Note MikØk2 - Externalities

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We recall our previous characterization of a household/consumer:

- A consumer is characterized by $(\succsim, X)$. Where $X \subset \mathbb{R}^n$ is a set of consumption bundles called the consumption possibilities, and contains the commodity bundles that are possible for the consumer to consume. Hence, whenever $x \in X$ then it is possible for the consumer to consume this commodity bundle. $\succsim$ is often assumed to be a reflexive and transitive preorder on X called the consumer's preference relation, such that if $x, y \in X$ then the relation $x \succsim y$ implies that the consumer prefers (weakly) x to y. We will often represent the consumer's preferences by a utility function $u \colon X \to R$, i.e., $u(x) \geq u(y)$ if and only if $x \succsim y$.

  In a market economy a consumer will have an income $m \in \mathbb{R}$ and all commodities $l = 1, \ldots, n$ can be traded on a market at a price $p_l \in \mathbb{R}$. A consumer can then choose to consume one of all the commodity bundles that satisfies

  1) $x \in X$, and
  2) $p \cdot x \leq m$

  The result of this model is that the consumer chooses $x^\star$ such that $x^\star = \arg\max\{u(x) \mid x \in X \wedge p \cdot x \leq m\}$. Thus, the consumer's optimal choice only of the prices, p, and the income, m, and we can derive the Walrasian demand function $x(p, m)$.

And a firm:

- A firm, in its most simple form, is characterized by a production technology, $Y \subset \mathbb{R}^n$, also called the production possibilities.

  In a market economy, the firm can trade all commodities, both it's input and output, $l = 1, \ldots, n$ on a market at prices $p_l \in \mathbb{R}$. Given these prices the firm will trade a production plan y on the market and receive a profit of $p \cdot y$. The result of this model is that the firm will choose $y^\star$ such that $y^\star = \arg\max\{p \cdot y \mid y \in Y\}$. Thus, the firm's optimal choice of production plan depends only on the prices, p, and we can derive the Walrasian supply function $y(p)$.

One fact and assumption which were not greatly emphasised was that the consumer's utility, or the firm's profit, only depended on the commodity bundle the consumer, or the firm, chose *himself*.

However, it is obvious that my utility does not *solely* depend on my own choices! my night sleep depends on my neighbours' need for loud music during the night; My enjoyment of a delicious dinner at a restaurant depends of the neighbour table's smoking habits; etc. All such situations are also called externalities.

Definition 1 (Externalities) *We say that there is an* **externality** *if one agent's behaviour affects another agent's utility or possibilities* directly*.*

It is important to note the adjective "directly"! The functioning of the market where the price balances supply and demand implies that through the market households and firms affects each other! For example an increase in demand which increases the price will affects the individual consumer and firm, which his/hers preferences have changed. But this effect is *in*direct, namely through the market.

1 Bilateral externalities

We refer to a **bilateral externality** if the externality only affects 2 agents.

1.1 Complete information

This could also be referred to as the classical theory of externalities who's most prominent name was Arthur Cecil Pigou, with his book *Wealth and Welfare* (1912). It is him who introduced the distinction between *private* and *social* costs.

Laize fair - Walrasian equilibrium

We consider an economy with 2 agents: a firm and a consumer. There are 3 commodities: a consumption good, labour time and pollution. The consumer and the firm is characterized as follows

- The consumption possibilities

$$X = \{(c, l, x) \mid c \geq 0, 0 \leq l \leq \bar{l}, x \geq 0\}$$

  and the utility function

$$u(c, l, x) = \tilde{u}(c, \bar{l} - l) - \phi(x)$$

- The production possibilities

$$Y = \{(y, n, h) \mid n \geq 0, y = h = f(n)\}$$

- The consumer owns the firm and thus receives the profit as dividends

We assume the following

- $\tilde{u}(\cdot, \cdot)$ is strictly monotone and quasi-concave, while $\phi(\cdot)$ satisfies $\phi'(x) > 0$ and $\phi''(x) > 0$ for all $x \geq 0$.
- $f'(n) > 0$ and $f''(n) < 0$ for all $n > 0$

