Alternative health insurance schemes:  
a welfare comparison  

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Abstract

In this paper, we present a simple model of health insurance with asymmetric information, where we compare two alternative ways of organizing the insurance market. Either as a competitive insurance market, where some risks remain uninsured, or as a compulsory scheme, where however, the level of reimbursement of loss is to be determined by majority decision. In a simple welfare comparison, the compulsory scheme may in certain environments yield a solution which is inferior to that obtained in the market. We further consider the situation where the compulsory scheme may be supplemented by voluntary competitive insurance; this situation turns out to be at least as good as either of the alternatives.

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1. Introduction

In recent years, several countries have experienced a growing concern about the organization of health care, its financing and its costs. In countries with a tax-financed health system, prolonged efforts in cost containment has led to severe strains in the organization of health care and the provision of its services, giving rise to waiting lists, queues, and failing ability to catch up with the development of new medical treatments. The understandable reluctance of the general public to accept an increased tax burden thus results in a general lowering of the quality of the services provided in the health care sector. This may very well
be at variance with the wishes of the same general public, who might be willing to spend
more on health if the money was paid in another way than as general income taxes.

In the present paper, we consider a simple model of compulsory health insurance. Our
model is a version of the by now classical model of adverse selection in the insurance
market. Under free market conditions, if the individual risks cannot be observed by the
insurer, then part of the public will choose no insurance. Assuming risk aversion, this will
result in a utility loss both to the non-insured and the insured, as compared with what could
have been obtained in the ideal situation where risks are observable by the insurer.

In view of this, it has been conjectured by Akerlof (1970) that compulsory insurance
may be welfare improving, and this question of whether compulsory insurance may lead
to a Pareto improvement has been investigated in detail by Dahlby (1981). In the present
work, we expand on the seminal work by Dahlby in two directions: first of all, we introduce
an equilibrium determination of the level of compulsory insurance, to be further explained
below, and secondly, we consider welfare comparisons which go beyond Pareto improve-
ments, such as utilitarian average utility comparisons and compensation criteria. The reason
for this extension of the analysis is that introduction of a compulsory insurance in most cases
implies a utility loss for the low risk individuals, so that Pareto improvement is ruled out
unless the initial situation has some particular properties (so that the low risk individuals
already choose the level of compulsory insurance).

To explain the need for an equilibrium determination of the compulsory insurance scheme,
we notice that the scheme must be acceptable to the general public in order to survive; we
incorporate this feature in our model by the condition that the coalition of individuals who
pay more for insurance than they get from it, should not be a majority, since otherwise the
system would probably be changed. It is assumed that the insurance scheme covers only
part of the outlays caused by illness; the size of this coverage is to be determined by the
general public by majority decision.

A compulsory scheme will force the low-risk individuals to have insurance at a cost which
is higher than their willingness to pay, but on the other hand, since it covers everyone, the
premium will be lower than if some people were left out. The final result is not clear,
although there might be some intuition in favour of the all-embracing, compulsory scheme.
We show that the compulsory system will not necessarily be superior to the market solution
with its well-known welfare losses; the majority decisions with respect to the coinsurance
part of the insurance scheme may lead to a result which is worse than the market solution in
the sense that the winners gain less than what is lost by the losers. This result depends on the
distribution of risk characteristics in the population, and it uses an interpersonal comparison
of utility (all individuals are assumed to have identical utility functions), but it shows that
universal participation in an insurance scheme does not by itself guarantee that the result is
welfare superior to what can be achieved in the free market.

As was perhaps to be expected, the situation changes when the compulsory insurance
scheme can be supplemented by individual insurance. In this case the mixed equilibrium,
where coverage of the compulsory scheme is determined by majority decision and the
cost of supplementary insurance by market conditions, is welfare superior (in the sense of
Hicks-domination) to the equilibrium in a market with no compulsory insurance.

Since political conditions of majority enters as an important part of our argument, we use
a model with an arbitrary number of different risk characteristics (rather than the two-type
model which is common in the literature). As a model of a comprehensive national health insurance system, our approach is admittedly crude; on the other hand, it allows for some welfare comparisons of alternative organizational principles, and in this way it may be seen as contributing the redesign of health financing based on insights from modern economic theory along with, e.g. Diamond (1992).

The paper is organized as follows: in Section 1, we introduce the basic model, and in Section 2 we discuss allocations obtained in the market; which is essentially a simple version of the by now classical adverse selection model introduced by Rothschild and Stiglitz (1976). In the Section 3, we consider the political equilibrium of a compulsory insurance scheme and compare it to the market solution. In Section 4, we consider a somewhat more refined model of the market, following Pauly (1974), Johnson (1977), and Dahlby (1981), where people can buy partial as well as full coverage, and show that the conclusions of the previous section carries over also to this case. In Section 5, we consider a mixed system, where there is a market for voluntary insurance on the top of the compulsory scheme, and Section 6 contains some concluding comments on the model and its possible extensions. Proofs of the propositions has been gathered into the final Section 7.

2. A simple model of health insurance

In the following, we consider a model of insurance against the expenses caused by illness which is a simple version of the classical health insurance model of Arrow (1963). All cases of illness and subsequent treatments are subsumed in a single uncertain loss of size \( L > 0 \).

The insurance scheme of our model is one where the individual pays a premium and is then reimbursed, fully or partially, in the case of loss. We let \( B \) denote the reimbursement, \( 0 \leq B \leq L \).

Individuals are characterized by their individual risk (which in our simple model is fixed and beyond the influence of the individual itself) and their utility function on income, which describes the attitude towards risk. The individual risk is given by the probability \( \pi \) of incurring the loss \( L \); \( \pi \) belongs to an interval \( E = [\pi_{\text{min}}, \pi_{\text{max}}] \) and has distribution function \( F \). Let \( \bar{\pi} \) denote the mean of this distribution, \( \bar{\pi} = \int_{\pi_{\text{min}}}^{\pi_{\text{max}}} \pi \, dF(\pi) \). It is a crucial feature of our model that there is asymmetric information: the individual risk is private knowledge of the individual.

