Exercises 1, suggested solutions - MikØk2

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Normalform games

Opgave 1.1

Consider the following game - called "Stone-Scissor-Paper":

There are 2 players, player 1 and 2. The two players are placed with their front against each other, with the one fist clenched. Simultanously the players rais their arm and counts to 3. Reaching 3 each player forms the fist of either: "Stone" (the fist is kept clenched), "Scissor" (the fore- and ring finger is unfolded and separated) or "Paper" (The entire fist is unfolded). The result is the: either player 1 wins, player 2 wins or it is a draw. A player who wins gets a payoff of på 1, the looser gets a payoff of -1 while a draw yields a player a payoff of 0. Who wins depends on the ordering: "Stone beats Scissor", "Scissor beats Paper" and "Paper beats Stone".

a) Formulate this as a game on normalform. Find the Bi-matrix.

b) What is a mixed strategi of player 1 in this game?

c) Do there exists a Nash equilibrium in pure strategies of this game?

Exercise 1.1 - suggested solution

a) Formulate this as a game on normalform. Find the Bi-matrix.

The set of players is $N = \{1, 2\}$, and the strategy sets are given by

$$S_1 = \{St, Sc, P\} = S_2$$

while the payoffs are given in the following bimatrix

	St	Sc	P
St	$(0,0)$	$(1,-1)$	$(-1,1)$
Sc	$(-1,1)$	$(0,0)$	$(1,-1)$
P	$(1,-1)$	$(-1,1)$	$(0,0)$

b) What is a mixed strategi of player 1 in this game?

A mixed strategy of player 1 is $\sigma_1 = (\alpha, \beta, \gamma)$, with $\alpha + \beta + \gamma = 1$, such that $\alpha \geq 0$ is the probability of playing St, $\beta \geq 0$ is the probability of playing Sc and $\gamma \geq 0$ is the probability of playing P.

c) Do there exists a Nash equilibrium in pure strategies of this game?

No, there exists no NE in pure strategies. Consider player 1,

- playing St, player 2 will play P but then player 1 wants to play Sc
- playing Sc, player 2 will play St, but then player 1 wants to play P
- playing P, player 2 will play Sc, but then player 1 wants to play St

Thus, whatever player 1 plays of pure strategies, and the best respond of player 2, then player 1 would like to deviate.

Opgave 1.2

Consider the following description of a game - called “Sharing a penny”:

There are 2 players, player 1 and 2. They each state an amount on a note which the other player cannot see. The amount is the share of $100 note the player wants. if the sum of shares stated by the two players exceeds 100% the players gets nothing, if not they are awarded their desired share.

a) Formulate this as a game on normalform. Find the Bi-matrix.

b) Find all Nash equilibria in this game

Exercise 1.2 - suggested solution

a) Formulate this as a game on normalform. Find the Bi-matrix.

The normal form game is given as $N = \{1, 2\}$ the set of players, $S_1 = \{100 \geq x \geq 0\} = [0, 100] = \{100 \geq y \geq 0\} = S_2$ the strategy sets and the payoff functions

$$u_1(x, y) = \begin{cases} x & x + y \leq 100 \\ 0 & x + y > 100 \end{cases}$$

$$u_2(x, y) = \begin{cases} y & x + y \leq 100 \\ 0 & x + y > 100 \end{cases}$$

b) Find all Nash equilibria in this game

We conjecture that any outcome of the form $(x, y) = (x, 100 - x)$ is a Nash equilibrium. Consider any $x \in [0, 100]$, if $y < 100 - x$ then the payoff to player 2 is then $u_2(x, y) = y < 100 - x = u_2(x, 100 - x)$. While, if $y > 100 - x$ then $u_2(x, y) = 0 \leq 100 - x = u_2(x, 100 - x)$. Thus, player 2 does not have any incentive to deviate. Symmetrically, player 1 does not have an incentive to deviate.