A Walrasian equilibrium is then given as

Definition 2 (Walrasian equilibrium) *A* **Walrasian equilibrium** *(WE) is a tupple*

$$((p^\star, w^\star), (c^\star, l^\star, x^\star), (y^\star, n^\star, h^\star))$$

such that

- $(y^\star, n^\star, h^\star)$ *solves:* $\max_{(y,n,h) \in Y} p^\star y - w^\star n$ *and* $\pi^\star = p^\star y^\star - w^\star n^\star$
- $(c^\star, l^\star)$ *solves:* $\max_{c,l} u(c, l, x^\star)$ *s.t.* $p^\star c \leq w^\star l + \pi^\star$
- $y^\star = c^\star$, $x^\star = h^\star$ *and* $l^\star = n^\star$.

This is the normal equilibrium principles of a market economy with private property: the firm maximizes profit given prices, the consumer maximizes utility given the budget constraint, and the markets clear. The only difference is that $x^\star$ affects the consumer's utility , but that the consumer cannot choose the amount of $x^\star$. This is solely up to the firm to decide.

Note that conceptually we have distinguished between the pollution of the firm, h, and the experienced pollution of the consumer, x. However, they are of course two sides of the same coin, but it turns out that it is conceptually advantageous as we extend our analysis to include public intervention and to be able to compare the different institutions.

What determines a Pareto efficient allocation in this economy? As always we only care about the welfare of the consumer:

Definition 3 (Pareto efficiency) *An allocation* $((y', n', h'), (c', l', x'))$ *is* **Pareto efficient** *if*

- *it is feasible, i.e.,* $y' = c'$, $y' = f(n')$, $n' = l' \leq \bar{l}$ *og* $f(n') = h' = x'$
- *There exists no feasbile allocation* $((y'', n'', h''), (c'', l'', x''))$ *such that* $u(c'', l'', x'') > u(c', l', x')$

It is easy to see that a Pareto efficient allocation solves the following maximization problem

$$\begin{array}{rl} \max_{(c,l,x)} & u(c,l,x) \\ \text{s.t.} & \begin{cases} l \geq 0 \\ l \leq \bar{l} \\ c = f(l) \\ x = f(l) \end{cases} \end{array}$$

One can interpret the optimization problem as follows: the consumer take control over the firm directly and chooses the production plan that is carried through. The chosen production plan will then at the same time be the consumption bundle of the consumer. Thus, the consumer is affected directly by his own decisions and only those! Note, that since u is strictly quasi-concave and since the constraints are convex there exists an *unique* Pareto efficient allocation.

We will now show that a interior solution of a Walrasian equilibrium will *never* be Pareto efficient.

Proposition 1 *It always holds that* $x' > x^\star$.

Proof:

Let $((p^\star, w^\star), (c^\star, l^\star, x^\star), (y^\star, n^\star, h^\star))$ be a Walrasian equilibrium and let $((y', n', h'), (c', l', x'))$ be the Pareto efficient allocation. We show that $x' \leq x^\star$.

We note that since $((c^\star, l^\star, x^\star), (y^\star, n^\star, h^\star))$ is a feasible allocation it must hold that

$$u(c', l', x') \geq u(c^\star, l^\star, x^\star)$$

and thus

$$\tilde{u}(c', l') - \tilde{u}(c^\star, l^\star) \geq \phi(x') - \phi(x^\star).$$

by applying the 1. welfare theorem we know that $(c^\star, l^\star, x^\star)$ solves

$$\begin{array}{rl} \max_{(c,l)} & \tilde{u}(c,l) \\ \text{s.t.} & \begin{cases} l \geq 0 \\ l \leq \bar{l} \\ c = f(l) \end{cases} \end{array}$$

and thus we can conclude that

$$\tilde{u}(c', l') - \tilde{u}(c^\star, l^\star) \leq 0.$$

This implies that $\phi(x') - \phi(x^\star) \leq 0$ and since ϕ is strictly increasing, $x^\star \geq x'$. □

This is in stark contrast to the content of the 1. welfare theorem in the case of no externalities: A Walrasian equilibrium is always (under some weak conditions on the utility function) Pareto efficient!

