Trade Policy Dynamics, Entry Costs and Exchange Rate Uncertainty

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Abstract

This paper analyzes trade policy dynamics when there are sunk costs of entry and demand uncertainty. A natural generalization of the classic export tax prescription for a domestic industry facing downward-sloping foreign demand is defined and implemented as a dynamic competitive equilibrium with fully rational firms. The optimal tax rate adjustment policy is a trigger strategy. This provides a rationale for infrequent revisions of trade policy in response to exogenous shocks.

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1 Introduction

Understanding the relationship between trade policy dynamics, demand uncertainty and the entries and exits of firms has gained renewed importance in view of volatile exchange rates and collapsing world trade that accompanied the financial turmoil of 2008.[1] In this paper, I address this interplay by extending a classical policy prescription for optimal trade policy to an economic environment with entry and exit costs and uncertainty about future demand conditions, a case which will be shown to be of particular relevance to developing countries.

A rich literature—starting from Baldwin (1988), Baldwin and Krugman (1989), and Dixit (1989a,b)—has suggested that sunk costs are a primary reason for sluggish adjustment of foreign trade flows *e.g.* to real exchange rate variations. The empirical relevance of sunk costs has been firmly established. Evidence exists both for developing and developed countries and for a number of different industries, *e.g.* food, textiles, paper and chemical exports from Colombia (Roberts and Tybout, 1997), and manufactured exports from the United States (Bernard and Jensen, 2004), Lower Saxony, a region in Northern Germany (Bernard and Wagner, 2001), and Germany (Kaiser and Kongsted, 2008). These studies estimate patterns of entries and exits that are consistent with the general predictions of sunk cost models. They find significant persistence in individual firms' export status and non-linearities in the relationship between relative prices and exports. In combination, such findings suggest the presence of hysteretic effects (Krasnosel'skii and Pokrovskii (1989), Amable, Henry, Lordon and Topol (1994, 1995), Cross (1994)). Essentially, trade flows do not completely revert should relative prices return to their initial values.

[1] See Baldwin and Evenett (2009) for a review of recent world trade developments and specific policy proposals, including warnings against so-called "murky protectionism" as a response to the crisis.

In this paper, I will apply the important insights derived from the sunk cost hysteresis literature to the analysis of trade policy dynamics. Specifically, I consider competitive home firms in a constant-returns industry that produce a homogenous good exclusively for exports. Firms face sunk costs of entry and exit and real exchange rate variations are the only source of uncertainty. The dynamic equilibrium for this industry is characterized by an upper exchange rate trigger which causes entry of firms and a lower trigger associated with exits from the market, see Dixit (1989a,b). These triggers delimit a wide band of inaction which is the source of inertia in the industry response to exchange rate changes. This band reflects a value associated with the opportunity to wait for new information about the exchange rate before committing oneself to incur a sunk cost, the value of waiting as defined by McDonald and Siegel (1986).

It has been emphasized by Dixit and Pindyck (1994, chapter 9), that the existence of a band of inaction does not constitute a reason for policy intervention *per se*. In the context of trade policy, however, a presumption remains from traditional static analysis that policy intervention may benefit the home country. In the classical case of a downward-sloping foreign demand curve which competitive domestic firms fail to take into account, optimal trade policy requires a tax (or a quota) be imposed on exports, see Helpman and Krugman (1989). I show how the policy prescription for the static case carries over to the dynamic framework with sunk costs and demand uncertainty. In particular, I extend a result by Leahy (1993) to show formally that the optimal dynamic policy is feasible and implementable as a competitive equilibrium.

As to the practical importance of the case of export taxation, a 2002 OECD study finds that export taxes are levied on a number of products such as forestry products, fishery products, mineral and metal products, leather, and hide and skin products. Although export taxes have increasingly been abolished as part of bilateral and regional trade agreements, there are recent examples of export taxes being introduced, including a tax imposed on textile exports by China in 2005[2] and the U.S.-Canada softwood lumber dispute (Anderson, 2006) that has lead to a 2006-agreement with the potential for a Canadian export tax.

In terms of the geographical reach, table 1 reports the number of countries which applied export taxes by regions and other groupings according to the above-mentioned OECD study. Export taxation is clearly extant, in particular in developing countries.

Insert Table 1 about here!

The OECD study cites two main reasons that countries impose taxes on exports: To create a reliable source of revenue in a situation where countries face difficulties in collecting domestic taxes; and to promote domesting processing industries by securing cheap raw materials. The former motive, in particular, is most likely to apply to developing countries. To keep the theoretical analysis tractable in a dynamic setting, I will disregard to latter motive and focus on a single industry that produces exclusively for exports. In keeping with a partial equilibrium perspective, I also choose to neglect any distributional or labour adjustment issues related to trade policy.[3]

There are two main contributions of this paper. First, I extend existing literature on optimal trade taxation to the empirically relevant case of sunk cost hysteresis. A second contribution is the extension of previous results of Dixit (1989b) and

[2]Press release from the Chinese Embassy, http://www.china-embassy.org/eng/xw/t177900.htm

[3]On the former, see San Vicente Portes (2009) who considers the effect of trade liberalization on inequality. On the latter, see Furusawa and Lai (1999) in a two-country trade liberalization context and Belke and Göcke (2001, 2005) for the analysis of links between uncertainty and employment in a model of sunk cost hysteresis.

Dixit and Pindyck (1993) to a case in which there is a distortion which warrants government intervention and can be corrected by implementing the optimal tax rate.

There are three main results of the analysis. First, the intuition of the static policy prescription, that the optimal policy imposes a positive tax while a subsidy is never relevant, carries over to the dynamic case. Second, the optimal adjustments reflect a zone of inaction. The optimal policy involves (potentially long) periods of inaction and brief episodes of transition during which the structure of the industry changes and the tax rate is revised. In this sense, the model provides a rationale for infrequent revisions of trade policy in response to exogenous shocks. Third, the existence of an inaction band does not in itself provide justification for policy intervention. Intuitively, the inertia inherent in industry dynamics is reflected one-to-one by optimal policy. Tax rates are only to be adjusted when the current profits of domestic firms reach levels which—from a static point of view—would seem inconsistent with a normal rate of return.