For later use, we introduce the quantity:

\[
C(\pi', B) = \frac{1}{1 - F(\pi')} \int_{\pi'}^{\pi_{\text{max}}} \pi \, B \, dF(\pi)
\]

which is average outlays of the insurer per insured individual with risk greater than or equal to \( \pi' \) when reimbursement is \( B \); \( C \) is differentiable in the second argument, and

\[
\frac{\partial C(\pi', B)}{\partial B} = \frac{1}{1 - F(\pi')} \int_{\pi'}^{\pi_{\text{max}}} \pi \, dF(\pi)
\]

does not depend on \( B \); its value taken at \((\pi_{\text{min}}, B)\) is \( \bar{\pi} \).

Each consumer is endowed with a von Neumann–Morgenstern utility function \( u \) on alternative levels of wealth; the amount \( P \) that an individual with risk parameter \( \pi \) is
willing to pay in order to obtain the amount $B$ as (partial) reimbursement of losses may be found as

$$\pi u(W - P - (L - B)) + (1 - \pi) u(W - P) = \pi u(W - L) + (1 - \pi) u(W)$$  \hspace{1cm} (2)

where $W$ is the initial wealth assumed to be the same for all individuals. The Eq. (2) defines $P = P(\pi, B)$ as a function of $\pi$ and $B$, and its derivatives with respect to $B$ can be found using the implicit function theorem as

$$\frac{\partial P}{\partial B} = \left[ 1 + \frac{1 - \pi}{\pi} \frac{u'(W - P)}{u'(W - P - (L - B))} \right]^{-1}$$  \hspace{1cm} (3)

The second partial derivative of $P$ w.r.t. $B$ is non-positive, so that $P(\pi, \cdot)$ is a concave function; also, we have that $P(\pi, kL) \geq kP(\pi, L)$ for $0 \leq k \leq 1$.

Let

$$\kappa(\pi) = 1 + \frac{(1 - \pi) u'(W)}{u'(W - L)}$$  \hspace{1cm} (4)

$\kappa(\pi)$ is the inverse of the marginal premium-coverage rate when no insurance is taken, and the it is small when the individual is sufficiently risk averse.

An insurance allocation is a pair $(B(\cdot), p)$, where $B(\cdot)$ is a function assigning a coverage $B(\pi)$ to each type $\pi$, and $p$ is a payment per unit of coverage. We assume that allocations are individually rational in the sense that no individual buys more coverage that according to her willingness to pay:

$$pB(\pi) \leq P(\pi, B(\pi)), \quad \text{all } \pi$$  \hspace{1cm} (5)

As a first step in our discussion of insurance allocations, we introduce a particular class of allocations $(B(\cdot), p)$, namely such where individuals must choose either $B(\pi) = L$ or $B(\pi) = 0$, so that the choice of the individual is between full insurance or no insurance at all. If furthermore, we demand that the payment for insurance should cover the cost, then the premium $P = pL$ to be paid if (full) insurance is chosen must be such that

$$\int_{\{\pi | P \leq P(\pi, L)\}} (p - \pi) L d F(\pi) = 0$$

so that the premium equals average cost of insurance over all individuals of type $\pi$ taking insurance.

An insurance allocation with these properties will be called a voluntary full insurance allocation. It should be emphasized that the word ‘full’ refers to the fact that each individual takes an insurance with full reimbursement if she takes any, and not to universal coverage in the sense that everyone has insurance, which is not necessarily the case. Indeed, a voluntary full insurance allocation is characterized by a parameter value $\pi^0$ such that

$$P = P(\pi^0, L) = C(\pi^0, L)$$  \hspace{1cm} (6)

individuals with risk parameter $\pi \geq \pi^0$ have $P(\pi, L) \geq P(\pi^0, L)$ (since $P(\cdot, L)$ is non-decreasing) and take insurance while the remaining individuals do not.
The problem of existence of a voluntary full insurance allocation is resolved rather trivially: if \( P(\pi_{\text{min}}, L) \geq C(\pi_{\text{min}}, L) \), then we have a full insurance equilibrium where everybody takes insurance at the premium \( P = C(\pi_{\text{min}}, L) \). Since \( P(\pi_{\text{max}}, L) > C(\pi_{\text{max}}, L) = \pi_{\text{max}} L \) (individuals are assumed risk averse), we get by continuity that there is some \( \pi \in [\pi_{\text{min}}, \pi_{\text{max}}] \) such that (6) is satisfied. It is not excluded that there can be voluntary full insurance allocations, namely if \( P(\pi, L) - C(\pi, L) \) is not monotonically decreasing in \( \pi \).

The voluntary full insurance allocation will be useful in comparisons of the different types of equilibria to be introduced in the sequel. We have refrained from calling it an "equilibrium" even though this terminology can be found in the literature (cf. e.g. (Hirshleifer and Riley, 1992)). The reason for this is that there are conceptual difficulties in describing realistic institutions such that the voluntary full insurance allocation would actually obtain as a result of equilibrium choices of insurance companies and individuals. Indeed, the condition (6) which takes the form of a zero-profit condition assuming that only contracts with full coverage is provided, but both conditions are questionable. Indeed, in the seminal contribution by Rothschild and Stiglitz (1976), there is a possibility of partial coverage, and insurance companies are profit maximizers. In a competitive market with possibility of proposing partial coverage, there would typically be several different contracts in the market, and in most cases there is a possibility of earning additional profits by proposing contracts differing from those already in the market, thus giving rise to problems of existence. These problems have been dealt with by several authors, cf. e.g. Wilson (1977), Spence (1978), Riley (1979).

