Exercises 2 - MikØk2

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1 Normal form games

Exercise 1

Consider the following situation:

There are 2 firms, an incumbent, player 1, and a potential entrant, player 2. The incumbent can choose to build a new factory "B" or not "D", while the potential entrant kan enter, "E", or remain passive, "D". The incumbent's construction costs, c, of building the new factory can either be high, c_H, or low, c_L, i.e. $c_H > c_L$.

If the construction costs are high the payoff will be as follows:

	E	D
B	0,-1	2,0
D	2,1	3,0

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while low construction costs will imply that the payoffs are as follows:

	E	D
B	3,-1	5,0
D	2,1	3,0

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a) What is the Nash equilibrium if $c = c_H$?

b) What is the Nash equilibrium if $c = c_L$?

Assume that only the incumbent knows his construction costs. The potential entrant believs that with p_1 the construction cost is high, $c = c_H$.

c) Formulate this as a Bayesian game

d) Find the Bayesian Nash equilibrium for $p_1 < \frac{1}{2}$

e) Find the Bayesian Nash equilibrium for $p_1 > \frac{1}{2}$

Exercise 2

Consider the following situation - called a "first-price, sealed bid auction":

There are 2 bidders, with a valuation v_i of bidder $i = 1, 2$ of a good. The good is indivisible and the supply is a single unit. The two bidders' valuation are independently, uniformly distributed on the interval $[0, 1]$. Each bidder only knows his own value of the good. If the bidder i obtains the good and pays a price of p, the value to the bidder is $v_i - p$. If he does not obtain the good the value is 0. The rules of the game is as follows: Each bidder simultanously submit a bid. The highest bidder is granted the good and pays the bid. If they bid the same the good is randomly allocated between the two, i.e., there is a probability of $\frac{1}{2}$ of each bidder getting the good.

a) Formulate this situation as a Bayesian game

b) Solve for a Bayesian Nash equilibrium (Hint: Assume that the strategy is of the affin form $b_i(v_i) = a_i + b_i v_i$. Solve then for a_i and b_i)

c) Do the bidders bid their value of the good?

2 Extensive form games

Exercise 3

Consider the following description of a game - called "The truth game":

There are 2 players, player 1 and 2, and a gamemaster. The gamemaster has a coin that is bent such that, flipped randomly, the coin will come up with "heads" 80% of the time. Both players know this. The game-master flips the coin and shows the result to player 1. Player 1 makes an announcement to player 2 what the result of the flip is, either "head" or "tail". Player 2 having heard player 1's announcement but not seen the result must guess what the result of the flip was. Player 1 has the following payoffs: if player 2 guess "head" payoff is 2 and payoff is 0 if the guess is "tail". Moreover, if player 1's announcement is correct player 1's payoff is increased by 1. For player 2 the payoff is 1 if he guesses right and 0 if his guess is wrong.

a) Formulate this game as a game on extensivform. What is the game tree.

b) What is the normalform representation of this game?

c) Find all Nash equilibria of this game

Exercise 4

Consider the following description of a game:

There are 2 players, player 1 and 2. There are the following rules:

- Player 1 chooses the action l or r, where l ends the game with payoff $(2, 0)$

- Player 2 observes player 1's action. If 1 chose r, the player 2 can choose between l' and r', where l' ends the game with payoff $(1, 1)$
- Player 1 observes player 2's action and recalls his own choice. If there has been chosen r and r' then player 1 can choose between l'' and r'', both ending the game. If player 1 chooses l'' payoffs are $(3, 0)$, while the payoffs from choosing r'' are $(0, 2)$

a) What is the game tree of this game

b) Find the normalform representation

c) Find all Nash equilibria

d) Find the Subgame perfect Nash equilibrium.

Extra exercises

Exercise 5

Assume that there are 2 players, A and B. Consider the following game: They have to divide a dollar and they bargain using the following rules

- First, player A propose a division
- Second, player B is told the proposition, and can then accept or reject the proposition
- If B accepts the proposition the division is distributed to each and the payoff is the share. If B rejects neither gets a penny, and their payoff is zero.

There is no discounting of payoffs.

a) Write this as an Extensive form game

b) Show that any division is a Nash equilibrium of this game

c) Find the unique SPE equilibrium of the game