The paper is organized as follows. Section 2 presents the basic model of a competitive export industry and the social planner's dynamic programming problem is solved in section 3. Implementation of the optimal policy via an export tax is the subject of section 4 which also discusses the dynamic consistency of the proposed policy. Section 5 calibrates a model to illustrate the dynamic characteristics of the industry and the associated trade policy for a case of linear demand. Section 6 contains my conclusions and some remarks regarding generalizations and limitations of the analysis.

2 The Model

This section presents the basic continuous-time model of a competitive export industry with sunk costs of entry and exit similar to Dixit (1989a,b) and Leahy (1993).

Consider a domestic industry producing a homogeneous export good. The industry is faced by the inverse demand function, $\tilde{P} = \tilde{D}(Q)$, where $\tilde{P}$ is the foreign currency market price of domestic exports and Q is the flow of exports per unit of time.[4] Assuming that $\tilde{D}$ is differentiable and that $\tilde{D}' < 0$, $Q\tilde{D}'' + 2\tilde{D}' < 0$, a well-defined static market equilibrium exists.

Firms are risk-neutral and form rational expectations. Future profits are discounted at the instantaneous rate $\rho > 0$. Firms employ identical technologies with real production costs $C > 0$ per unit of output. Assuming constant returns at the level of the industry, variations of industry output leave real unit production costs unaffected. Firms are assumed to be infinitesimal and total industry output, Q, is treated as a continuous variable. The output of an individual firm, however, is assumed to be fixed similar to the basic sunk cost model.[5] Then, by a proper normalization, Q can be regarded as a measure of the 'number' of firms. This specification ignores potential heterogeneity among firms. See Dixit and Pindyck (1993, chapter 9) for generalizations of the sunk cost model to heterogenous competitive firms and Melitz (2003) on a model in which monopolistically competitive firms differ in terms of productivity and entry costs.

The firm has a binary choice between being active or inactive. A sunk cost of k per unit is associated with entering the industry and a cost of l per unit is incurred when exiting. We take $k + l > 0$ in order to rule out a profitable strategy of repeated entries and exits. Either the entry cost or the exit cost could be zero or negative while leaving qualitative results unaffected. Two characteristics of entry and exit costs are important. First, entry and exit costs at the level of the industry

[4] Domestic and foreign price levels are normalized at one so that the exchange rate, prices and costs can be interpreted in real terms. Throughout, a tilde denotes a variable denominated in foreign currency.

[5] A model of variable production at the firm level is considered by Pindyck (1988) and by Dixit and Pindyck (1994, section 9.1B). Implications for the present analysis are discussed in section 6.

arise linearly with the change in Q. From the perspective of a firm with a fixed level of production, however, the costs of exit and entry appear lump-sum. Second, entering the market can be regarded as a partially irreversible investment. Partial irreversibility stems from the fact that the amount 'invested' when a firm enters is lost if the firm should leave the market again, *i.e.*, there is immediate 'rusting' of the tangible or intangible assets acquired upon entry. Due to the non-zero flow production cost, C, flow profits may turn negative and since costless suspension of operations is assumed to be impossible, firms will leave the industry only if current market conditions become sufficiently unfavorable.

Examples of entry costs according to this specification are the cost of setting up a specific production line or establishing a distribution network for the foreign market. Exit costs which are perhaps less important in a foreign trade context can be thought of as clean-up costs and firing costs. *A priori*, the costs of adjusting foreign trade flows may be fairly limited, but a main conclusion of the literature, *e.g.*, Dixit (1989a), holds that small entry and exit costs have sizable effects on industry dynamics in combination with demand uncertainty.

The stochastic element of the model is the real exchange rate, R, which is measured in units of domestic currency per unit of foreign currency. The domestic currency price is $R\tilde{P}$, and the exchange rate thus enters as a multiplicative shock to foreign demand. R follows geometric Brownian motion,

$$dR = \mu R\, dt + \sigma R\, dw. \tag{1}$$

μ is the drift parameter, $\sigma > 0$ is the volatility parameter, and dw is the increment of a standard Wiener process so that $E(dw) = 0$ and $E(dw^2) = dt$, see Malliaris and Brock (1982, p. 36).

This is the continuous-time analogue to a logarithmic random walk with drift in which shocks to the process have permanent effect. This specification is considered a first approximation to the case of a floating real exchange rate.[6] The exchange rate process is taken as exogenous for the analysis of trade policy, *i.e.*, the government is guided by objectives that are independent of this particular industry if in fact it actively influences the exchange rate. For the case of a country with a fixed nominal exchange rate, (1) must be interpreted as driven by shocks to the relative price levels in the home and the foreign country. The specification in (1) also allows a more general interpretation as a shock to the profit function, see Dixit and Pindyck (1993, chapter 9).

3 Optimal exit and entry of firms

As a benchmark for the taxing activities of the government, I will first consider the industry from a social planner's perspective. This allows the government to directly determine the number of firms in the exporting industry. I establish some formal properties of the optimal dynamic policy and provide intuition for the optimal plan by a numerical example.

The social planner maximizes the expected present value of the flow of profits from home exporting firms with a correction for the costs associated with adjusting the number of firms in the industry. The current flow of real profits from exports is $R\,\tilde{D}(Q) - C$ per unit and the objective function is defined as

$$E\Big(\int_0^\infty (R\,\tilde{D}(Q) - C)Q\,e^{-\rho t}dt - \sum_i (k\,dQ_+ + l\,dQ_-)e^{-\rho i}\Big). \tag{2}$$

An increase (a decrease) in the number of firms at an instant i is denoted by $dQ_+\,(dQ_-)$. The control variable, Q, is chosen so as to maximize (2) subject to (1) and the initial state (Q_0, R_0).

[6] For an analysis of firms' entry and exit decisions under a purely temporary exchange rate process, see Franke (1991) who showed that the dynamic industry equilibrium has properties similar to those derived for (1).

Consider for a moment a case without frictions. Then, the problem is separable over time and the optimal value, Q_f, is continuously defined by the standard static first-order condition, $R\,\widetilde{MR}(Q_f) = C$. Here $\widetilde{MR}(Q) \equiv \tilde{D}(Q) + Q\,\tilde{D}'$ is the social real marginal revenue function. The assumption of a well-defined static equilibrium implies that $\widetilde{MR}' < 0$ and that Q_f is an increasing function of R.