The main advantage of our approach is of course its simplicity, and our main conclusion is the following. In a voluntary full insurance allocation, individuals belonging to \( \{\pi | P(\pi, L) < C(\pi, L)\} \) choose to remain uninsured; they would have taken an insurance geared to their own risk parameter (which, unfortunately, is unobservable to others), but they do not want to pay what it costs to insure the more risky individuals. This is the phenomenon of adverse selection, leading to a welfare loss since everyone is risk averse and wants insurance, but some individuals may not get any.

3. The political equilibrium

The suboptimality of the voluntary full insurance allocation is to some extent due to the fact that some individuals do not participate, and it might therefore be conjectured that this problem could be overcome by making health insurance compulsory. Improvement of the market solution by the introduction of a compulsory scheme has been considered by several authors, cf. Akerlof (1970), Pauly (1974), Johnson (1977), Dahlby (1981), Cave (1985). In this section, we consider a compulsory insurance scheme where the coverage is chosen by majority decision and compare it to the voluntary full insurance allocation of Section 2.

A compulsory insurance scheme has the obvious advantage that universal participation gives a lower premium than when some individuals stay outside the scheme. However, the low-risk consumers may still not be satisfied with it; when insurance costs too much compared to what it gives, the unsatisfied individuals will presumably want less of it, in our model expressed by a small value of the reimbursement level \( B \). For a given value of \( B \), the individuals who regard the compulsory insurance scheme as too expensive have risk
parameters in the set:

\[ E_B = \{ \pi | P(\pi, B) < C(\pi, B) \} \]

Remaining individuals find the insurance scheme advantageous.

Since insurance is no more a matter of individual decisions in the market, it seems reasonable that the forces regulating the contract should be subject to political control. We formulate this in a very simplistic way by the assumption that the insurance scheme should not be rejected by a majority:

\[ \int_{E_B} dF(\pi) \leq \frac{1}{2} \quad (7) \]

Formally, we define a political equilibrium of our model as a reimbursement level \( B^* \) satisfying (7) and maximal with this property. Thus, in the equilibrium compulsory insurance is provided up to the level where an increase in reimbursement would be rejected by a majority.

The existence question is now non-trivial and needs a special investigation; also, we are interested in cases where the political equilibrium satisfies \( B^* L \). The results are listed in Proposition 1 below; its proof can be found in Section 7.

**Proposition 1.** Assume that \( \kappa(\pi) < \frac{1}{\bar{\pi}} \) for all \( \pi \). Then a political equilibrium exists. Moreover, if \( P(\pi_m, L) < C(\pi, L) \), where \( \pi_m \) is the median of the distribution of \( \pi \), then \( B^* L \) in the political equilibrium.

The condition for the reimbursement in the political equilibrium to be smaller than \( L \) implies that the median \( \pi_m \) is smaller than any \( \pi^0 \) characterizing a voluntary full insurance allocation. Indeed, suppose that \( \pi_m \) exceeds the smallest such \( \pi^0 \), for which we must have \( C(\pi, L) = P(\pi^0, L) \); using monotonicity in \( \pi \) of both functions, we get that

\[ C(\pi, L) \leq C(\pi^0, L) = P(\pi^0, L) \leq P(\pi_m, L) \]

contradicting the condition \( C(\pi, L) > P(\pi_m, L) \), so \( \pi_m \leq \pi^0 \).

It is possible to get some intuitive feeling for what goes on in the model by elaborating slightly on the graphical representation of the adverse selection model given in Hirshleifer and Riley (1992), as shown in Fig. 1. We have the parameter space on the horizontal axis and premiums or average costs along the vertical axis. The density function of the distribution is not shown in the figure.

With full insurance, the average outlays of the insurance company to individuals with risk parameter \( \pi \) is given by the straight line \( L\pi \); the average reimbursement to individuals with risk \( \pi \) is the curve \( C(\pi, L) \), and finally, willingness to pay for the individual of type \( \pi \) is given by \( P(\pi, L) \). The voluntary full insurance allocation is defined by the intersection between \( P(\cdot, L) \) and \( C(\cdot, L) \), which occurs at \( \pi^0 \). Agents with risk parameter below \( \pi^0 \) do not insure, agents with risk parameter above \( \pi^0 \) get full coverage.

Consider now the political equilibrium. Since \( P(\pi_m, L) < C(\pi_m, L) \), there is a majority which considers the premium paid for an all-embracing compulsory full insurance too high (this is seen from the fact that the premium \( C(\pi_m, L) \) is greater than the willingness to pay
for all $\pi$ smaller than the value at corresponding to the point $A$ in the figure). Consequently, it will be reduced to some $B^* < L$, such that

$$P(\pi_m, B^*) = C(\pi_{\min}, B^*)$$

In the figure, the straight line $L \pi$ becomes $B^* \pi$, the reimbursement $C(\pi, L)$ is changed to $C(\pi, B^*)$, and willingness to pay reduces to $P(\pi, B^*)$. The reimbursement level $B^*$ is determined in such a way that the horizontal line at $C(\pi_{\min}, B^*)$ and $P(\cdot, B^*)$ intersects in the point $B$ at $\pi = \pi_m$. Clearly, $B^*$ is then a political equilibrium.

To achieve this equilibrium, the $P$- and $C$-curves had to move downwards from the situation with full reimbursement. That this will eventually take us to an equilibrium is shown in the proof of Proposition 1. Intuitively, the reason why it works is that when reimbursement levels are reduced, the cost of insurance is reduced in a linear way, while the willingness to pay (which is given by a concave function) reduces by less than cost, so that eventually they become equal.

Comparing the voluntary full insurance allocation and the political equilibrium the most obvious difference is of course that some people were left with no insurance (or rather chose not to be covered) in the first, while in the political equilibrium the insurance scheme covers everyone, albeit not fully. This is not surprising, since the model was constructed in this way. It is more interesting to notice that the gains and losses are unevenly distributed: the low risks have obtained insurance at an acceptable cost, whereas the high risks have to accept that only part of their losses are reimbursed. In our simple model, we may consider...
the overall effect by looking at average expected utility. The following result tells us that a transition from a voluntary full insurance allocation to a political equilibrium may be welfare reducing.

