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PASCAL'S AND TABARROK'S WAGERS

ABSTRACT. In a recent paper A. Tabarrok [Believe in Pascal's Wager? Have I Got a Deal for You!, *Theory and Decision* 48, 123–128, 2000] argued that a believer who accepts Pascal's Wager should in addition accept payment of any given fee in return for a given increase in the probability of reaching God. However the conclusion is obtained from manipulations of infinities which are not valid in an expected utility model. In this note, an alternative model is formulated in which Tabarrok's conclusion can be obtained.

KEY WORDS: Pascal's Wager, Tabarrok's Wager

1. INTRODUCTION

Pascal's Wager is the name given to an argument due to Blaise Pascal for believing in God. The idea is that, making the assumption that believing in God results in infinite utility if God exists, it pays off to believe in God in an expected utility sense, as long as the probability that God exists is greater than zero. See Hájek (2004) for historical background and a bibliography.

In a recent paper, Tabarrok (2000) argued that a believer who accepts Pascal's Wager should in addition accept any finite payment of money in return for an (arbitrarily small) increase in the probability of "reaching God".

Tabarrok's conclusion is obtained from certain manipulations of infinities, which, however, are not valid in relation to the expected utility model that seems to underlie the analysis.

In this note, we formulate an alternative model where preferences are lexicographic in the probability of obtaining "infinitely happy life", and in which Tabarrok's conclusion is valid.

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2. PASCAL'S WAGER

Pascal's Wager can be modelled as follows: Assume that there are two states of the world: "God exists" (s_1) and "God does not exist" (s_2), and let p be the probability that "God exists". There are two possible actions, "Believe in God" (a_1) and "Do not believe in God" (a_2).

Let $f(a_i, s_j)$ be the utility of action i under state j, where $f(a_1, s_1) = \infty$ and finite otherwise.[1] Expected utility of action i is then

$$E(a_i) = pf(a_i, s_1) + (1 - p)f(a_i, s_2),$$

and a preference relation $\succ$ (on actions) is governed by expected utility if

$$a_i \succ a_j \Leftrightarrow E(a_i) > E(a_j).$$

There is of course nothing wrong in defining preferences this way, but it should be noticed that standard models such as the von Neumann–Morgenstern expected utility model do not allow for infinite utilities (see e.g. Kreps, 1988).

But taking this model for granted for any $p > 0$ we now have $a_1 \succ a_2$. In words, if a person believes that there is any chance that God exists, the action "Believe in God" is strictly dominant.

3. TABARROK'S WAGER

Tabarrok was not very explicit about the underlying model, and some of the analysis also seems to involve probability distributions where the entries do not sum to 1, but if our interpretation is correct, the idea is the following: A third action is introduced which we can call "Believe in God and pay a fee" (a_3). By combining believing in God with a payment of a given fee, a person can increase the probability of reaching God from p to p'. More specifically, we have

$$p' > p, \text{ if } p > 0,$$
$$p' = 0, \text{ if } p = 0.$$

Expected utility is then defined to be $E(a_3) = p'f(a_1, s_1) + (1 - p')f(a_1, s_2) - w$, where w is the finite fee. Since $E(a_2) > E(a_3)$ for $p = 0$ and $E(a_2) = E(a_3) = \infty$ for $p > 0$, a_3 is never better than a_2.

Tabarrok reaches the opposite conclusion, a_3 is infinitely better than a_2 for any $p > 0$, by reducing the expression $E(a_3) - E(a_2)$ to $p' \times \infty - p \times \infty$ and then again to ∞. To see why such manipulations make no sense in this context, consider for example the following two sequences of pseudo-manipulations: $p' \times \infty - p \times \infty = (p' - p) \times \infty = \infty$, and $p' \times \infty - p \times \infty = p' \times \infty - (p' + p) \times \infty = (p' - p') \times \infty - p \times \infty = -\infty$, a contradiction.

4. LEXICOGRAPHIC PREFERENCES

It is possible to reach Tabarrok's conclusion from a model where preferences are lexicographic in the probability of reaching God.

An action a_i basically determines three factors: The probability p_i of establishing a contact to God, the level of utility $u_i^1 \in \mathbb{R} \cup \{\infty\}$ realized if a contact to God is established (where infinite utility corresponds to "infinitely happy life"), and the utility level $u_i^2 \in \mathbb{R}$ realized otherwise (assumed to be finite).

Define the preferences as follows:

$$a_i \succ_L a_j \Leftrightarrow \begin{bmatrix} [u_i^1 = \infty, u_j^1 = \infty \text{ and } p_i > p_j], \text{or} \\ [u_i^1 = \infty, u_j^1 = \infty, p_i = p_j \text{ and } u_i^2 > u_j^2], \text{or} \\ [p_i u_i^1 + (1 - p_i)u_i^2 > p_j u_j^1 + (1 - p_j)u_j^2] \end{bmatrix}.$$

It is not possible to represent the preference relation $\succ_L$ by a real-valued utility function (see Kreps, 1988), and reasoning using an expected utility argument is not valid. But it is clear that if $\succ_L$ represents preferences, Tabarrok's Wager should be accepted.

5. FINAL REMARK

This note has formulated simple models which are compatible with the desired conclusions from Pascal's and Tabarrok's

Wagers, and in particular it could be noticed that even accepting Pascal's Wager it does not follow that Tabarrok's Wager should be accepted.

In order to avoid misunderstandings, it should be emphasized that focus here has been on modelling, and an attempt to evaluate the relevance of decision theoretic arguments of this kind has not been made. It could seem rather limited, however, since the nature of the utilities and probabilities involved is far from clear, let alone the question of measurement and observability of these factors.

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NOTES

1. We follow here Tabarrok and consider "utilities" as primitives of the analysis; what they represent seems open to interpretation, but it appears from Tabarrok's calculations that utilities are commensurable with income and even measured in the same units. In the following, we can therefore think of utilities as income (consumption), allowing for infinite utility in case that "infinitely happy life" is obtained.

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