Re-introducing the entry and exit costs, the problem is to decide when to add a marginal unit, dQ_+, at a lump-sum cost, $k\,dQ_+$, or to eliminate a marginal unit at the cost $l\,dQ_-$. This defines a so-called optimal stopping problem regarding the marginal production unit. The optimal dynamic policy is defined by a pair of trigger values of the exchange rate, the upper trigger, $R_H(Q)$, at which a marginal unit is added, and the lower trigger, $R_L(Q)$, at which the unit is eliminated. This means that units are not added or eliminated continuously as the exchange rate varies because adjustment costs are lump-sum for units of given size. In the range between $R_L(Q)$ and $R_H(Q)$, there is no adjustment to exchange rate variations. The reason for this zone of inaction is quite intuitive: Adjustment costs are linear in the change of Q (with a kink at zero) and thus of first-order in dQ; on the other hand, the benefits from adjusting around a social optimum are of second order in dQ by differentiability of $\tilde{D}(Q)$, also see Dixit (1993, section 4.6).

To build some further intuition for the dynamic social optimum, I will use a graphical representation. It is the linear demand case examined in detail in section 5 where the choice of calibration is motivated in a developing country context. Figure 1 shows the trigger functions, $R_H(Q)$ and $R_L(Q)$, in (Q, R) space. The optimal plan implies that an infinitesimal unit is added whenever R rises to $R_H(Q)$. For a given value of Q, the exchange rate would have to drop considerably to $R_L(Q)$ in order to induce exit of the same unit. In particular, $R_L(Q)$ is well below the level at which the current marginal profit at Q is zero as indicated by the curve that shows the frictionless level, $R(Q_f)$.

A horizontal perspective on figure 1 shows the inaction range in terms of Q. Evidently, for the parameter values applied in this example quite large deviations of Q from Q_f are implied by the social optimum.

Insert Figure 1 about here!

Formally, the problem is analogous to the case considered in Dixit (1989b) except for the fact that the industry in question produces an exportable. It is therefore profits rather than total surplus which is being maximized here. Intuitively, the social planner should act as a monopolist and equate marginal revenue to marginal cost. The dynamic problem is solved by standard methods, see Malliaris and Brock (1982), Dixit (1989b), and Dixit and Pindyck (1993, chapter 9). The general solution for a given value of Q is

$$W(Q,R) = A(Q)R^{-\alpha} + B(Q)R^{\beta} + [\frac{R\tilde{P}}{\rho - \mu} - \frac{C}{\rho}]Q. \qquad (3)$$

$A(Q)$ and $B(Q)$ are constants to be determined for each Q from boundary conditions at the trigger values $R_H(Q)$ and $R_L(Q)$. The parameters $-\alpha$ and β are the solutions to the characteristic equation $\frac{1}{2}\sigma^2\xi(\xi - 1) + \mu\xi - \rho = 0$.

The social value of an established exporting industry, can be divided into three components from (3). The term in square brackets is the expected present value of future profits keeping Q constant.[7] The fact that the problem is two-sided means that there are two further terms, both of which are associated with option values. This is contrary to the basic case of irreversible investments considered e.g. by Pindyck (1988). The remaining terms represent the value of being able to vary optimally the number of units in response to future real exchange rate variations. Because it is an option value, $A(Q)R^{-\alpha} + B(Q)R^{\beta}$ should be positive or zero for

[7]The rate of discount applied to future export revenues reflects a possibly non-zero drift of the real exchange rate process. Evidently, $\rho > \mu$ is necessary for $W(Q,R)$ to be finite.

all possible values of R, implying that $A(Q) \geq 0$ and $B(Q) \geq 0$. Boundedness of the option values is established as follows. $A(Q)R^{-\alpha}$ is the value of the option to close down units at a cost of l per unit in order to avoid operating losses when market conditions become unfavorable. The value of this term is bounded by the present value of the operating costs avoided by closing the whole industry, $C\,Q/\rho$, minus the total cost of shut-down, $l\,Q$. That would be the net benefit of closing the industry if R fell to a level arbitrarily close to zero. The second part, the option to add further production units at a cost of k per unit, becomes more valuable when the real exchange rate R is high because the flow profit is an increasing function of R. For this term to remain bounded, the social marginal revenue should decrease at a sufficient rate as Q increases. This holds, for example, for a linear demand specification and also for iso-elastic demand if the price elasticity of demand is less than β (in numerical value).

It remains to solve for $A(Q)$, $B(Q)$ and the trigger exchange rates, $R_L(Q)$ and $R_H(Q)$, for given values of Q. They are determined by the boundary conditions of the problem, the so-called value-matching and smooth-pasting conditions, which will be stated formally from Dixit and Pindyck (1994, chapter 9).

$R_H(Q)$ is the critical value which triggers the introduction of an infinitesimal unit if reached by the real exchange rate. At this trigger, the marginal change of the value function from adding an infinitesimal unit equals the cost of entry, $W_Q(Q, R_H(Q)) = k$, which is the value-matching condition. Inserting the general expression for $W(Q,R)$ from (3) and defining $a(Q) \equiv A'(Q)$ and $b(Q) \equiv -B'(Q)$ I obtain the condition

$$a(Q)R_H(Q)^{-\alpha} - b(Q)R_H(Q)^{\beta} + R_H(Q)\frac{\widetilde{MR}(Q)}{\rho-\mu} - \frac{C}{\rho} - k = 0. \tag{4}$$

A further condition for $R_H(Q)$ equates the derivatives of the marginal value and the entry cost with respect to the exchange rate, $W_{QR}(Q, R_H(Q)) = 0$, implying that

$$-\alpha a(Q) R_H(Q)^{-\alpha-1} - \beta b(Q) R_H(Q)^{\beta-1} + \frac{\widetilde{MR}(Q)}{\rho - \mu} = 0. \tag{5}$$

This is the smooth-pasting condition.