**Proposition 2.** Assume that the mean of the distribution of \( \pi \) does not exceed the median; then average expected utility is higher in any full insurance equilibrium than in the political equilibrium.

**Remarks.**

(1) It can be seen from the proof of Proposition 2 that the result holds even if \( \bar{\pi} \) slightly exceeds \( \pi_m \); if \( \pi_m \geq \bar{\pi} \), then any voluntary full insurance allocation gives higher average utility than the political equilibrium. By continuity, there is \( \varepsilon > 0 \) such that the result remains true when \( \| \pi_m - \bar{\pi} \| < \varepsilon \).

(2) In the case of a symmetric distribution (\( \pi_m = \bar{\pi} \)), we have equality in (9), so that aggregate utility in the political equilibrium equals aggregate utility without any insurance. Intuitively, the gain which an individual with lower-than-average risk obtains by having some insurance in the political equilibrium is exactly offset by the loss of the symmetric high-risk individual paying too much for the insurance.

(3) If \( \bar{\pi} \) exceeds \( \pi_m \), then the political equilibrium is better (on the average) than no insurance, but not necessarily better than the voluntary full insurance allocation; however, for \( \pi_m \) large enough, this will be the case.

Thus, in cases where the distribution of risk parameters in the population is such that the mean and the median are close, we can show that the insurance scheme of the political equilibrium is inferior to that given by a voluntary full insurance allocation. The result may hold for other distributions as well but it will depend on the exact form of the distribution.

The result obtained is as yet incomplete, since we compare an equilibrium allocation (namely that of political equilibrium) with an allocation which might not be sustainable in a reasonable institutional framework, and indeed we have only performed the first step—to be followed by another one in the next section—towards a meaningful welfare comparison of equilibria. It is, however, remarkable that the attempt to solve allocation problems by what could be considered as democratic decision (majority voting) may be rather unsatisfactory from a welfare point of view.

4. Compulsory insurance and the Pauly–Johnson equilibrium

As mentioned in Section 2, the notion of a voluntary full insurance allocation which was the point of departure for the welfare comparisons in Section 3, is somewhat artificial; it might seem more reasonable, and indeed in accordance with the literature in the field, e.g. Pauly (1974), Johnson (1977), Dahlby (1981), that individuals are allowed to buy insurance of arbitrary coverage at a given competitive price rather than that the choice should be only between buying full coverage or nothing at all. In this section, we consider a particular type of equilibrium with free choice of coverage by individuals and free entry by insurance firms. It is shown that equilibrium allocations will be Pareto-superior to voluntary full
insurance allocations, a result which then may be used to complete the the comparison with
the political equilibrium of Section 3.

Following Dahlby (1981), we introduce equilibria given by the premium-coverage ratio
$p$, at which the individuals may buy coverage, so that a coverage $B$ will entail a premium
of $pB$. An allocation $(B(\cdot), p)$ is said to be a Pauly–Johnson equilibrium, if:

(i) it is individually optimal in the sense that for each $\pi$, $B(\pi)$ is the amount of coverage
which maximizes utility of type $\pi$ at the premium-coverage rate $p$, that is

$$B(\pi) = \arg\max_{B \in [0, L]} [\pi u(W - pB - (L - B)) + (1 - \pi)u(W - pB)]$$

(ii) it satisfies the no-profit condition

$$\int_{\pi_{\text{min}}}^{\pi_{\text{max}}} (p - \pi)B(\pi) dF(\pi) = 0$$

The first-order conditions for individual optimum in (i) are

$$\frac{\partial}{\partial B} [\pi u(W - pB - (L - B)) + (1 - \pi)u(W - pB)]$$
$$= \pi u'(W - pB - (L - B)) + (1 - \pi)u'(W - pB) \geq 0$$

for an optimum satisfying $0 < B \leq L$, with inequality in the case of $B = L$, where the
condition reduces to

$$\pi \geq p$$

While the present notion of equilibrium opens for contracts with less than full reimburse-
ment, it is subject to the same shortcomings as those mentioned in Section 2 with respect to
sustainability as equilibrium in an insurance market. In particular, it should be noticed that
the equilibrium conditions require only that insurers break even on each contract, without
considering whether another insurer might enter the market, cream skim and make this
contract non-viable.

In a voluntary full insurance allocation characterized by $\pi^0$, only the individuals with
risk parameter $\pi \geq C(\pi^0, L)/L$ will buy full coverage if they can buy any coverage at the
premium-coverage ratio $p = C(\pi^0, L)/L$. All other individuals will buy less than full
coverage, and this of course has an impact on the average cost of insurance. Thus, going
from a voluntary full insurance allocation to a Pauly–Johnson equilibrium, the situation of
the individuals is improved by their increased possibility of choice, but this gain might be
offset by a rise in the cost of insurance due to the reduced coverage. The following result
shows that this does not happen in a reasonably broad class of situations.

**Proposition 3.** Let a voluntary full insurance allocation be given by $\pi^0$, and assume that
at the premium-coverage ratio $p^0 = C(\pi^0, L)/L$.

(i) all individuals prefer some coverage to no insurance, and
(ii) individuals of type $\bar{\pi}$ prefer partial to full coverage.
Then there exists a Pauly–Johnson equilibrium in which everyone is at least as well off as in the voluntary full insurance allocation.

We may combine Propositions 2 and 3 to get the following corollary.

**Corollary.** Assume that the mean of the distribution of \( \pi \) does not exceed the median, and that the voluntary full insurance allocation is uniquely defined and has the following properties:

(i) all individuals prefer some coverage to no insurance, and
(ii) individuals of type \( \bar{\pi} \) will prefer partial to full coverage.

Then there is a Pauly–Johnson equilibrium such that average utility is greater than in the political equilibrium.