Opgave 1.3

Consider the following game given by their Bimatrix

	L	C	R
T	2,0	1,1	4,2
M	3,4	1,2	2,3
B	1,3	0,2	3,0

a) Is the strategy "B" a dominated strategy for player 1?

b) Find the strategies that survives an iterativ deletion of dominated strategies

c) Find Nash equilibrium i pure strategies

Exercise 1.3 - suggested solution

a) Is the strategy "B" a dominated strategy for player 1?

We see that playing B yields payoffs

- 1 if player 2 plays L
- 0 if player 2 plays C
- 3 if player 2 plays R

while playing T yields payoffs

- 2 if player 2 plays L
- 1 if player 2 plays C
- 4 if player 2 plays R

and thus T strictly dominates B.

b) Find the strategies that survives an iterativ deletion of dominated strategies

Eliminating player 1's strategy, implies that for player 2, the strategy C is now dominated by R. Having eliminated C, we cannot eliminate any more strategies.

c) Find Nash equilibrium i pure strategies

There are two Nash equilibria in this game, namely (M, L) and (T, R).

Opgave 1.4

Consider the following game given by their Bimatrix

	O	F
O	2,1	0,0
F	0,0	1,2

a) Find Nash equilibria in pure strategies

b) Find Nash equilibria in mixed strategier

Exercise 1.4 - suggested solution

a) Find Nash equilibria in pure strategies

There are two Nash equilibria in pure strategies: (O, O) and F, F.

b) Find Nash equilibria in mixed strategier

In order to find a Nash equilibrium in mixed strategies, we let $\sigma_1 = (\alpha, 1-\alpha)$ and $\sigma_2 = (\beta, 1-\beta)$ denote the mixed strategies of each player. We then use the necessary condition of a mixed equilibrium

$$U_1(O, \sigma_2) = 2\beta + 0(1-\beta) = 0\beta + 1(1-\beta) = U_1(F, \sigma_2)$$

which implies that $\beta = \frac{1}{3}$. Equivalently, we use that

$$U_2(\sigma_1, O) = 1\alpha + 0(1-\alpha) = 0\alpha + 2(1-\alpha) = U_2(\sigma_1, F)$$

and obtains $\alpha = \frac{2}{3}$. Thus, we obtain a Nash equilibrium in (non-trivial) mixed strategies as $(\frac{2}{3}O \oplus \frac{1}{3}F, \frac{1}{3}O \oplus \frac{2}{3}F)$.

Opgave 1.5

Consider the following situation:

There are 2 firms, an incumbent, player 1, and a potential entrant, player 2. The incumbent can choose to build a new factory "B" or not "D", while the potential entrant kan enter, "E", or remain passive, "D". The incumbent's construction costs, c, of building the new factory can either be high, c_H, or low, c_L, i.e. $c_H > c_L$.

If the construction costs are high the payoff will be as follows:

	E	D
B	0,-1	2,0
D	2,1	3,0

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while low construction costs will imply that the payoffs are as follows:

	E	D
B	3,-1	5,0
D	2,1	3,0

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a) What is the Nash equilibrium if $c = c_H$?

b) What is the Nash equilibrium if $c = c_L$?

Assume that only the incumbent knows his construction costs. The potential entrant believes that with p_1 the construction cost is high, $c = c_H$.

c) Formulate this as a Bayesian game

d) Find the Bayesian Nash equilibrium for $p_1 < \frac{1}{2}$

e) Find the Bayesian Nash equilibrium for $p_1 > \frac{1}{2}$

Exercise 1.5 - suggested solution

See the next weeks exercises solutions

Extra exercises

These exercises are only relevant if the exercises above are finished before the time runs out

Exercise 1.X1

Show that

- If $s^\star \in S$ is a NE, then it will always survive an iterative deletion of strictly dominated strategies
- There exists games in which NE are deleted in an iterated deletion of weakly dominated strategies
- If s_i is a dominated strategy of i, then in any mixed NE $\sigma^\star$ we must have that $\sigma_i^\star(s_i) = 0$