Similarly, the level of the exchange rate, $R_L(Q)$, makes it optimal to eliminate a marginal unit satisfies a value-matching condition,

$$a(Q) R_L(Q)^{-\alpha} - b(Q) R_L(Q)^{\beta} + R_L(Q) \frac{\widetilde{MR}(Q)}{\rho - \mu} - \frac{C}{\rho} + l = 0 \tag{6}$$

and a smooth-pasting condition,

$$-\alpha a(Q) R_L(Q)^{-\alpha-1} - \beta b(Q) R_L(Q)^{\beta-1} + \frac{\widetilde{MR}(Q)}{\rho - \mu} = 0. \tag{7}$$

The solution to (4)-(7) in terms of $R_H(Q)$, $R_L(Q)$, $a(Q)$, and $b(Q)$ characterizes the optimal adjustment rule for a given value of Q. Properties of similar problems have been studied extensively, see Dixit (1989a), Dixit (1993), and Dixit and Pindyck (1994).[8]

4 Optimal Trade Policy

I now turn to analyzing a decentralized implementation of the social optimum by means of a trade tax. A natural generalization of the standard static policy prescription is suggested and shown to replicate the social optimum. Establishing feasibility

[8] In particular, a unique solution exists so that R_L, R_H, A, and B are defined as functions of Q. Moreover, the optimal rule for regulating the number of units implies that $R_H(Q) > R_L(Q)$, see Dixit (1989a, appendix A). In addition to these results, it can be shown that $R_L(Q)$, $R_H(Q)$, $A(Q)$, and $B(Q)$ are continuously differentiable functions and that both trigger functions are strictly increasing in Q.

of this policy under real exchange rate uncertainty *vis-à-vis* a competitive industry with entry and exit costs is the main subject. The analysis proceeds in three steps. First, I examine the behavior of a so-called myopic firm that disregards the possibility of future entries, exits, or changes of the tax rate. The optimal choice of triggers by this firm based on the *after-tax price* is shown to correspond to the trigger *exchange rates* that characterize the social plan optimum. Next, it is argued that rational firms may indeed ignore the possibility of further entry, exits, or tax changes in competitive equilibrium. Finally, I consider dynamic consistency.

4.1 Export Tax

Recall the textbook analysis of trade policy in a competitive export market, *e.g.*, Helpman and Krugman (1989, chapter 2). The optimal export tax is non-negative and positive if the demand faced jointly by competitive home firms is finitely elastic. The static social optimum implies $R\,\widetilde{MR}(Q) = C$ and may be implemented by the optimal *ad valorem* export tax, τ, so that the domestic currency after-tax price, $P \equiv R\tilde{P}/(1+\tau)$, equals $R\,\widetilde{MR}(Q)$ which in turn equals marginal social costs, C, *i.e.*,

$$\tau = \tilde{D}(Q)/\widetilde{MR}(Q) - 1, \tag{8}$$

where $\widetilde{MR}(Q) > 0$. This yields the standard formula, $\tau = 1/(\epsilon - 1)$, where ϵ is (the numerical value of) the price elasticity of demand.

The equivalent dynamic setting is the frictionless case discussed in section 3. A social optimum implies that τ is varied so that $P = R\,\tilde{P}/(1+\tau) = C$ holds continuously. Consider the effect on τ of a parametric change, dR. This affects the optimal number of firms by shifting the social optimum along the demand schedule. From (8) the optimal tax rate is a continuously differentiable function of Q_f for the frictionless case and $Q_f/(1+\tau)\ d\tau/dQ_f = \frac{\epsilon-\psi}{\psi\epsilon}$ where $\psi = -\widetilde{MR}(Q)/(Q\widetilde{MR}'(Q)) > 0$ is the inverse of the elasticity of social marginal revenue with respect to Q, defined in numerical value so as to be comparable to the price elasticity of demand, ϵ.

The optimal tax rate in general depends on Q_f. If the inverse demand schedule is less convex than the constant elasticity case, *e.g.*, the case of linear demand, we obtain $d\tau/dQ_f > 0$. Only for the case of constant elasticity of demand is $d\tau/dQ_f = 0$. Moreover, we noted in section 3 that Q_f varies positively and continuously with R in the frictionless case so that $d\tau/dR > 0$ if inverse demand is less convex than the constant elasticity case.

Returning to the case of entry and exit costs, the social dynamic optimum implied that the optimal value of Q stays constant while R varies within the interval $(R_L(Q), R_H(Q))$, and the ratio $\tilde{D}(Q)/\widetilde{MR}(Q)$ is also unaffected. This observation suggests a simple generalization of the static export tax policy to the dynamic case:

Definition 1 *Take* $\tau = \tilde{D}(Q)/\widetilde{MR}(Q) - 1$ *for the initial value,* Q_0*. The optimal path of the tax rate is then defined by:*

1. $d\tau = 0$ *if* $R_L(Q) \leq R \leq R_H(Q)$ *;*
2. $\frac{d\tau}{dR}\frac{R}{1+\tau} = \frac{\epsilon-\psi}{\psi\epsilon}\frac{R}{Q}\frac{1}{R_H'(Q)}$ *if* $R = R_H(Q)$ *and* $dR > 0$ *;*
3. $\frac{d\tau}{dR}\frac{R}{1+\tau} = \frac{\epsilon-\psi}{\psi\epsilon}\frac{R}{Q}\frac{1}{R_L'(Q)}$ *if* $R = R_L(Q)$ *and* $dR < 0$*.*

The elasticity of τ with respect to R thus depends on ϵ, ψ, and the inverse elasticity of R with respect to Q along R_H and R_L. By this definition, the after-tax price facing exporters, P, replicates the socially optimal path of $R\,\widetilde{MR}(Q)$.

Note that the rate of export tax set according to Definition 1 will reflect a zone of inaction similar to that associated with the number of exporting firms. The model thus embodies "trade policy hysteresis" in the sense that the return of the real exchange rate to some initial value does not necessarily reverse the effect on the optimal rate of export taxation.

I next turn to the decentralized implementation of the social dynamic optimum based on the after-tax price process resulting from Definition 1.