**Proof.** By Proposition 2, the voluntary full insurance allocation yields greater average utility than the political equilibrium, and by Proposition 3, there is a Pauly–Johnson equilibrium such that every individual is even better off. □

It should be emphasized that the inferiority of the political equilibrium was obtained under specific assumptions on the distribution of characteristics, and if these assumptions are not satisfied, the political equilibrium may indeed be better than Pauly–Johnson equilibria. That this is indeed the case can be seen from an example given by Dahlby (1981) of an economy with two types of risk where the political equilibrium is a Pareto-improvement of the Pauly–Johnson equilibrium.

5. The case of compulsory plus voluntary insurance

In the previous sections, we have compared the two extreme cases of voluntary insurance and compulsory insurance; in real life what is encountered is usually a mixture of the two schemes. There is a compulsory scheme the coverage of which is fixed by majority decision, but individuals may buy additional insurance in the market. Indeed, since the compulsory insurance scheme may provide some individuals with a lower level of reimbursement than they desire, a natural extension of the institutional setup considered would be a mixed solution, combining the compulsory scheme with voluntary insurance contracts. Thus, agents may buy insurance contracts which cover some or all of the loss remaining after payment of the reimbursement from the compulsory scheme. We shall investigate how this setup compares with the two extreme cases of only voluntary and compulsory insurance.

To describe the outcome in a situation with both compulsory and voluntary insurance, we shall need to consider pairs of insurance allocations, one defined by the compulsory insurance, where we need only to specify the coverage \( B \), since the payment per unit of coverage follows as \( C(\pi_{\min}, B)/B \), and an additional insurance allocation \( (\hat{B}(\cdot), \hat{p}) \) from the voluntary insurance market. In order to define an equilibrium, which in the present context will be called a mixed equilibrium, we need to combine the conditions discussed...
previously for each of the two types of insurance provision. Thus, the compulsory insurance should not be considered too expensive by a majority, and the coverage chosen in the voluntary insurance market should be optimal for each $\pi$ at the premium-coverage ratio $\hat{\rho}$; each of these conditions must hold given the choices made in the other insurance market.

In order to formulate the equilibrium conditions, we need an extension of the notion of willingness to pay introduced in Section 2. Given that an individual of type $\pi$ buys coverage $\hat{B}(\pi)$ and pays the premium $\hat{\rho}\hat{B}(\pi)$, the willingness to pay for the compulsory insurance with coverage $B$, $P(\pi, B; (\hat{B}(\pi), \pi\hat{B}(\pi)))$, is defined as the solution $P$ to

$$\pi u(W - \hat{\rho}\hat{B}(\pi) - P - (L - \hat{B}(\pi) - B)) + (1 - \pi)u(W - \hat{\rho}\hat{B}(\pi) - P)$$

$$= \pi u(W - \hat{\rho}\hat{B}(\pi) - (L - \hat{B}(\pi))) + (1 - \pi)u(W - \hat{\rho}\hat{B}(\pi))$$

which is the obvious extension of (2). The quantity $P(\pi, B)$ used in the previous sections becomes $P(\pi, B; (0, 0))$ in our new formulation.

Now, we may state the equilibrium condition pertaining to compulsory insurance in the mixed equilibrium $(B, (\hat{B}(\cdot), \hat{\rho}))$: the level $B$ of coverage should be maximal among all $B$ satisfying:

$$\int_{E_{\hat{B}}} dF(\pi) \leq \frac{1}{2}$$

where $E_{\hat{B}} = \{\pi \mid P(\pi, B; (\hat{B}(\pi), \hat{\rho}\hat{B}(\pi))) < C(\pi_{\min}, B)\}$, should be a majority. For voluntary insurance, we have that the two conditions of the Pauly–Johnson equilibrium should be satisfied, in the present case assuming the level of compulsory insurance to be given, so that

(i) $\hat{B}(\cdot)$ is individually optimal in the sense that for each $\pi$, $\hat{B}(\pi)$ is given by

$$\hat{B}(\pi) = \arg\max_{\hat{B} \in [0, L]} [\pi u(W - C(\pi_{\min}, B) - \hat{\rho}\hat{B} - (L - B - \hat{B})) + (1 - \pi)u(W - C(\pi_{\min}, B) - \hat{\rho}\hat{B})]$$

(ii) It satisfies the no-profit condition

$$\int_{\pi_{\min}}^{\pi_{\max}} (\hat{\rho} - \pi)\hat{B}(\pi) dF(\pi) = 0$$

Intuitively, it should be expected that the present setup combines the positive aspects of the compulsory scheme, notably the low cost of insurance, with those of the market (freedom of choice), and therefore the mixed equilibrium should be welfare superior in some sense to each of the previously introduced equilibria. In the remaining part of this section we investigate how far this intuition can actually be proved. For the comparison with the political equilibrium, the situation is rather simple and the result correspondingly unambiguous:

**Proposition 4.** For any political equilibrium there is a Pareto improving mixed equilibrium.
The comparison with the Pauly–Johnson equilibrium (without compulsory insurance) is less straightforward. Clearly some individuals may be better off if compulsory insurance is abandoned, namely at least those taking no voluntary insurance in the mixed equilibrium. However, the remaining individuals are hurt by the fact that some of the low-risk individuals no longer participate so that insurance becomes more expensive. The final outcome will depend on both the underlying utility function and on the distribution of risk types. The following proposition shows that in some situations, depending on the utility function specifying the attitude towards risk and the distribution of risk in the population, the market solution given by the Pauly–Johnson equilibrium may indeed be better than the mixed equilibrium according to the Kaldor criterion:

**Proposition 5.** Let \((B, (\hat{B}(\cdot), \hat{p}))\) be a mixed equilibrium, and let \(E_B = \{\pi | P(\pi, B) < C(\pi_{\min}, B)\}\). Further, let \((B(\cdot), p)\) be a Pauly–Johnson equilibrium. Assume that

\[
\bar{\pi}B > [1 - P_F(E_B)]pB + \int_{E_B} pB(\pi) dF(\pi)
\]

Then the individuals in \(E_B\) gaining from a transition from the mixed equilibrium to the Pauly–Johnson equilibrium can compensate the remaining individuals.