We note that in a WE it must hold that

$$\frac{\tilde{u}_l'(c^\star, l^\star)}{\tilde{u}_c'(c^\star, l^\star)} = \frac{w^\star}{p^\star} = f'(l^\star)$$

while in a Pareto efficient allocation

$$\frac{\tilde{u}_l'(c', l')}{\tilde{u}_c'(c', l') - \phi'(x')} = f'(l').$$

This illustrates the concepts of private og social costs: The private costs the firm is facing is $\frac{w}{p} = \frac{\tilde{u}_l'}{\tilde{u}_c'}$, while the social costs, i.e., the costs of the consumer of providing the extra unit of output is $\frac{\tilde{u}_l'}{\tilde{u}_c' - \phi'}$. Thus we see that the social costs, ceritus paribus, exceeds the private costs, $\frac{\tilde{u}_l'}{\tilde{u}_c'} < \frac{\tilde{u}_l'}{\tilde{u}_c' - \phi'}$!

public intervention

I proposition 1 we have seen that in a Laize fair market economy the resulting allocation is not efficient. Thus, it could be desirable for the government to intervene and correct this failure. We shall consider the following interventions:

- taxes, and
- pollution rights.

We recall that we have assumed complete information, i.e., that the government knows the firms production possibilities and the utility function of the consumer. Thus, the government also knows the Pareto efficient allocation.

Let us first consider the case of introducing taxes on pollution

Definition 4 (Walrasian equilibrium with taxes) *A* **Walrasian equilibrium with taxes** t *is a tupple*

$$((p^\star, w^\star), (c^\star, l^\star, x^\star), (y^\star, n^\star, h^\star))$$

such that

- $(y^\star, n^\star, h^\star)$ *solves:* $\max_{(y,n,h)\in Y} p^\star y - th - w^\star n$ *and* $\pi^\star = p^\star y^\star - th^\star - w^\star n^\star$
- $(c^\star, l^\star)$ *solves:* $\max_{c,l} u(c, l, x^\star)$ *such that at* $p^\star c \leq w^\star l + \pi^\star + th^\star$
- $y^\star = c^\star$, $x^\star = h^\star$ *og* $l^\star = n^\star$.

Such a tax as in Definition 4 is also called a Pigouvian tax. We note that the revenue to the government, $th^\star$, is paid out to the consumer. This implies that the government runs a balanced budget. But is it desirable to impose this tax? Well, this depends of course on the level of the tax. It turns out that choosing the appropriate tax it is possible to implement the Pareto efficient allocation.

Proposition 2 *Let* $t = \frac{p^\star \phi'(x^\star)}{\tilde{u}'_c(c^\star, l^\star)}$ *then* $x' = x^\star$.

Proof:

We know that the firm will choose the production plan (y, n, h) which solves

$$(p - t) f'(n) = w \tag{1}$$

taking (p, w, t) as given. But we also know that $n = n' = l'$ must satisfy

$$\frac{\tilde{u}'_l(c', l')}{\tilde{u}'_c(c', l') - \phi'(x')} = f'(l').$$

Furthermore, $\frac{\tilde{u}'_l(c^\star, l^\star)}{\tilde{u}'_c(c^\star, l^\star)} = \frac{w^\star}{p^\star}$ from the utility maximization of the consumer. From equation 1 we obtain that

$$\begin{aligned} f'(n^\star) &= \frac{w^\star}{p^\star - t} = \frac{w^\star}{p^\star} \frac{1}{1 - \frac{\phi'(x^\star)}{\tilde{u}'_c(c^\star, l^\star)}} = \frac{\tilde{u}'_l(c^\star, l^\star)}{\tilde{u}'_c(c^\star, l^\star)} \frac{\tilde{u}'_c(c^\star, l^\star)}{\tilde{u}'_c(c^\star, l^\star) - \phi'(x^\star)} \\ &= \frac{\tilde{u}'_l(c^\star, l^\star)}{\tilde{u}'_c(c^\star, l^\star) - \phi'(x^\star)} \end{aligned}$$

such that $n^\star = l'$ and $x' = x^\star$.