4.2 A Myopic Firm

Consider a firm which is myopic in the sense of Leahy (1993). In the context of trade policy, I extend the notion of myopia to imply that the firm neglects the possibility of future changes in the number of firms as well as the associated changes in the rate of export taxation. However, the firm is rational in regard to the process driving the real exchange rate, (1). Assume that the number of firms currently in the market is $\hat{Q}$. The path of domestic currency after-tax price as perceived by the myopic firm is then

$$\begin{aligned} d\hat{P} = \tilde{D}(\hat{Q})/(1+\hat{\tau})dR &= \tilde{D}(\hat{Q})/(1+\hat{\tau})[\mu R dt + \sigma R dw] \\ &= \mu \hat{P} dt + \sigma \hat{P} dw, \end{aligned} \tag{9}$$

where $\hat{\tau}$ follows from (8) for $Q = \hat{Q}$. Due to the multiplicative nature of the exchange rate shock the after-tax price $\hat{P}$ as perceived by a myopic firm follows geometric Brownian motion with the same parameters as R. The myopic firm solves an optimal stopping problem equivalent to Dixit (1989a) to which I refer for details. To simplify calculations, assume that individual firms produce one unit of output.

Briefly, I obtain the value function of a myopic firm which is currently not in the industry (but is contemplating entry). Define $V^O(\hat{P})$ as the value of the idle firm at the current price, $\hat{P}$, with the general solution

$$V^O(\hat{P}) = b_1 \hat{P}^{-\alpha} + b_2 \hat{P}^{\beta}, \tag{10}$$

where α and β remain defined exactly as above because the parameters of (1) and (9) are identical.

The value function for a myopic firm that is already active in the industry (but contemplating exit) can be similarly obtained as

$$V^I(\hat{P}) = a_1\hat{P}^{-\alpha} + a_2\hat{P}^{\beta} + [\frac{\hat{P}}{\rho - \mu} - \frac{C}{\rho}]. \tag{11}$$

Boundary considerations eliminate a_2 and b_1 whereas a_1 and b_2, together with the critical prices, $\hat{P}_L$ and $\hat{P}_H$, are determined by arguments that run parallel to those applied in solving the social planner's problem in section 3. At the optimal price of entry, $\hat{P}_H$, the value-matching condition, $V^I(\hat{P}_H) - k = V^O(\hat{P}_H)$, must hold which by inserting from (10) and (11) implies that

$$a_1\hat{P}_H^{-\alpha} - b_2\hat{P}_H^{\beta} + \frac{\hat{P}_H}{\rho - \mu} - \frac{C}{\rho} - k = 0. \tag{12}$$

A further condition on the value functions at the entry trigger is the smooth pasting condition, $V_P^I(\hat{P}_H) = V_P^O(\hat{P}_H)$, or

$$-\alpha a_1\hat{P}_H^{-\alpha-1} - \beta b_2\hat{P}_H^{\beta-1} + 1/(\rho - \mu) = 0. \tag{13}$$

Similar conditions hold for the value functions at the exit trigger, $\hat{P}_L$, at which the myopic firm decides to leave the market: $V^I(\hat{P}_L) = V^O(\hat{P}_L) - l$, and $V_P^I(\hat{P}_L) = V_P^O(\hat{P}_L)$.

So far this is a completely standard Dixit (1989a)-type problem. I will now provide the link to the social planner's problem specifically for the case of an exporting industry. Here, the trigger *prices* of the myopic firm based on the myopic after-tax price process, (9), and the trigger *exchange rates* of the social planner, $R_L(\hat{Q})$ and $R_H(\hat{Q})$, can be linked by substituting $R\,\tilde{D}(\hat{Q})/(1+\hat{\tau}) = R\,\widetilde{MR}(\hat{Q})$ for $\hat{P}_H$ in (12), (13), and the similar conditions involving the exit trigger, $\hat{P}_L$. From the value matching condition,[9]

$$a_1(R\widetilde{MR})^{-\alpha} - b_2(R\widetilde{MR})^{\beta} + R\frac{\widetilde{MR}}{\rho-\mu} - \frac{C}{\rho} - k = 0, \tag{14}$$

implying that

$$\hat{a}R^{-\alpha} - \hat{b}R^{\beta} + R\frac{\widetilde{MR}}{\rho-\mu} - \frac{C}{\rho} - k = 0, \tag{15}$$

where $\hat{a} \equiv a_1\widetilde{MR}^{-\alpha}$ and $\hat{b} \equiv b_2\widetilde{MR}^{\beta}$. Substituting from these definitions in the smooth pasting condition, (13), I obtain

$$-\alpha(\hat{a}\,\widetilde{MR}^{\alpha})(R\,\widetilde{MR})^{-\alpha-1} - \beta(\hat{b}\,\widetilde{MR}^{-\beta})(R\,\widetilde{MR})^{\beta-1} + 1/(\rho-\mu) = 0, \tag{16}$$

and multiplying through by $\widetilde{MR}$ I finally get

$$-\alpha\hat{a}R^{-\alpha-1} - \beta\hat{b}R^{\beta-1} + \frac{\widetilde{MR}}{\rho-\mu} = 0. \tag{17}$$

Obviously, (15) is equivalent to the value matching condition, (4), of the social planner's problem, and (17) corresponds to the smooth pasting condition, (5). Similar substitutions will work for the optimality conditions at the myopic firm's exit trigger price, $\hat{P}_L$, leaving $\hat{a}$ and $\hat{b}$ as defined above.

Thus, the exchange rate implicit in the optimal entry trigger, $R = \hat{P}_H/\widetilde{MR}(\hat{Q})$, satisfies the social planner's problem for $Q = \hat{Q}$ and similarly for the exit trigger. By uniqueness of the solution I have established the equivalence:

Proposition 1 *If τ is set according to Definition 1, a myopic firm faced with the after-tax price process, $\hat{P}$, will enter and exit at price levels which are equivalent to the optimal exchange rate triggers of the government's planning problem.*

This analysis, however, is based on the assumption of myopia. To establish feasibility of the policy it must be implemented in a competitive industry of rational firms.

[9] The argument of Q has been suppressed in (14) - (17).

4.3 The Competitive Equilibrium

In a competitive industry, firms are actively making decisions regarding entry and exit, and moreover, continue to do so. A rational firm will thus realize that future real exchange rate variations almost surely trigger entries and exits. These adjustments, in turn, limit the variations of the domestic currency price. In addition, the optimal policy as suggested in Definition 1 implies that the future path of the domestic currency after-tax price almost surely will be affected by changes of the rate of export tax.

