Exercises 3 - MikØk2

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1 Normal form games

Exercise 1

Consider the following stage game

	O	F
O	3,3	0,4
F	4,0	1,1

Let $\frac{1}{2} < \delta < 1$ be the discount rate.

a) Draw the payoffs in a diagram with u_1 as the horisontal axis and u_2 as the vertical axis

b) What is the minimax payoff of player 1? of player 2?

c) What are the feasible average payoffs? What are the feasible rational payoffs?

d) Suggest a strategy pairs such that $(3, 3)$ is the average payoff of a SPE of the infinitely repeated game. Show that it is actually a SPE.

Assume that the players use the following strategies:

player 1 : start play O. In stage t, t odd, if the immediate predecessor is (F, O) then play O and else play F infinitely. In stage t, t even, if the immediate predecessor is (O, O) then play F and else play F infinitely.

player 2 : start play O. In stage t, t odd, if the immediate predecessor is (F, O) then play O and else play F infinitely. In stage t, t even, if the immediate predecessor is (O, O) then play O and else play F infinitely.

e) What is the discounted payoff to each player of the outcome

$$(O, O), (F, O), (O, O), (F, O), \ldots?$$

f) What is the discounted payoff to each player of the outcome

$$(F, O), (O, O), (F, O), (O, O), \ldots?$$

g) Show that this is a SPE of the game when δ is sufficiently close to 1. Find an approximate value of the $\underline{\delta} < 1$ such that this is a SPE. What are the average payoffs of each player in this SPE?

Exercise 2

Assume that there are 2 players, A and B. Consider the following game: They have to divide a dollar and they bargain using the following rules

- First, player A propose a division
- Second, player B is told the proposition, and can then accept or reject the proposition
- If B accepts the proposition the division is distributed to each and the payoff is the share. If B rejects neither gets a penny, and their payoff is zero.

There is no discounting of payoffs.

a) Write this as an Extensive form game

b) Show that any division is a Nash equilibrium of this game

c) Find the unique SPE equilibrium of the game

Exercise 3

Let there be 2 depositors each with a deposit of $D > 0$ in a bank. Then bank has invested the deposit of both depositors in a project which can be liquidated in the short- or long-run. If the project is liquidated in the short run the project is worth $2r$, while a liquidation in the long run yields a dividend of $2R$. We assume that $r < D < R$. The game proceed as follows:

stage 1:
- if both withdraw their deposit, the bank must liquidate and both gets a payoff of r, and the game ends
- if one withdraw her deposit, but the second keeps the deposit, the bank must liquidate and the withdrawer gets D, and the other gets $2r - D$, and the game ends
- if neither withdraws, the game proceed to the next stage

stage 2
- if both withdraw their deposit, the bank must liquidate and both gets a payoff of R, and the game ends

- if one withdraw her deposit, but the second keeps the deposit, the bank must liquidate and the withdrawer gets D, and the other gets $2R - D$, and the game ends
- if neither withdraws, their deposit is now R each

a) Find the bimatrix of the stage 2 game

b) What is the Nash equilibrium of the stage 2 game? What is the payoff in this outcome?

c) Find the bimatrix of the stage 1 game, in which the payoff of the action profile $(nowithdraw, nowithdraw)$ is the Nash equilibrium payoff of stage 2

d) Find the SPE of this game

e) If we interprete an early withdraw as a bank run, what does the model tells about the rationality of bank runs?

Exercise 4

Consider 2 firms, firm 1 and 2, who are the only producers of a commodity with the invers demand function

$$P(q) = a - bQ$$

where Q is the total quantity of the commodity, $a, b > 0$ are constants. The firms have identical, constant marginal costs $c > 0$. The firms compete ala Cournot, hence their strategic variable is the produced quantity. Assume that the firms can only produce in a few quantities, either the competitive q_i^c, the Cournot $q_i^\star$ or half the monopoly quantity $\frac{q^m}{2}$.

- What is the normal form game and the bimatrix of this game?
- Find the Nash equilibrium of the game