

Exercises 1 - MikØk2

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Normalform games

Opgave 1.1

Consider the following game - called "Stone-Scissor-Paper":

There are 2 players, player 1 and 2. The two players are placed with their front against each other, with the one fist clenched. Simultaneously the players raise their arm and counts to 3. Reaching 3 each player forms the fist of either: "Stone" (the fist is kept clenched), "Scissor" (the fore- and ring finger is unfolded and separated) or "Paper" (The entire fist is unfolded). The result is the: either player 1 wins, player 2 wins or it is a draw. A player who wins gets a payoff of $\frac{1}{2}$, the loser gets a payoff of $-\frac{1}{2}$ while a draw yields a player a payoff of 0. Who wins depends on the ordering: "Stone beats Scissor", "Scissor beats Paper" and "Paper beats Stone".

- a) Formulate this as a game on normalform. Find the Bi-matrix.
- b) What is a mixed strategy of player 1 in this game?
- c) Do there exist a Nash equilibrium in pure strategies of this game?

Opgave 1.2

Consider the following description of a game - called "Sharing a penny":

There are 2 players, player 1 and 2. They each state an amount on a note which the other player cannot see. The amount is the share of \$100 note the player wants. If the sum of shares stated by the two players exceeds 100% the players get nothing, if not they are awarded their desired share.

- a) Formulate this as a game on normalform. Find the Bi-matrix.
- b) Find all Nash equilibria in this game

Opgave 1.3

Consider the following game given by their Bimatrix

	L	C	R
T	2,0	1,1	4,2
M	3,4	1,2	2,3
B	1,3	0,2	3,0

- Is the strategy "B" a dominated strategy for player 1?
- Find the strategies that survives an iterativ deletion of dominated strategies
- Find Nash equilibrium i pure strategies

Opgave 1.4

Consider the following game given by their Bimatrix

	O	F
O	2,1	0,0
F	0,0	1,2

- Find Nash equilibria in pure strategies
- Find Nash equilibria in mixed strategier

Opgave 1.5

Consider the following situation:

There are 2 firms, an incumbent, player 1, and a potential entrant, player 2. The incumbent can choose to build a new factory "B" or not "D", while the potential entrant kan enter, "E", or remain passive, "D". The incumbent's construction costs, c , of building the new factory can either be high, c_H , or low, c_L , i.e. $c_H > c_L$.

If the construction costs are high the payoff will be as follows:

	E	D
B	0,-1	2,0
D	2,1	3,0

while low construction costs will imply that the payoffs are as follows:

	E	D
B	3,-1	5,0
D	2,1	3,0

- What is the Nash equilibrium if $c = c_H$?

b) What is the Nash equilibrium if $c = c_L$?

Assume that only the incumbent knows his construction costs. The potential entrant believes that with p_1 the construction cost is high, $c = c_H$.

c) Formulate this as a Bayesian game

d) Find the Bayesian Nash equilibrium for $p_1 < \frac{1}{2}$

e) Find the Bayesian Nash equilibrium for $p_1 > \frac{1}{2}$

Extra exercises

These exercises are only relevant if the exercises above are finished before the time runs out

Exercise 1.X1

Show that

- If $s^* \in S$ is a NE, then it will always survive an iterative deletion of strictly dominated strategies
- There exists games in which NE are deleted in an iterated deletion of weakly dominated strategies
- If s_i is a dominated strategy of i , then in any mixed NE σ^* we must have that $\sigma_i^*(s_i) = 0$