Exercises 5 - MikØk2

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Exercise 1

Consider the simple competitive model of adverse selection of chapter 13.B MWG.

Assume that $r(\cdot)$ is a continuous and strictly increasing function and that there exists $\hat{\theta} \in (\underline{\theta}, \overline{\theta})$ such that $r(\theta) < \theta$ for every $\theta < \hat{\theta}$ and $r(\theta) > \theta$ for every $\theta > \hat{\theta}$. Let $f(\cdot)$ be the density function of workers, with $f(\theta) > 0$ for every $\theta \in [\underline{\theta}, \overline{\theta}]$. Show that a competitive equilibrium with unobservable worker types necessarily involves a Pareto inefficient outcome.

Exercise 2

Consider the simple competitive model of adverse selection.

Let $\Theta = [0, 2]$. Compute the competitive equilibrium with asymmetric information in the following cases

a) $f(\theta) = \frac{1}{2}$ and $r(\theta) = -1 + \theta$

b) $f(\theta) = \frac{1}{2}$ and $r(\theta) = 1 + \sqrt{\theta}$

Exercise 3

Consider two agents: a firm and a worker. The firm can hire the worker in which case the worker can produce y units of output and the output is sold at a market at a unit price of p. The quantity of output produced by the worker is unknown to the firm, and can take the values $y \in \{y_L, y_H\}$ with $y_H > y_L$. The probability of high productivity is $q > 0$. The worker could alternatively to be employed be on social benefits leaving him with a reservation wage r. Recall that the reservation wage is exactly the wage paid to the worker that would made him indifferent between being un- or employed. Assume that a type with productivity y_H has a reservation wage r_H, and similarly r_L of workers with type y_L, $r_H > r_L$ and let $r = qr_H + (1 - q) r_L$.

Assume that the firm is the only firm to hire the worker.

a) If the worker did not know his productivity, for which values of q, p, y_L, y_H and r would the firm hire the worker? When would the worker accept the offer?

We next assume that the worker knows his own productivity.

b) For each of the following cases of parameter cases provide the equilibrium of the model and evaluate whether it is efficient

   1) $r_H \geq py_H$ and $r_L \geq py_L$
   2) $r_H \geq py_H$ and $r_L < py_L$
   3) $r_H < py_H$ and $r_L < py_L$
   4) $r_H < py_H$ and $r_L \geq py_L$

Exercise 4

Consider the commodity of a loan. The demanders for loans is called the borrowers, while the supplier are called lenders. The market price of a loan is r the interest rate, while the "quality" of the loan is denoted by q. The quality of the loans is the likelihood of the borrower not defaulting, and hence repaying the loan. The supply of loans is a function $S(r,q)$ in which the supply of loans, i.e., the amount that a loaner is willingly to sell given the price r and the quality q, where $\frac{\partial S}{\partial r} > 0$ and $\frac{\partial S}{\partial q} > 0$. The demand for loans is a simple function $D(r)$, which is downward sloping, $\frac{dD}{dr} < 0$. Assume that there exists a relationship between quality and price, given by $Q(r) = q$.

a) Can you think of situations in which $Q(\cdot)$ is a decreasing function?

b) Show that there does not necessarily exist a price r and q such that $D(r) = S(r,q)$. Draw an example.