Exercises 5 - MikØk2

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Exercise 1

Consider the simple competitive model of adverse selection of chapter 13.B MWG.

Assume that $r(\cdot)$ is a continuous and strictly increasing function and that there exists $\hat{\theta} \in (\underline{\theta}, \overline{\theta})$ such that $r(\theta) < \theta$ for every $\theta < \hat{\theta}$ and $r(\theta) > \theta$ for every $\theta > \hat{\theta}$. Let $f(\cdot)$ be the density function of workers, with $f(\theta) > 0$ for every $\theta \in [\underline{\theta}, \overline{\theta}]$. Show that a competitive equilibrium with unobservable worker types necessarily involves a Pareto inefficient outcome.

Exercise 1 - solution

Proof: Let $(w^\star, \Theta^\star)$ be a competitive equilibrium. We know then that

$$\begin{aligned} \Theta^\star &= \{\theta \mid r(\theta) \leq w^\star\} \\ w^\star &= \mathrm{E}[\theta \mid \Theta^\star] \end{aligned}$$

An efficient outcome satisfies that $\Theta^{\star\star} = \{\theta \mid r(\theta) \leq \theta\} = [\hat{\theta}, \overline{\theta}]$, but since $r(\cdot)$ is increasing we must have that for any w

$$\Theta(w) = \{\theta \mid r(\theta) \leq w\} = \{\theta \mid \theta \leq r^{-1}(w)\}$$

and hence $(\Theta \setminus \Theta^{\star\star}) \cap \Theta(w^\star) \neq \emptyset$ for any $w^\star$.

Exercise 2

Consider the simple competitive model of adverse selection.

Let $\Theta = [0, 2]$. Compute the competitive equilibrium with asymmetric information in the following cases

a) $f(\theta) = \frac{1}{2}$ and $r(\theta) = -1 + \theta$

b) $f(\theta) = \frac{1}{2}$ and $r(\theta) = 1 + \sqrt{\theta}$

Exercise 2 - solution

a) We have that

$$\begin{aligned} F(\theta_0) &= \frac{\theta_0}{2} \\ \mathrm{E}[\theta \mid \theta \leq \theta_0] &= \frac{1}{F(\theta_0)} \int_0^{\theta_0} \theta f(\theta)\ d\theta \\ &= \frac{\theta_0}{2} \end{aligned}$$

and thus since

$$\begin{aligned} \Theta(w) &= \{\theta \mid r(\theta) \leq w\} \\ &= \{\theta \leq w+1\} \end{aligned}$$

such that and equilibrium should satisfy that

$$\begin{aligned} \Theta^\star &= \{\theta \leq w^\star + 1\} \\ w^\star &= \mathrm{E}\left[\theta \mid \theta \leq w^\star + 1\right] = \frac{w^\star + 1}{2} \end{aligned}$$

and hence $w^\star = 1$ and $\Theta^\star = \Theta$.

b) We have that

$$\begin{aligned} F\left(\theta_0\right) &= \frac{\theta_0}{2} \\ \mathrm{E}[\theta \mid \theta \leq \theta_0] &= \frac{1}{F(\theta_0)} \int_0^{\theta_0} \theta f(\theta)\ d\theta \\ &= \frac{\theta_0}{2} \end{aligned}$$

and thus since

$$\begin{aligned} \Theta\left(w\right) &= \{\theta \mid r\left(\theta\right) \leq w\} \\ &= \left\{\theta \leq \left(w-1\right)^2\right\} \end{aligned}$$

such that and equilibrium should satisfy that

$$\begin{aligned} \Theta^\star &= \left\{\theta \leq \left(w^\star - 1\right)^2\right\} \\ w^\star &= \mathrm{E}\left[\theta \mid \theta \leq \left(w^\star - 1\right)^2\right] = \frac{\left(w^\star - 1\right)^2}{2} \end{aligned}$$

Thus there are two solutions

$$w^\star = \begin{matrix} \sqrt{3} + 2 \\ 2 - \sqrt{3} \end{matrix}$$

and the corresponding employment sets are

$$\begin{aligned}\Theta_1 &= \Theta\left(\sqrt{3}+2\right) = \left\{\theta \mid \theta \leq \left(\sqrt{3}+1\right)^2\right\} = \Theta \\ \Theta_2 &= \Theta\left(-\sqrt{3}+2\right) = \emptyset\end{aligned}$$

Exercise 3

Consider two agents: a firm and a worker. The firm can hire the worker in which case the worker can produce y units of output and the output is sold at a market at a unit price of p. The quantity of output produced by the worker is unknown to the firm, and can take the values $y \in \{y_L, y_H\}$ with $y_H > y_L$. The probability of high productivity is $q > 0$. The worker could alternatively to be employed be on social benefits leaving him with a reservation wage r. Recall that the reservation wage is exactly the wage paid to the worker that would made him indifferent between being un- or employed. Assume that a type with productivity y_H has a reservation wage r_H, and similarly r_L of workers with type y_L, $r_H > r_L$ and let $r = qr_H + (1-q)\, r_L$.