In order to determine the resulting *endogenous* price process we recall the definition of the after-tax price, $P = R\,\tilde{D}(Q)/(1+\tau) \equiv D^*(Q,R)$, where the export tax rate is set according to Definition 1. The change of the rate of export tax at the entry trigger can go either way depending on the convexity of the demand function, but the definition of the optimal trade policy implies that $P = R\,\widetilde{MR}(Q)$ so that $D^*_Q = R\,\widetilde{MR}' < 0$. The fact that the government implements the social dynamic optimum ensures that the change of the after-tax price resulting from entry—directly or indirectly through changes in the trade tax—is unambiguously negative. This effectively bounds the variation of P and the actual price process facing the rational firm is given by

$$\begin{aligned} dP &= D^*_Q dQ + \widetilde{MR}(Q)[\mu R dt + \sigma R dw] \\ &= D^*_Q dQ + \mu P dt + \sigma P dw. \end{aligned} \tag{18}$$

Comparing (18) to the myopic process, (9), the extra term captures the effect of entry and exit on the after-tax price either directly or via a change of the trade tax.

Despite the fundamental difference between expectations formation in (9) and (18), Leahy (1993) shows that the optimal entry and exit levels of a myopic firm taking (9) as exogenous and a rational firm recognizing (18) are identical. The equality is established by a fixed-point argument parallel to Dixit and Pindyck (1994, sec. 8.3).

I can thus summarize the main result on optimal trade policy in this model:

Proposition 2 *The export tax policy of Definition 1 implements the social dynamic optimum. A competitive firm faced with the resulting after-tax price process enters and exits at the trigger levels of the exchange rate defined in the social planner's problem. The policy is feasible as the entry-exit policies of competitive firms define a rational-expectations equilibrium.*

4.4 Dynamic Consistency

Finally, some remarks on dynamic consistency of the trade policy in Definition 1. First, we note that the social optimum derives from a dynamic programming argument, *i.e.*, at each point in time and given the current state, (Q, R), dQ is chosen optimally. Second, the social optimum is implemented by the policy defined in Definition 1 so that the government has no incentive to renege on the policy at any point. Third, the entry-exit policies of firms define a rational-expectations equilibrium. We can thus conclude that the policy of Definition 1 is dynamically consistent according to the Kydland and Prescott (1977) definition.[10]

An important implication of the trade policy is that adverse exchange rate developments apparently are aggravated by trade policy. If, for example, the current exchange rate, R, is such that $P = R\tilde{D}(Q)/(1+\tau) < C$ while $R\tilde{D}(Q) > C$, a pre-tax profit opportunity is turned into a current loss by the export tax. Of course, by the above equivalence results, firms are willing to stay within the industry despite incurring a current loss. Taking a broader perspective, however, the political economy of tax formation could make it hard for the government to stick to the consistent policy, especially in periods where that policy inflicts a negative current profit flow on domestic firms. For a moment, take the exit and entry triggers of

[10] See Karp (1987) for an analysis of dynamic consistency for the case of an import tariff.

competitive firms as fixed at P_L and P_H, respectively. Then τ can indeed be varied within a range that corresponds to the price zone of inaction,

$$R\,\tilde{D}(Q)/P_H - 1 < \tau < R\,\tilde{D}(Q)/P_L - 1, \tag{19}$$

without causing any entries or exits. However, such discretionary changes of trade policy in response to exchange rate variations would be anticipated by firms and cause excessive entry and delay exits, *i.e.*, both the entry trigger and the exit trigger would be adjusted downwards. As long as $\tau > 0$ in all periods the resulting industry evolution would still be preferred to the laissez-faire equilibrium. If in any period $\tau < 0$, that is no longer unambiguously true and a policy that implies $\tau < 0$ throughout, *i.e.*, an export subsidy, cannot be preferred to laissez-faire.

5 Industry and Taxation Dynamics: A Numerical Illustration

To illustrate the dynamic properties of trade policy in this model, I will consider two cases: A developing country (LDC) case which is calibrated as a high-interest rate, high-exchange rate volatility scenario; and a developed country (DC) case with lower values of the discount factor and the volatility parameter. The parameters can be interpreted on a 'quarterly' basis.[11] Parameters that are common to both scenarios are: The full cost per period of entering the market and producing one unit forever which is normalized at $C + \rho k = 1$; the drift parameter for the stochastic process (1) which is set at $\mu = 0$; the inverse demand function which is linear, $\tilde{P} = 1.25 - Q/200$, and is associated with the social marginal revenue function $\widetilde{MR} = 1.25 - Q/100$; and finally, the cost of entry is $k = 8$ while there is no cost associated with leaving the market, $l = 0$.

[11] The parameters of the DC case are similar to those applied for a 'yearly' model of Dixit (1989b).

The LDC case has a discount rate of $\rho = 0.012$ corresponding to an annual interest rate of approximately 5 per cent. The volatility parameter of the exchange rate process in the LDC case is $\sigma = 0.1$. From any given point, R_0, the standard deviation of R/R_0 will be increasing at the rate of $\sqrt{t}$ so this specification implies an annual standard deviation of 20 per cent. The DC case has both ρ and σ fixed at half these values.[12]

Consider first the dynamics of the LDC case. As an example, take the central case $Q = 25$ such that $\widetilde{MR}(Q) = 1$. The upper trigger of the optimal policy, $R_H(25)$, is 1.470, while the lower trigger, $R_L(25)$, is 0.639, see figure 1. The marginal value in domestic currency of adding a further unit, $R_H(25)\,\widetilde{MR}(25)$, must be 47 per cent above full cost before τ is changed, and similarly $R_L(25)\,\widetilde{MR}(25)$ is 29 per cent below flow production costs. The zone of policy inaction is approximately 9 times wider than the static "Marshallian" interval, $(0.904, 1)$. The entry and exit trigger *prices* of a competitive firm in a market regulated according to the optimal policy can also be computed, $P_H = R_H(Q)\widetilde{MR}(Q)$ and $P_L = R_L(Q)\widetilde{MR}(Q)$. Because the production and entry costs are independent of Q, the price triggers are constants, $P_H = 1.470$ and $P_L = 0.639$, respectively.

