

# Exercises 7 - MikØk2

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## Exercise 1

Consider the following situation: There is two firms, a fisherman, denoted  $f$ , and a steel mill, denoted  $s$  located in the same area. The fisherman make his daily catch in a medium sized lake, which the steel mill is located at the shores about. The steel mill discharges chemicals and waste into the lake as a by-product of its activity.

Both firms sell their output on competitive markets, letting  $p_f$  be the price of fish and  $p_s$  the price of steel.

The steel mill in producing an amount  $x \geq 0$  of steel and discharging an amount  $h \geq 0$  of chemicals and waste, has a cost of  $c_s(x, h) \geq 0$ , with the following assumptions:  $\frac{\partial c_s(x, h)}{\partial x} > 0$ ,  $\frac{\partial c_s(x, h)}{\partial h} \leq 0$ ,  $\frac{\partial^2 c_s(x, h)}{\partial x^2} \leq 0$ ,  $\frac{\partial^2 c_s(x, h)}{\partial h^2} \leq 0$  and  $\frac{\partial^2 c_s(x, h)}{\partial h \partial x} < 0$  for every  $x, h \geq 0$ . Also, for every  $x$  there exists  $\bar{h}$  such that  $\frac{\partial c_s(x, \bar{h})}{\partial h} = 0$ .

The fisherman in producing an amount of  $f \geq 0$  of fish while the discharging of  $h \geq 0$  amount of chemicals and waste, endures a cost of  $c_f(f, h) \geq 0$ , with the following assumptions:  $\frac{\partial c_f(f, h)}{\partial f} > 0$ ,  $\frac{\partial c_f(f, h)}{\partial h} > 0$  and  $\frac{\partial^2 c_f(f, h)}{\partial f^2} < 0$  for every  $f, h \geq 0$ .

- State the profit maximization problem of the fisherman and the steel mill respectively
- State the necessary first order conditions characterizing the solution of  $x^*$ ,  $h^*$  and  $f^*$ . Are they also sufficient conditions?
- Find conditions which characterize the solution  $x^{**}$ ,  $h^{**}$  and  $f^{**}$  that maximize the total profits of both the fisherman and the steel mill.
- Show that  $h^{**} < h^*$ . Argue that the laiss-afaire situation is not Pareto efficient.

Assume next that the government knows the true cost functions,  $c_s(\cdot, \cdot)$  and  $c_f(\cdot, \cdot)$ , and they levy a tax  $t \geq 0$  on the dischargement of chemicals and waste. The revenue is paid out to the fisherman as a compensation for the damage.

- State the profit maximization problem of the fisherman and the steel mill respectively

- f) State the necessary first order conditions characterizing the solution of  $x^t$ ,  $h^t$  and  $f^t$ .
- g) How should  $t$  be chosen so that  $h^t = h^{**}$ ?

Assume next that the government setup a market for pollution permissions, and that they grant the steel mill the right to issue these pollution permissions. Assume that they both firms know the other firm's cost function. The trade of pollution permissions is conducted by means of bargaining between the steel mill and the fisherman. The bargaining takes place as follows:

1. The steel mill propose a contract,  $(q, z)$  where the fisherman pays  $q \geq 0$  per pollution permission and he obtains  $z$  pollution permissions
  2. The fisherman then accept or rejects. If the fisherman accepts the transfer of money is carried through and the steel mill cannot pollute more than  $z$ . If the fisherman rejects the proposal the steel mill can freely pollute any amount.
- h) Which contracts  $(q, z)$  will the fisherman accept?
  - i) Show that it will always be optimal to offer a contract which gives the fisherman exactly a profit equal to  $p_f f^* - c(f^*, h^*)$ .
  - j) Show that in any contract  $(q, z)$  in a SPE we must have that  $z = h^{**}$ .

## Exercise 2

Consider 2 firms. Firm 1 produces an output  $x$  which it sells in a competitive market. However, the production of  $x$  imposes a cost  $e(x)$  on firm 2. The production of  $x$  output units costs firm 2 an amount  $c(x)$ . We assume that both  $e(\cdot)$  and  $c(\cdot)$  are continuous differentiable, strictly increasing and convex. Let  $p$  be the price of the output of firm 1. Firm 2 obtains a profit gross pollution of  $\bar{\pi}_2$ .

- a) What is the profit of each of the firms
- b) Find the FOC that maximizes firm 1's profit, and denote by  $x_q$  the optimal solution

Assume that the social planner wishes to maximize the total profit of the firms

- c) State the problem of the social planner formally and determine the FOC, and denote  $x_e$  the efficient amount
- d) Show that  $x_q > x_e$

Assume that the social planner in order to obtain the efficient outcome introduce a tax,  $t > 0$ .

- e) What is the profit of each of the firms
- f) Find the FOC that maximizes firm 1's profit with taxes  $t$
- g) Find the level of  $t$  that implements the Pareto efficient amount  $x_e$