The condition in Proposition 5 will be satisfied in rather particular cases, where \(\hat{B}(\pi)\) is close to \(B\) for all \(\pi\) so that \(p\) differs only slightly from \(\bar{\pi} = C(\pi_{\min}, B)/B\). In this case, the loss due to less participation when the compulsory insurance is abandoned is small, and in suitably constructed cases it is smaller than what individuals in \(E_B\) gain from the smaller coverage.

It seems more in accordance with intuition that the mixed equilibrium should be better than the Pauly–Johnson equilibrium after suitable income transfers, and indeed it is easy to construct examples where this is the case. It is however less straightforward to give a simple criterion in the spirit of that given in Proposition 5 above, since individuals with type in \(E_B\) may pay more than the difference between the cost of insurance in the mixed equilibrium and in the Pauly–Johnson equilibrium, as they are strictly better off in the latter.

Comparisons are facilitated in the case where these individuals do not buy insurance unless they have to; this is of course an extreme case, but the result will hold if the amount that they do want to buy is sufficiently small.

**Proposition 6.** Let \((B, (\hat{B}(\cdot), \hat{p}))\) be a mixed equilibrium with \(E_B = \{\pi | P(\pi, B) < C(\pi_{\min}, B)\}\), and let \((B(\cdot), p)\) be a Pauly–Johnson equilibrium. Assume that \(\hat{B}(\pi) = 0\) for \(\pi \in E_B \cap \text{supp}F\) and \(\hat{B}(\pi) = L - B\) for \(\pi \in \text{supp}F \setminus E_B\) (where \(\text{supp}F\) is the support of the distribution of \(\pi\)). Then there is no system of transfers which will compensate the losers in a transition from \((B, \pi^*(B))\) to this voluntary full insurance allocation.

In terms of classical welfare economics, the mixed equilibrium is Hicks-better than the pure market equilibrium—those preferring a change back to the market cannot compensate those who do not. This is not too surprising since the combination of some compulsory insurance with voluntary supplementing insurance tends to combine the virtues of each of the two.
6. Concluding remarks

In this paper, we have considered a simple model of insurance under conditions of adverse selection. It has been shown that the well-known shortcomings of the market solution under asymmetric information, where some individuals choose not to have insurance, may not be solved by a compulsory scheme, if the coverage of the scheme is part of the equilibrium and fixed by majority decision. On the contrary, for some distributions of the risk parameters of the individuals, the market solution may be better than a compulsory scheme, at least when measured by average utility.

The model considered is primitive in several aspects. First of all, individuals have identical utility functions on money outcomes, and this feature is important in the sense that the welfare analysis in terms of average utility would hardly make sense otherwise. However, the basic point of our paper, that all-embracing compulsory insurance is not necessarily superior to the market solution, could be obtained also without common utilities, even if the formulation would be less simple.

Other shortcomings of the model may be more serious in the sense that the model fail to cover important aspects of reality. With respect to the way in which the political choice has been modeled it should be mentioned that the median voter model used is only one of several possible models. In particular, models of regulatory capture suggest that providers are likely to play a significant role in the design of the compulsory insurance package, which might therefore be more generous than desired by the median voter.

Finally, it should be mentioned that the model treats only one type of asymmetric information, namely adverse selection. In particular, there is no moral hazard in the model, whereas in real life, adverse selection and moral hazard come together. The joint effect of adverse selection and moral hazard is not understood if the two phenomena are considered separately; mechanisms which reduce the effect of either adverse selection or moral hazard may simultaneously work against the other type of market failure, cf. Dionne and Doherty (1992).

In order to treat such mechanisms, the model would most likely have to be extended from a one-period to a multi-period context, cf. e.g. Dionne and Lasserre (1985). A model of health insurance contracts which may depend on past performance may indeed be an important extension of the present model. However, this will be a topic of future research.

7. Proofs of propositions

In this section, we have collected the proofs of the Propositions 1–4 stated in the previous sections.

**Proof of Proposition 1.** If \( C(\pi_{\text{min}}, L) \leq P(\pi_m, L) \), then \( L \) is a political equilibrium; indeed, since \( P(\pi, L) \) is non-decreasing in \( \pi \), we have \( C(\pi_{\text{min}}, L) \leq P(\pi_m, K) \) for \( \pi_m \leq \pi \leq \pi_{\text{min}} \), so that weight of the set of individuals belonging to \( E_L = \{ \pi | P(\pi, L) < C(\pi_{\text{min}}, L) \} \) is at most 1/2.

Suppose now that \( C(\pi_{\text{min}}, L) > P(\pi_m, L) \). The function \( \Phi \) defined by

\[
\Phi(B) = C(\pi_{\text{min}}, B) - P(\pi_m, B)
\]

...
is defined and differentiable for all \( B \) in an open interval containing \([0, L]\), and by our assumption, \( \Phi(L) > 0 \). Moreover, \( \Phi(0) = 0 \).

For \( B = 0 \), we have

\[
\Phi'(0) = \left. \frac{\partial C(\pi_{\text{min}}, \cdot)}{\partial B} \right|_{B=0} - \left. \frac{\partial P(\pi_{\text{m}}, \cdot)}{\partial B} \right|_{B=0}
\]

We know already that the derivative of \( C(\pi_{\text{min}}, B) \) w.r.t. \( B \) equals \( \bar{\pi} \) for all values of \( B \). For the second term, we have that

\[
\left. \frac{\partial P(\pi_{\text{m}}, \cdot)}{\partial B} \right|_{B=0} = \left[ 1 + \frac{1 - \pi_{\text{m}}}{\pi_{\text{m}}} \frac{u'(W - P)}{u'(W - (L - B))} \right]^{-1} > \frac{1}{\kappa} > \bar{\pi}
\]

where we have used our assumption in (5) on risk aversion. It follows that \( \Phi'(0) < 0 \).

