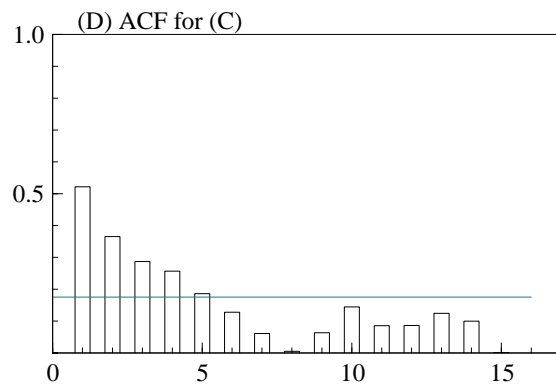
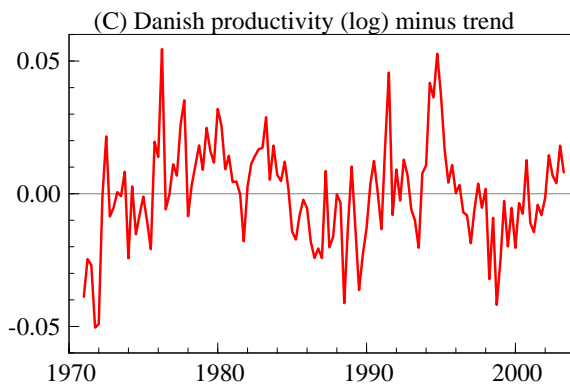
$$y_t^* = y_t - \mu_0 - \mu_1 t,$$

might be stationary. In this case, y_t is called **trend-stationary**.

The de-trended variable can be found as the estimated residual in the linear regression

$$y_t = \mu_0 + \mu_1 t + y_t^*.$$

As an example look at the Danish productivity.

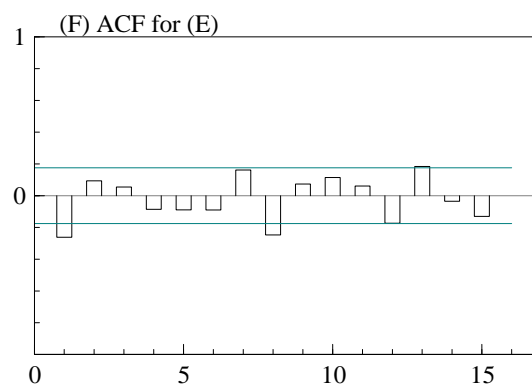
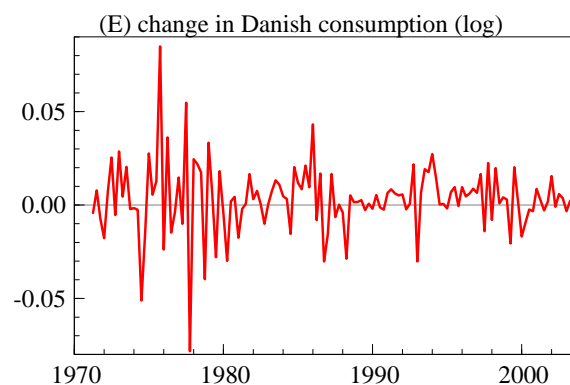


(B) Alternatively it might turn out that y_t is non-stationary while the first difference,

$$\Delta y_t = y_t - y_{t-1},$$

is stationary. In this case, y_t is **difference stationary** or **integrated of first order**, $I(1)$.

As an example look at Danish private consumption.



(C) Finally, it might turn out that two variables, y_t and x_t , are non-stationary, but related so that a linear combination

$$z_t = y_t - \beta \cdot x_t,$$

is stationary. Here y_t and x_t are $I(1)$ but so-called **co-integrated**.

As an example consider consumption, c_t , and income, y_t . Both are $I(1)$ and have no equilibrium. They are related, however, and the savings rate,

$$s_t = y_t - c_t,$$

seems to be stationary and equilibrium corrects.

