Hand-in exercises - MikØk2

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This is part of the exam of MikØk2 in the Mat.Øk education and it is mandatory to be signed up to the exam.

The solution must be handed in by 28. May 2009 to the instructor.

The solution can be written in either danish or english.

The weight of each exercise in the overall grade is stated above.

3 pages - 3 exercises

Exercise 1 - 30%

Consider the following situation - called a "*second-price, sealed bid auction*":

There are 2 bidders, with a valuation v_i of bidder $i = 1, 2$ of a good. The good is indivisible and the supply is a single unit. The two bidders' valuation are independently, uniformly distributed on the interval $[0, 1]$. Each bidder only knows his own value of the good. If the bidder i obtains the good and pays a price of p, the value to the bidder is $v_i - p$. If he does not obtain the good the value is 0. The rules of the game is as follows: Each bidder simultanously submit a bid. The highest bidder is granted the good and pays the second highest bid. If they bid the same the good is randomly allocated between the two, i.e., there is a probability of $\frac{1}{2}$ of each bidder getting the good.

a) Formulate this situation as a Bayesian game

b) Show that it is a Nash equilibrium to bid one's valuation, i.e., $b_i^\star = v_i$

c) Show that it is a weakly dominating strategy to bid one's valuation, i.e., $b_i = v_i$

d) Is it always optimal for a seller to announce that he sells at the highest bid?

Exercise 2 - 30 %

Consider the following stage game

	L	C	R
T	1,1	1,0	6,-1
M	-1,2	3,3	5,1
B	0,5	2,6	4,4

Assume that the game is repeated $T_0 < \infty$ times. The discountingfactor of payoffs is δ.

a) Find every Nash equilibrium in pure strategies in the stage game.

b) Show that
$$(T, L), (T, L), \ldots, (T, L)$$
is a SPE outcome of the repeated game.

c) Show that
$$(M, C), (M, C), \ldots, (M, C)$$
is a SPE outcome of the repeated game.

Consider the following strategies:

player 1 : start play B. In stage $t < T_0$, if the immediate predecessor is (B, R) then play B and else play T until the end. In stage T_0, if the immediate predecessor is (B, R) then play M and else play T.

player 2 : start play R. In stage $t < T_0$, if the immediate predecessor is (B, R) then play R and else play L until the end. In stage T_0, if the immediate predecessor is (B, R) then play C and else play L.

d) Show that when δ is sufficiently large then this is a SPE by using Backward induction. Find the value of $\underline{\delta} < 1$.

Exercise 3 - 40 %

Consider a market with 2 firms, firm 1 and 2. The demand is given by an inverse demand function $P(Q) = a - bQ$, with $a, b > 0$. Each firm produces with identical, constant return to scale technology, and with marginal costs $c > 0$. Firm 1 is a new, dynamic firm with a fast and effective working procedure. While firm 2 is an olde, hierarchical working procedure. This implies that firm 1 can respond faster to new information, therefor the game rules are as follows:

1) Firm 1 decides upon a quantity q_1

2) Then firm 2 observes firm 1's choice, and then choose it's quantity q_2

3) Each firms payoff is it's profits

a) What is each firms' payoff function?

b) Given q_1, find the optimal quantity of firm 2

c) What is firm 2's reaction function, $R_2(\cdot)$?

d) Find the optimal q_1 given that firm 2 reacts according to $R_2(\cdot)$.

e) Explain why the solution is a SPE of this game