Having calculated the parameters of the optimal policy, the dynamic properties of the competitive industry can be characterized. A particular sample path of the exchange rate process is shown in figure 2, panel A. The initial values of state variables are $R_0 = 1$ and $Q_0 = 25$ and (1) is approximated by a geometric random walk defined in $J = 64$ subperiods of each "quarter", $\Delta R_j = \sigma/\sqrt{J}\,R_{j-1}\,Z_j$, where $Z_j \sim N(0, 1)$. A sample consisting of every Jth observation of this 'daily' process is then taken as the 'quarterly' realization of the exchange rate.

For the particular exchange rate realization, R remains within the zone of inaction

[12] This is comparable to the post-Bretton Woods quarterly volatility of major real exchange rates, see Gagnon (1993, table 1).

corresponding to the initial number of firms in periods 1 through 35. Then follows periods of transition interspersed with 8-10 periods of no change in the number of firms. Finally, the optimal number of firms gets more than halved following a dramatic fall of the real exchange rate.

Insert Figure 2 about here!

The evolution of the industry from the point of view of a rational, competitive firm is considered in panel B. The domestic currency after-tax price, P, is the solid curve limited to the range (P_L, P_H). Periods of transition correspond to revisions of the optimal tax rate in this decentralized implementation (panel D). As demand is linear, the entries after period 35 imply an increase in τ, and the late period of transition is associated with a substantial lowering of the tax rate.

While the conclusions drawn from figure 2 in part depend on the particular sample path of R, more general properties of industry dynamics such as the expected time until the first policy action and its probable direction can be determined from the results of Dixit (1993, section 6.1). In the LDC case, the probability that the upper boundary is reached first can be calculated at 0.434. Thus, the lower limit was in fact slightly more likely to be hit first. The expected time until either boundary is hit is 34.4 periods. From a position approximately at the center of the inaction zone it will take 8 1/2 years on average for R to reach one of the boundaries implying a constant tax rate in that period. This particular realization yields a time of the first hit close to average.

Insert Figure 3 about here!

Figure 3 illustrates the corresponding dynamics of the DC case. I have used identical realizations of the real exchange rate process although its standard deviation is halved as compared to figure 2. But also the inaction band, $(R_H(Q), R_L(Q))$, is much narrower in this case. In fact, the first transition which is again upwards in terms of Q, happens around period 35. This is above the expected value of 27 periods for the DC case. Overall, the qualitative features of the evolution of the industry and the optimal rate of taxation are very similar with long periods of inaction and brief episodes of transition.

6 Concluding Remarks

Three main conclusions emerge from the analysis. First, the traditional policy prescription remains valid in the dynamic case: The optimal policy imposes a positive tax while a subsidy is never relevant for the case of an export industry in which competitive domestic firms jointly face a downward-sloping export demand function.

Second, the optimal tax rate adjustment reflects the inaction zone of the social dynamic optimum. The optimal policy involves (potentially long) periods of inaction and brief episodes of transition during which the structure of the industry changes and the tax rate is revised. In this sense, the model provides a rationale for infrequent revisions of trade policy in response to exogenous shocks.

Third, trade policy should not alter an inaction band caused by costs of entry and exit. Instead, the inertia inherent in industry dynamics should also characterize optimal policy. Tax rates are only to be adjusted when the current profits of domestic firms reach levels which from a static point of view would seem inconsistent with a normal rate of return. Still, considering the equivalence results, competitive firms will earn exactly the normal rate of return if they enter and exit at the trigger levels outlined above.

The model can be generalized in some directions while the validity of results in the paper is limited in others. The assumption of constant returns at the industry level is not crucial. The model of Dixit (1989b) has decreasing returns which in terms of our model would imply that the trigger levels of social marginal revenue are increasing in Q. This complicates the solution but the basic equivalence between the social plan, the behavior of a myopic firm, and the competitive equilibrium remains following Leahy (1993). Essentially, this equivalence requires that a normal rate of return is obtained by entering.

A second, possible generalization regards the stochastic process, (1), which may be interpreted as any multiplicative shift to demand. A general demand shift interpretation is probably more satisfactory in the case of *aggregate* exports. In addition, while the assumption of a geometric Brownian motion seems reasonable in the context of exchange rate fluctuations, a result of Leahy (1993, proposition 2) shows that the main equivalence between the social planner's problem and the competitive equilibrium generalizes to time-independent diffusion processes.

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Table 1: Number of WTO members applying export taxes.