Now we have that the differentiable function \( \Phi \) satisfies \( \Phi(0) = 0 \) and \( \Phi'(0) < 0 \), so there must be some \( \tilde{B} > 0 \) such that \( \Phi(\tilde{B}) < 0 \). Since \( \Phi(L) > 0 \), there must be some \( B^* \) with \( B^* < B^* < L \) such that \( \Phi(B^*) = 0 \) or \( C(\pi_{\text{min}}, B^*) = P(\pi_{\text{m}}, B^*) \), giving a political equilibrium with the desired properties. \( \square \)

**Proof of Proposition 2.** In the political equilibrium with reimbursement level \( B^* \), the average expected utility is

\[
\int_{\pi_{\text{min}}}^{\pi_{\text{m}}} [\pi u(W - P(\pi_{\text{m}}, B^*) - (L - B^*)) + (1 - \pi)u(W - P(\pi_{\text{m}}, B^*))] \, dF(\pi)
\]

\[
= \bar{\pi} u(W - P(\pi_{\text{m}}, B^*) - (L - B^*)) + (1 - \bar{\pi})u(W - P(\pi_{\text{m}}, B^*))
\]

By our assumption, we have that \( \pi_{\text{m}} \geq \bar{\pi} \); it follows that \( P(\pi_{\text{m}}, B^*) \geq P(\bar{\pi}, B^*) \) since \( P(\cdot, B^*) \) is non-decreasing, and by monotonicity of \( u \), we conclude that

\[
\bar{\pi} u(W - P(\pi_{\text{m}}, B^*) - (L - B^*)) + (1 - \bar{\pi})u(W - P(\pi_{\text{m}}, B^*))
\]

\[
\leq \bar{\pi} u(W - P(\bar{\pi}, B^*) - (L - B^*)) + (1 - \bar{\pi})u(W - P(\bar{\pi}, B^*))
\]

\[
= \bar{\pi} u(W - L) + (1 - \bar{\pi})u(W)
\]

where the last equality sign follows from the definition of \( P(\bar{\pi}, B^*) \). Now the last expression may alternatively be written as

\[
\int_{\pi_{\text{min}}}^{\pi_{\text{m}}} [\pi u(W - L) + (1 - \pi)u(W)] \, dF(\pi)
\]

(12)

which is the average expected utility without any insurance. However, in the voluntary full insurance allocation part of the consumers choose insurance, which therefore must leave them better off than if they had no insurance at all. This means that average expected utility in the market solution is greater than (12), and consequently greater than the left hand side of (10). \( \square \)

**Proof of Proposition 3.** First of all we show that at the premium-coverage rate \( p^0 \), if every individual chooses optimal coverage \( B(\pi, p^0) \), then

\[
\int_{\pi_{\text{min}}}^{\pi_{\text{m}}} (p^0 - \pi)B(\pi, p^0) \geq 0
\]
Indeed, since every individual takes some coverage, and since $B(\pi, p)$ is non-decreasing in $\pi$, we have that
\[\int_{\pi_{\min}}^{\pi_{\max}} (p^0 - \pi) B(\pi, p^0) \, dF(\pi) \geq \int_{\pi_{\min}}^{\pi_{\max}} (p^0 - \pi) B(\pi_{\min}, p^0) \, dF(\pi) = (p^0 - \bar{\pi}) B(\pi_{\min}, p^0) \geq 0 \] (13)
where the last inequality follows from the fact that individuals of type $\bar{\pi}$ do not want full coverage, so that $\bar{\pi} \leq p^0$.

If (13) holds with equality, then $p^0$ defines a Pauly–Johnson equilibrium, and we are done. Suppose therefore that the integral on the right hand side is strictly positive. It may be checked (using e.g. the maximum theorem, cf. Hildenbrand (1974), Theorem 0.B.III.3) that the integral $\int_{\pi_{\min}}^{\pi_{\max}} (p - \pi) B(\pi, p) \, dF(\pi)$ is a continuous function of $p$ in the interval $[0, p^0]$. Since for $p = 0$, the value of the integral is $-\int_{\pi_{\min}}^{\pi_{\max}} \pi B(\pi, 0) \, dF(\pi) = -\int_{\pi_{\min}}^{\pi_{\max}} \pi L \, dF(\pi) = -\bar{\pi} L < 0$ where we have used that $B(\pi, p) \geq B(\pi, p^0)$ for each $\pi$ (the optimal coverage level is non-increasing in $p$), we get by continuity that there is $\hat{p} \in [0, p^0]$ such that $\int_{\pi_{\min}}^{\pi_{\max}} (\hat{p} - \pi) B(\pi, \hat{p}) \, dF(\pi) = 0$. Clearly $\hat{p}$ is a Pauly–Johnson equilibrium, and since for each $\pi$, individuals with risk parameter $\pi$ are at least as well off with coverage $B(\pi, p)$ as with $B(\pi, p^0)$, we have the desired result. □

**Proof of Proposition 4.** We show that if $(B, (\hat{B}(\cdot), \hat{p}))$ is a mixed equilibrium, then $B$ also defines a political equilibrium (so that the availability of voluntary supplementary insurance does not change the political equilibrium conditions). Since, by the remark following the proof of Proposition 1, the political equilibrium is uniquely defined, we get that the mixed equilibrium differs from the political equilibrium only in that the individuals have access to some additional improvement of their situation, and the statement of proposition follows.