Assume that the firm is the only firm to hire the worker.

a) If the worker did not know his productivity, for which values of q, p, y_L, y_H and r would the firm hire the worker? When would the worker accept the offer?

We next assume that the worker knows his own productivity.

b) For each of the following cases of parameter cases provide the equilibrium of the model and evaluate whether it is efficient

1) $r_H \geq py_H$ and $r_L \geq py_L$
2) $r_H \geq py_H$ and $r_L < py_L$
3) $r_H < py_H$ and $r_L < py_L$
4) $r_H < py_H$ and $r_L \geq py_L$

Exercise 3 - solution

a) The profit of the firm is $\Pi = p\,(qy_H + (1-q)\, y_L) - w$ given a wage w. The firm will offer the job for any wage $w \leq p\,(qy_H + (1-q)\, y_L)$. Since the worker does not know his own type, he will accept to be employed iff $w \geq r$, but then for any combination of parameters such that $r \leq p\,(qy_H + (1-q)\, y_L)$ there will be an employment.

b) For each of the following cases of parameter cases provide the equilibrium of the model and evaluate whether it is efficient

1) $r_H \geq py_H$ and $r_L \geq py_L$: Here, we have that $r \geq p\left(qy_H + (1-q)\, y_L\right)$ and thus the firm will never hier any workers at all, which is also efficient.

2) $r_H \geq py_H$ and $r_L < py_L$: Here, the firm would only hire low productivity workers, letting $w = r_L$ and obtain an expected profit of $\Pi = (1-q)\left(py_L - r_L\right)$, which is again efficient

3) $r_H < py_H$ and $r_L < py_L$: If the firm offers the contract $w = r_L$ only the low productivity workers will accept and he obtains an expected profit of $(1-q)\left(py_L - r_L\right)$. If the firm offers the contract $w = r_H$ both types accept the job, and the expected profit is $qpy_H + (1-q)\, py_L - r_H$. Thus, this is profitable iff

$$qpy_H + (1-q)\, py_L - r_H \geq (1-q)\left(py_L - r_L\right) \Leftrightarrow q\left(py_H - r_H\right) \geq (1-q)\left(r_H - r_L\right)$$

that is if the expected profit of hiring a high productive worker exceeds the extra wage to be paid to convince the high productive to accept. In this case, the low productive obtains an information rent, $\rho = w - r_L = r_H - r_L$, which is a benefit which arises because the firm cannot distinguish between high and low productive. If it holds that $q\left(py_H - r_H\right) < (1-q)\left(r_H - r_L\right)$ then the equilibrium will be $w = r_L$ and only the low productive will be employed. If the last case holds, then the equilibrium is inefficient. In the first case it is efficient: every body gets employed.

4) $r_H < py_H$ and $r_L \geq py_L$: In this case, it is never optimal for the firm to offer $w = r_L$. Thus, the firm offers $w = r_H$ iff

$$qpy_H + (1-q)\, py_L - r_H \geq 0 \Leftrightarrow qpy_H + (1-q)\, py_L \geq r_H$$

But then this situation is inefficient iff $qpy_H + (1-q)\, py_L < r_H$.

Exercise 4

Consider the commodity of a loan. The demanders for loans is called the borrowers, while the supplier are called lenders. The market price of a loan is r the interest rate, while the "quality" of the loan is denoted by q. The quality of the loans is the likelihood of the borrower not defaulting, and hence repaying the loan. The supply of loans is a function $S(r, q)$ in which the supply of loans, i.e., the amount that a loaner is willingly to sell given the price r and the quality q, where $\frac{\partial S}{\partial r} > 0$ and $\frac{\partial S}{\partial q} > 0$. The demand for loans is a simple function $D(r)$, which is downward sloping, $\frac{dD}{dr} < 0$. Assume that there exists a relationship between quality and price, given by $Q\left(r\right) = q$.

a) Can you think of situations in which $Q\left(\cdot\right)$ is a decreasing function?

b) Show that there does not necessarily exist a price r and q such that $D(r) = S(r, q)$. Draw an example.

Exercise 4 - solution

a) Assume that the interest rate increases, why should the likelihood of default increase? The reason is that the higher the interest rate, the higher is the required return of the projects that are launched. But if high return projects are also more risky, the default risk increases, and hence the quality of the loans decreases.

b) We find that since the quality of the supply depends on the price r, we have that the slope of the supply curve $\bar{S}(r) = S(r,)$is

$$\frac{d\bar{S}}{dr} = \frac{\partial S}{\partial r} + \frac{\partial S}{\partial q}\frac{dQ}{dr}$$

which is negative iff

$$\frac{\partial S}{\partial r} < -\frac{\partial S}{\partial q}\frac{dQ}{dr}$$

that is when $\frac{dQ}{dr}$ is sufficiently large negatively. Thus, we could find an example as follows: