Exercises 4 - MikØk2

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Exercise 1

Consider a monopolist who produces to a market with invers demand function

$$P(q) = a - q.$$

The monopolist does not know the parameter $a > 0$, but knows that it takes the values $\{5, 15\}$ with probability of $a = 5$ being $\frac{1}{2}$. The monopolist has constant marginal costs, $c = 1$.

a) Formulate the profit maximization problem when a is unknown.

b) Solve the profit maximization problem when a is unknown.

c) What is the maximal expected profit?

d) What is the optimal strategy when $a = 5$ and the monopolist observes this? resp. when $a = 15$? Find the corresponding profit levels.

e) If the monopolist could observe a before choosing q, what is then the expected profit?

f) What is the "informations cost" for the monopolist?

Exercise 2

Consider 2 firms, firm 1 og 2, who are the only producers of a commodity with the invers demand function

$$P(q) = a - bQ$$

where Q is the total quantity of the commodity, and $a, b > 0$ are constants. Firm 1 has either marginal costs $c_H > c_L$, while firm 2's marginal costs c are publicly known. The firms compete ala Cournot, hence their strategic variable is the produced quantity.

a) Find the Nash equilibrium if $c = c_H$ and firm 2 knows this.

b) Find the Nash equilibirum if $c = c_L$ and firm 2 knows this.

Assume now that Firm 2 does not know the marginal cost c_1 of firm 1 but believs that firm 1 has $c = c_H$ with probability $\pi > 0$, and firm 1 knows this.

c) Formulate this as a Bayesian game

d) Show that in a Bayesian Nash equilibrium the strategies solves the equations

$$\begin{aligned} a - c_L - 2bq_1(c_L) - bq_2 &= 0 \\ a - c_H - 2bq_1(c_H) - bq_2 &= 0 \\ a - c - 2b(\pi q_1(c_L) + (1-\pi)q_1(c_H)) - bq_2 &= 0 \end{aligned}$$

e) Find the Bayesians Nash equilibrium in pure strategies.

Exercise 3

Consider 2 firms, firm 1 and 2, who are the only producers of a commodity with the invers demand function

$$P(q) = a - bQ$$

where Q is the total quantity of the commodity, $a, b > 0$ are constants. The firms have identical, constant marginal costs $c > 0$. The firms compete ala Cournot, hence their strategic variable is the produced quantity. Assume that they compete in T periods which are mere repeatitions of the static game. The firms have a common discount factor $\delta < 1$.

a) Find the Nash equilibrium in the stage-game and the corresponding stage-payoff, denote it by π^C.

b) What is the maximal total stage-payoff the firms could obtain? Denote it by π^M.

c) Let $T < \infty$. Show that $\pi = \pi^C$ is the unique SPE in this game. (*Hint: Use backward induction*)

d) Let $T = \infty$. Show that $\pi = \frac{\pi^M}{2}$ is possible to obtain in a SPE when δ is sufficiently close to 1.

e) Find a treshold for δ in question d). How does this treshold compare to the case of price-competition? Explain the difference.