□

Of course, if we choose a different tax level, $t \neq t^{po}$, then the result will not be Pareto efficient. It is easy to see that $t > t^{po}$ would yield less pollution than efficiency requires, $n^\star < n'$. While $t < t^{po}$ would yield pollution more than efficiency requires. Than some pollution is efficiency may also come to a surprise to somebody. but the reason why *some* pollution is good is that this is required in order to obtain the consumption good, which is a good for the consumer. The choice is then how we should balance between wealth in terms of consumption and disutility from pollution.

Note also that in order for the government to actually implement the Pareto efficient allocation through taxes it is necessary to know the utility function and production function. This it needs to compute the magnitudes $\phi'(\cdot)$,$\tilde{u}_c'(\cdot,\cdot)$ at the point $(c^\star, l^\star, x^\star)$. The price $p^\star$ is public available and thus pose no extra requirement of the government to observe. This is however not the case of the mentioned information. It is difficult to see how the government should know this information. It could ask each to state the required information. However, they are not likely to state their true values! The consumer in order to obtain a large compensation for any pollution would overstate his cost of pollution, while the firm in order to decrease the demands for pollution cuts would overstate his costs of decreasing the pollution. It remains to have revelation mechanisms to reveal that informations. Such exists but we shall not consider them here.

We next turn to the other public intervention: the introduction of pollution rights. We shall grant the consumer the right to issue pollution rights which the firm needs to buy in order to pollute.

Definition 5 (Walrasian equilibrium with pollution rights) *A* **Walrasian equilibrium with pollution rights** *is a tupple*

$$((p^\star, w^\star, r^\star), (c^\star, l^\star, x^\star), (y^\star, n^\star, h^\star))$$

such that

- $(y^\star, n^\star, h^\star)$ *solves:* $\max_{(y,n,h)\in Y} p^\star y - r^\star h - w^\star n$ *and* $\pi^\star = p^\star y^\star - r^\star h^\star - w^\star n^\star$
- $(c^\star, l^\star, x^\star)$ *solves:* $\max_{c,l,x} u(c, l, x)$ *such that* $p^\star c \leq w^\star l + r^\star x + \pi^\star$
- $y^\star = c^\star$, $l^\star = n^\star$ *and* $x^\star = h^\star$*.*

It is easy to see the consequences of creating a market for pollution will imply that the market equilibrium will once again be Pareto efficient. This follows directly from the fact that we now have the normal Walrasian equilibrium and thus we can apply the 1. welfare theorem. In our model this can be seen directly: It holds that

$$\frac{\tilde{u}_c'}{\tilde{u}_x'} = \frac{p^\star}{r^\star} \Leftrightarrow r^\star = \frac{p^\star \phi'}{\tilde{u}_c'}$$

which is the optimal tax derived above, and thus implements the Pareto efficient allocation.

We state the result in a proposition for later references

Proposition 3 *In a Walrasian equilibrium with pollution rights we have that* $x^\star = x'$.

Note that in the definition of the equilibrium we have assigned the property rights of the pollution rights to the consumer. We could of course alternatively have assigned the rights with the firm; this is only a distributional, and hence political, decision. The result is always efficient.

This result is the basis of the interpretation of externalities as a poorly assigned property rights, as emphasised by Coase. The solution will then be that the government should be more active in protecting these property rights, more than intervening through taxes. This solution has at least one obvious advantage, since the government do not know the information needed to implement the optimal tax - informations that the agents, the firm and the consumer, do not have the right incentives to reveal. Thus, the advice is as follows: create a market for pollution rights, instead of government control using taxes. However, this raises the issue of incomplete information which we shall next turn our attention towards in section 1.2

1.2 Incomplete information

The Analysis in section 1.1 was carried out under the assumption of complete information: hence, the firm knows the consumer's utility function, the consumer knows the firm's production function, and the government knows both the firm's and the consumer's characteristics. This implied an equivalence between the different policies: taxes and pollution rights.

We shall abandon the general equilibrium framework and move towards a partial equilibrium model. This is because we want to simplify an otherwise complex situation, to obtain the result.

Pollution rights - revisited

Consider the following model:

- There are 2 commodities: money, T, and pollution, h
- Pollution is either high $\bar{h}$ or low 0
- A **firm** can be of type $\theta \in \mathbb{R}$ with the payoffs $\pi(h, \theta) + T$ where $h \in \{0, \bar{h}\}$ is the amount of pollution and T is a money amount
- A **consumer** can be of type $\eta \in \mathbb{R}$ with the payoffs $\phi(h, \eta) + T$ where $h \in \{0, \bar{h}\}$ is the pollution and T is a money amount. We assume that $\phi(0, \eta) \geq 0$ for every η

- There is a density function $\tilde{g}$ and distribution function $\tilde{G}$ for θ, and a density function $\tilde{f}$ for η with distribution function $\tilde{F}$.
- The firm knows his type θ, but not the consumer's type η - only the distribution $\tilde{F}$
- The consumer knows his type η, but not the firm's type θ - only the distribution $\tilde{G}$
- The consumer has the right to issue pollution rights

We can define some important magnitudes

$$b(\theta) \equiv \pi(\bar{h}, \theta) - \pi(0, \theta) \tag{2}$$
$$c(\eta) \equiv \phi(0, \eta) - \phi(\bar{h}, \eta) \tag{3}$$

where $b(\theta)$ is firm of type θ's gain by having obtained the pollution rights, while $c(\eta)$ is consumer of type η's gain by being not exposed to the pollution - or the consumer's cost from pollution. We then have the distribution of the variable b given by

$$G(b) = \text{Prob}\{b(\theta) \leq b\} = \int_{-\infty}^{b} g(\beta)\ d\beta$$

while the distribution of c is

$$F(c) = \text{Prob}\{c(\eta) \leq c\} = \int_{-\infty}^{c} f(\gamma)\ d\gamma,$$

where $g(b) \geq 0$ and $f(c) \geq 0$. Of course the densities f and g are both derived from the densities $\tilde{f}$ and $\tilde{g}$ respectively. However, we do not know the exact form of f and g, since we have not specified the form of $\tilde{f}$ and $\tilde{g}$. We further assume that $\left|\frac{dg(b)}{db}\right| \leq 2g(b)$ for every θ.

The consumer and the firm engage in a bargaining using the following procedure

- The consumer proposes a contract (T, h) where the firm can pollute the amount h and pay a compensation on T to the consumer
- The firm can then either accept or reject
- If the firm accept the contract the firm of type θ receives payoffs $\pi(h, \theta) - T$ while the consumer receives payoffs $\phi(h, \eta) + T$. If the firm declines the firm receives a payoff of $\pi(0, \theta)$ and the consumer $\phi(0, \eta)$.

