Hand-in exam - MikØk2 2009

This is part of the exam of MikØk2 in the Mat.Øk education.

The exam is handed out on 22. June 2009 at 9:00 am and the solution must be handed in on 23. June 2009 at 5:00 pm.

The solution can be written in either danish or english.

All questions weight equally in the overall grade.

4 pages - 4 exercises

Exercise 1

Consider the following situation: Two animals are fighting over some prey. Each can behave like a dove or like a hawk. The best outcome for each animal is that it acts like a hawk while the other acts like a dove; In this case, the hawkish animal obtains a payoff of 4 and the other gets 1. The worst outcome is that both animals act like hawks, in which case they obtain 0 payoff each. If they both act dovish they obtain a payoff of 2 each. We denote playing "dove" by "D" and playing "hawk" by "H".

a) What is the Normal form game of this situation? Find the corresponding bi-matrix.

b) Find all pure Nash equilibria

c) Is there a Nash equilibrium in mixed strategies? If yes, what are the corresponding expected payoffs?

Assume that they repeat the game infinitely, and let $\delta < 1$ be the discount rate.

d) Suggest a SPE in which they both play D for some δ sufficiently high. Find the threshold $\underline{\delta}$ for which all $\delta \geq \underline{\delta}$ this is a SPE.

e) Suggest a SPE in which they alternate between playing outcome (D, H) and (H, D) for some δ sufficiently high. Find the threshold $\underline{\delta}$ for which all $\delta \geq \underline{\delta}$ this is a SPE.

f) Find the set of feasible average payoff

Exercise 2

Consider 2 firms, firm 1 og 2, which are the only producers of a commodity for which the invers demand function is

$$P(q) = a - bQ$$

where Q is the total quantity of the commodity, and $a, b > 0$ are constants. Both firms have constant marginal costs $c > 0$ which is publicly known. The firms compete a la Cournot, hence their strategic variable is the produced quantity. The economy experiences economic bulls and recessions, which is modelled as a taking the values a_L or a_H, with $a_H > a_L$. The a priori probability of $a = a_L$ is $\frac{1}{2}$.

a) Find the Nash equilibrium if $a = a_H$ and both firms know this.

b) Find the Nash equilibrium if $a = a_L$ and both firms know this.

c) Compare the prices and profits in the two equilibria. How do they respond to changes in demand?

Assume that neither of the firms know the value of a.

d) Describe this as a Bayesian game

e) Find the BNE of this game

Assume that firm 1 knows the true value of a, while firm 2 knows only that a takes the value a_L with probability of $\frac{1}{2}$.

f) Describe this as a Bayesian game

g) Find the BNE of this game

Exercise 3

Consider the following situation: A bank has a bankier hired in its portfolio management department, which has the function of maximizing the return of the bank's assets. The bankier has to choose between 2 alternatives: A speculative estate project of exclusive apartments, denoted alternative a, or in a portfolio consisting of equity shares and bonds, denoted alternative b. The bank has equipped the bankier with an amount of $I > 0$ to invest in one of the projects, but he has to choose only one. The bank does not have the administrative capacity to monitor all its bankiers, and thus it cannot control which of the projects the bankier chooses; it can only observe the earnings of the bankier.

Each project has a probability π_i of success, if success the value is V_i and failure the value is $-V_i$ for $i \in \{a, b\}$. Project a is thus characterized by parameters (π_a, V_a) and likewise for project b (π_b, V_b).

We make the following assumptions on the parameter values

$$\pi_b V_b - \pi_a V_a > \frac{1}{2}(V_b - V_a) \tag{1}$$

$$V_a > V_b \tag{2}$$

$$\pi_b V_b < \pi_a V_a. \tag{3}$$

Consider the following wage contract offered by the bank to the bankier: Regardless of the firm's earnings, Π, you get paid an amount $w > 0$, and any *positive* realized earnings of the bank $\Pi \geq 0$ on the bankier's portfolio adds an amount $\alpha\Pi > 0$ to the wage. If the earnings of the bank on the portfolio is negative, he only gets a wage of w.

Assume that the bankier is risk-neutral.

a) Find specific parameter values of π's and V's that satisfies the assumptions of $(1) - (3)$

b) Which project would the bank choose if they had direct control of the investment decision?

c) Find the expected wage of the bankier to each project. Which project will the bankier choose to invest in?

d) Find the expected profit net of wage of the bank to each project. Which project will the bank prefer to have the bankier invest in?

e) Suggest a wage contract implementing the bank's preferred project.

f) What is the Pareto efficient project to implement?

Exercise 4

Suppose that two agents, $i = 1, 2$, are deciding how fast to drive their cars. Agent i chooses his speed $x_i \geq 0$ and gets utility $u_i(x_i)$ from this choice; we assume that $u_i'(x_i) = \frac{du_i(x_i)}{dx_i} > 0$ for every $x_i \geq 0$. However, the faster the agent drives, the more likely it is that they are involved in an accident. Let $p(x_1, x_2)$ be the probability of an accident, assumed to be increasing in each argument, and let $c_i > 0$ be the cost that the accident imposes on agent i. Assume that each agent's utility is linear in money, i.e., the utility from speed x_i and money w is $v_i(x_i, w) = u_i(x_i) + w$. The agents' have expected utility preferences on risky outcomes, $U_i = \mathrm{E}[v_i]$. All functions are assumed to be continuous differentiable.

a) Formulate this as a normal form game, i.e, state the strategy sets and the payoff functions

b) Find necessary conditions for $(x_1^\star, x_2^\star)$ being the Nash equilibrium of the game?

Assume that there is a social planner who wishes to maximize the weighted average, $U_1 + U_2$, by choosing x_1 and x_2

c) State formally the maximization problem of the Social planner

d) Find necessary conditions for (x_1', x_2') being the solution to Social planner's problem

e) Show that both car drivers has an incentive to drive too fast (compared with the optimal solution). Is the eksternality negative or positive?

Assume that in the case of accident driver i is fined an amount $t_i > 0$ which the government confiscate

f) What is the optimal choice of t_i's?

g) If the optimal t_i's are being used, what are the total cost including fines paid by the agents? How does this compare to the total cost of the accident?