Thus, let $(B, (\hat{B}(\cdot), \hat{p}))$ be a mixed equilibrium, and let $\tilde{\pi} = \inf\{\pi | \hat{B}(\pi) > 0\}$. We claim that $\hat{p} \geq C(\pi_{\min}, B)/B$, that is the voluntary insurance is at least as expensive per unit of coverage as the compulsory insurance. Indeed, if $\pi' \geq \pi$, then $\hat{B}(\pi') \geq \hat{B}(\pi)$ (the greater risk, the more insurance is optimal at the given prices), and for $\tilde{\pi} > \tilde{\pi}$ we have
\[\frac{\int_{\tilde{\pi}}^{\pi_{\max}} \pi \hat{B}(\pi) \, dF(\pi)}{\int_{\tilde{\pi}}^{\pi_{\max}} \hat{B}(\pi) \, dF(\pi)} \geq \frac{[1 - F(\hat{\tilde{\pi}})] \int_{\tilde{\pi}}^{\pi_{\max}} \pi B(\hat{\tilde{\pi}}) \, dF(\pi)}{B(\hat{\tilde{\pi}})} = [1 - F(\hat{\tilde{\pi}})] \int_{\tilde{\pi}}^{\pi_{\max}} \pi \, dF(\pi) \geq \tilde{\pi},\]
so that in the limit, we get from the zero-profit condition that
\[\hat{p} = \frac{\int \pi \hat{B}(\pi) \, dF(\pi)}{\int B(\pi) \, dF(\pi)} = \lim_{\tilde{\pi} \to \tilde{\pi}} \frac{\int_{\tilde{\pi}}^{\pi_{\max}} \pi \hat{B}(\pi) \, dF(\pi)}{\int_{\tilde{\pi}}^{\pi_{\max}} B(\pi) \, dF(\pi)} \geq \tilde{\pi}\]
and since $\tilde{\pi} = C(\pi_{\min}, B)/B$, this proves our claim.
Now, since voluntary insurance is more expensive than compulsory insurance per unit of coverage, we have $P(\pi, B; (\hat{B}(\pi), \pi)) > C(\pi_{\text{min}}, B)$ and a fortiori $P(\pi, B) > C(\pi_{\text{min}}, B)$. If $\hat{B}(\pi) = 0$, then $P(\pi, B; (\hat{B}(\pi), \hat{p})) = P(\pi, B)$, and therefore

$$P(\pi, B; (\hat{B}(\pi), \hat{p})) \leq C(\pi_{\text{min}}, B)$$

for all $\pi$, so that $B$ satisfies the conditions of a political equilibrium.

\[ \square \]

**Proof of Proposition 5.** If each of the individuals with $\pi \notin E_B$ are paid an amount allowing them to buy the first $B$ units of coverage at the price $p$ rather than at the price $C(\pi_{\text{min}}, B)/B = \tilde{\pi}$, then these individuals will be at least as well off in the Pauly–Johnson equilibrium $(B(\cdot), p)$ as in the mixed equilibrium, since the remaining coverage can be obtained at a price $p$ which is no greater than $\hat{p}$ (the fact that $p \leq \hat{p}$ can be seen by the same reasoning as in the proof of Proposition 4).

Now, this payment must equal $(1 - P_F(E_B))[p - \tilde{\pi}]B$. On the other hand, the individuals with $\pi \in E_B$ have a gain of

$$\int_{E_B} [C(\pi_{\text{min}}, B) - pB(\pi)]dF(\pi) = P_F(E_B)\tilde{\pi}B - \int_{E_B} pB(\pi)dF(\pi)$$

and from the assumption of the theorem, we get that

$$\tilde{\pi}B = (1 - P_F(E_B))\tilde{\pi}B + P_F(E_B)\tilde{\pi}B > [1 - P_F(E_B)]pB + \int_{E_B} pB(\pi)dF(\pi)$$

or

$$(1 - P_F(E_B))[p - \tilde{\pi}]B < P_F(E_B)\tilde{\pi} - \int_{E_B} pB(\pi)dF(\pi)$$

showing that the compensation is indeed feasible.

\[ \square \]

**Proof of Proposition 6.** Since

$$C(\pi_{\text{min}}, B) + C(\pi, L - B) \leq C(\pi, L)$$

and

$$P(\pi, L - B, (B, C(\pi_{\text{min}}, B))) + P(\pi, B, (L - B, \hat{p}(L - B))) \geq P(\pi, L)$$

for all $\pi$, we have that if $P(\pi, L) \geq C(\pi, L)$, then individuals of type $\pi$ will take full insurance also in the mixed equilibrium, and they will pay less (since it is cheaper due to the element of the compulsory risk sharing), and conversely, if individuals of type $\pi$ take full insurance in the mixed equilibrium and none in the voluntary full insurance allocation, then they are worse off in the latter.

We have therefore that if an individual becomes better off by the change from the mixed equilibrium to the voluntary full insurance allocation, then the type of the individual is in $E_B$, and the compensation which such individuals are willing to pay to the rest in order to change to a the market solution cannot exceed

$$\int_{E_B} [C(\pi_{\text{min}}, B) - P(\pi, B)]dF(\pi) < \int_{E_B} [C(\pi_{\text{min}}, B) - \pi B]dF(\pi)$$

(14)
where we have replaced the actual willingness to pay with that of a risk neutral individual with the same risk. Reformulating the integral, we get that

\[
\int_{E_B} \left[ C(\pi_{\text{min}}, B) - \pi B \right] dF(\pi) = C(\pi_{\text{min}}, B) P_F(E_B) - \int_{E_B} \pi B \, dF(\pi)
\]

\[
= C(\pi_{\text{min}}, B) P_F(E_B) - C(\pi_{\text{min}}, B) + \int_{E \setminus E_B} \pi B \, dF(\pi)
\]

\[
= \int_{E \setminus E_B} \pi B \, dF(\pi) - (1 - P_F(E_B)) C(\pi_{\text{min}}, B)
\]

(where we have used the notation \( P(E_B) = \int_{E_B} dF(\pi) \)). The last integral in the above expression can be recognized as \( 1 - P_F(E_B) \) times the average cost of insuring the reimbursement \( B \) when only individuals with type not in \( E_B \) participate. This amount is exactly what is needed to compensate the individuals with type \( \pi \notin E_B \) for the rise in price of insurance of the reimbursement \( B \). However, using (14) we see that amount which may be paid by individuals with \( \pi \in E_B \) falls short of this amount, and we have the conclusion. \( \square \)

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