We shall solve for a SPE of the game. The firm given a proposed contract (T, h). Consider a contract with $h = \bar{h}$. A firm of type θ will accept this contract if $T \leq b(\theta)$. This the consumer knows, but the consumer does not know the type of the firm, however, since he knows the distribution G og b he can form expectations on the payoffs given the contract $(T, \bar{h})$ which is

$$\begin{aligned} \mathrm{E}[u(T,\bar{h})|\eta] = & \quad \mathrm{Prob}\{b(\theta) \leq T\}\phi(0,\eta) + \mathrm{Prob}\{b(\theta) > T\}(\phi(\bar{h},\eta) + T) \\ = & \quad G(T)\phi(0,\eta) + (1 - G(T))(\phi(\bar{h},\eta) + T) \end{aligned}$$

since we know that $\mathrm{Prob}\{b(\theta) > T\} = 1 - \mathrm{Prob}\{b(\theta) \leq T\} = 1 - G(T)$. The optimal contract the consumer can offer is then given by the FOC

$$-g(T^\star)(T^\star + \phi(\hat{h},\eta) - \phi(0,\eta)) + 1 - G(T^\star) = 0 \Leftrightarrow T^\star = c(\eta) + \frac{1 - G(T^\star)}{g(T^\star)}$$

thus $T^\star > c(\eta)$. The second order condition is satisfied since $-g'(T - c(\eta)) - 2g \leq 0$. Note further, that $\frac{d\,\mathrm{E}[u(c,\bar{h})|\eta]}{dT} = 1 - G(c(\eta)) > 0$, i.e., the marginal utility of demanding $c(\eta)$ is strictly positive. It holds obviously that $\mathrm{E}[u(c,\bar{h}) \mid \eta] = \phi(0,\eta) \geq 0$ such that $\mathrm{E}[u(T^\star,\bar{h}) \mid \eta] > 0$.

What is Pareto efficient: it is easy to see that this requires that $h = \bar{h} > 0$ iff $c(\eta) < b(\theta)$, that is, it is efficient to allow pollution if the gain by the firm of pollution exceeds the consumer's cost of pollution.

Proposition 4 *The equilibrium with pollution rights and incomplete information is not Pareto efficient.*

Proof:

When $G(T^\star) - G(c(\eta)) > 0$ there exists a set of firm types, Θ_0, with strictly positive measure such that $c(\eta) < b(\theta) < T^\star$ for all $\theta \in \Theta_0$, who will not accept the contract.

□

This result implies that the proposition 3 only holds with perfect information. Note, however, that we no longer have considered a model with perfect competitive agents. This implies that we consider a bargaining situation, and, in reality, have we given the consumer a monopoly right of issuing pollution rights. This could be any different with bilateral eksternalities.

2 Multilaterale eksternaliteter

We now return to the case of complete information, bu twe add a second consumer, who is also exposed to pollution by the firm. An externality that involves more than 3 parties is called a **multilateral externality**. The most important difference between bilateral and multilateral externalities is the efficiency of pollution rights. Where pollution rights were efficient in the case of bilateral incomplete information externalities as shown in proposition 3 this is no longer the case of multilateral externalities. The reason is that having multilateral externalities, the trading of pollution rights induces a new externality, namely, a consumption externality between consumers: by selling a pollution right to the firm, the selling consumer is compensated, but the second consumer is not compensated! The question then is however, if the situation is improved, and this depends on the costs imposed by the new externality compared to the old.

We let $u_i(c_i, l_i, x) = \tilde{u}(c_i, l_i) - \phi_i(x)$ the utility function of consumer i. note that the elementar utility function $\tilde{u}(\cdot, \cdot)$ are identical for both consumers, while the discomfort of pollution of pollution differs. We assume for the sake of simplicity that each consumer owns a half of the firm. It has no effect on the results.

We next define a Walrasian-Nash equilibrium with pollution rights

Definition 6 (Walrasian-Nash equilibrium) *A* **Walrasian-Nash equilibrium** *(WNE) is a tuppel*

$$((p^\star, w^\star, r^\star), ((c_i^\star, l_i^\star, x_i^\star))_{i=1}^2, (y^\star, n^\star, h^\star))$$

such that

- $(y^\star, n^\star, h^\star)$ *solves:* $\max_{(y,n,h)\in Y} p^\star y - w^\star n - r^\star h$ *and* $\pi^\star = p^\star y^\star - w^\star n^\star - r^\star h^\star$
- *for* $i = 1, 2$ *the* $(c_i^\star, l_i^\star, x_i^\star)$ *solves:* $\max_{c_i, l_i, x_i} u_i(c_i, l_i, x_i + x_{-i}^\star)$ *such that* $p^\star c_i \le w^\star l + \frac{1}{2}\pi^\star + r^\star x_i$
- $y^\star = c^\star$, $l_1^\star + l_2^\star = n^\star$ *og* $h^\star = x_1^\star + x_2^\star$.

We note that consumer i only partly determines the level of pollution. Therefor we have added the subject *Nash* in the name of the equilibrium.
An allocation $(((c_i, l_i, x_i))_{i=1}^2, (y, n, h))$ is **feasible** in this economy if

$$\begin{aligned} c_1 + c_2 &= y = f(n) \\ l_1 + l_2 &= n \\ x_1 &= x_2 = h = f(n) \end{aligned}$$

Definition 7 *An allocation* $(((c_i, l_i, x_i))_{i=1}^2, (y, n, h))$ *is Pareto efficient if*

- *it is feasible, and*

- *there exists no other feasible allocation $(((c_i', l_i', x_i'))_{i=1}^2, (y', n', h'))$ such that $u_i(c_i', l_i', x_i') \geq u_i(c_i, l_i, x_i)$ for $i = 1, 2$. And at least one with strict inequality.*

It is shown in chapter MWG 16.E the following characterisation of Pareto efficient allocations:

Proposition 5 *An allocation $(((c_i, l_i, x_i))_{i=1}^2, (y, n, h))$ is Pareto efficient iff there exists $\lambda_i > 0$, $i = 1, 2$ which solves*

$$\begin{array}{rl} \max_{(c,l,x)} & \lambda_1 u_1(c_1, l_1, h) + \lambda_2 u_2(c_2, l_2, h) \\ s.t. & \begin{cases} l_i \geq 0 \\ l_i \leq \bar{l} \\ c_1 + c_2 = f(l_1 + l_2) \\ h = f(l_1 + l_2) \end{cases} \end{array} \tag{4}$$

Once more, it is easy to see that in a Walrasian-Nash equilibrium it holds that

$$\frac{\frac{\partial \tilde{u}_i}{\partial l}}{\frac{\partial \tilde{u}_i}{\partial c}} = \frac{w^\star}{p^\star}$$

and

$$\frac{\phi_i'(h^\star)}{\frac{\partial \tilde{u}_i}{\partial l}} = \frac{r^\star}{w^\star},$$

While the firm's FOC states that

$$(p^\star - r^\star) f'(n^\star) = w^\star.$$

On the contrary, in the case of Pareto efficiency it holds that

$$\frac{\frac{\partial \tilde{u}_i}{\partial l}}{\frac{\partial \tilde{u}_i}{\partial c}} = \frac{\mu_1 + \mu_2}{\mu_2} f'(n')$$

for some $\mu_1 > 0$ and $\mu_2 > 0$ being the Lagrange multipliers in the solution of the maximization problem in (4) above. Hence we can conclude that with pollution rights we must have that $f'(n^\star) > \frac{\frac{\partial \tilde{u}_i}{\partial l}}{\frac{\partial \tilde{u}_i}{\partial c}}$, while in the case of the Pareto efficient allocation it holds that $\frac{\frac{\partial \tilde{u}_i}{\partial l}}{\frac{\partial \tilde{u}_i}{\partial c}} > f'(n')$. Thus, we conclude that $n^\star \neq n'$, and there is too much pollution, $h^\star > h'$.

Why does the introduction of pollution rights not solve the problem in the case of multilateral externalities? The problem is that the introduction of tradeable pollution rights creates a new externality, namely that *between consumers*! When one consumer sells a pollution right to the firm, this right also affects the other consumer negatively: it is exposed to more pollution but is not compensated.