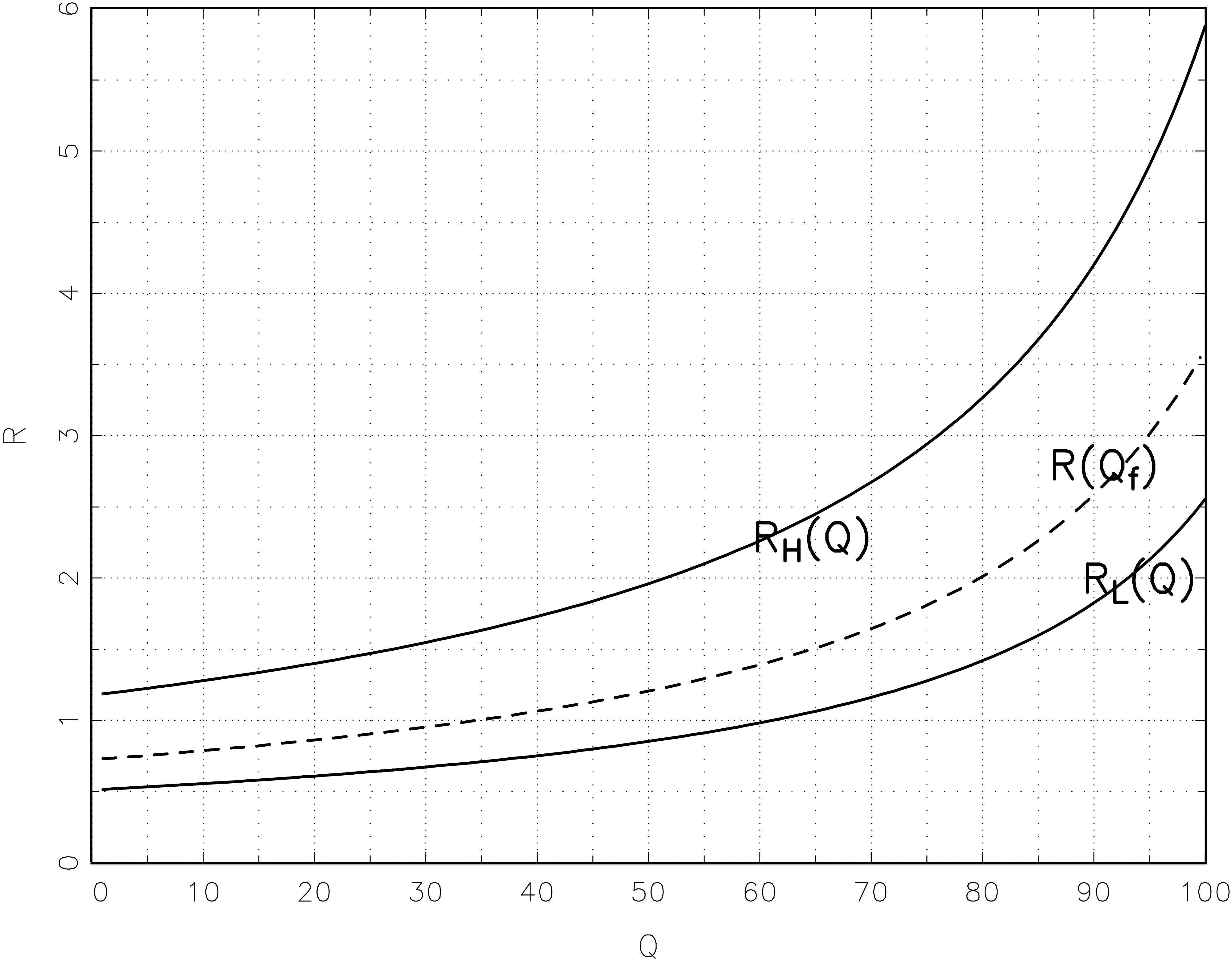
	Countries reviewed	Countries with export tax
Europe/Middle East	29	2
America	26	9
Asia/Pacific	19	11
Africa	26	17
Total	100	39
Of which:		
LDC	15	10
OECD	30	3
Others	55	26

Source: OECD (2002), Analysis of non-tariff measures: The case of export duties, Working Party of the Trade Committee. http://www.oecd.org/trade.html.

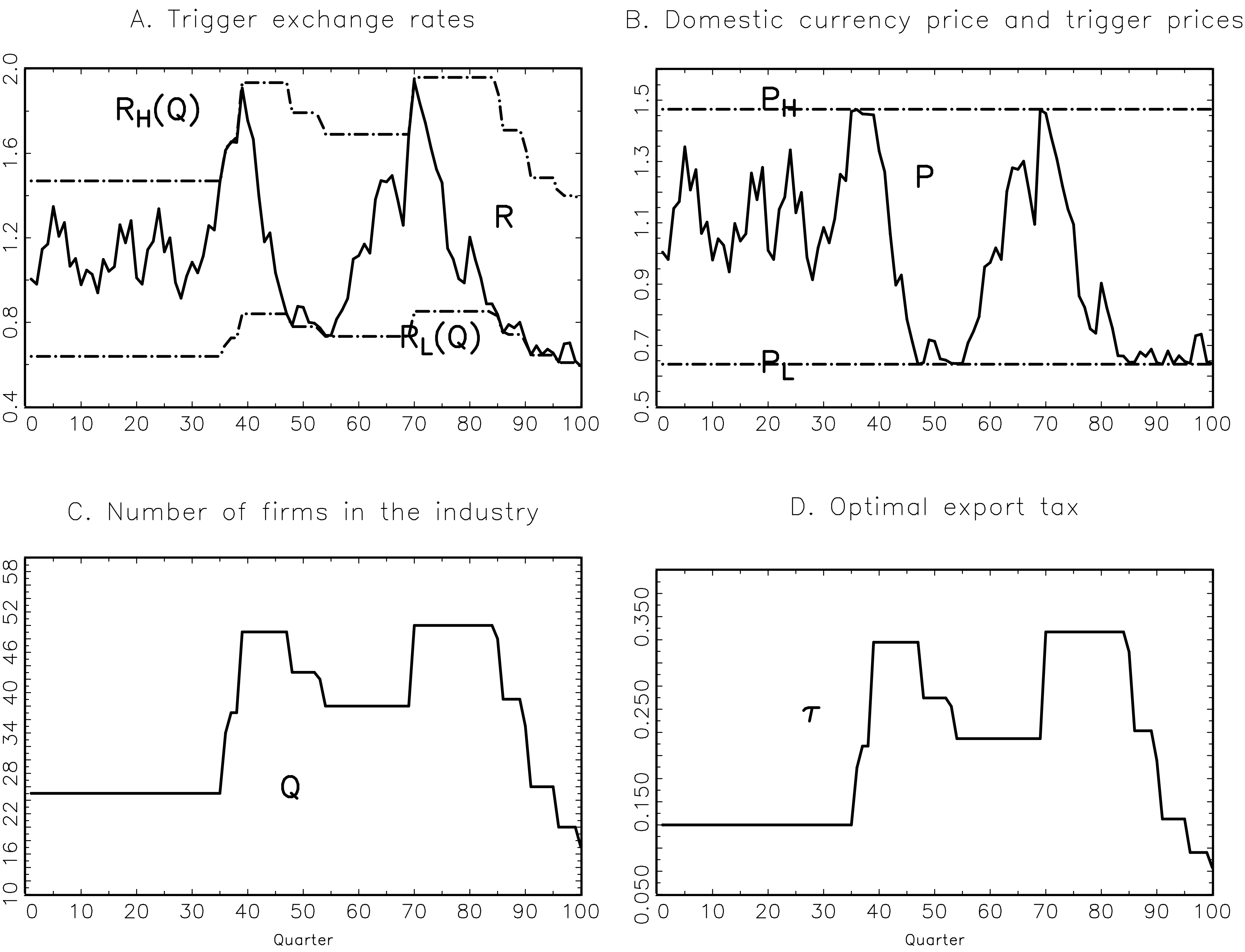
Figure 1: Trigger exchange rates: Case with entry costs and frictionless case

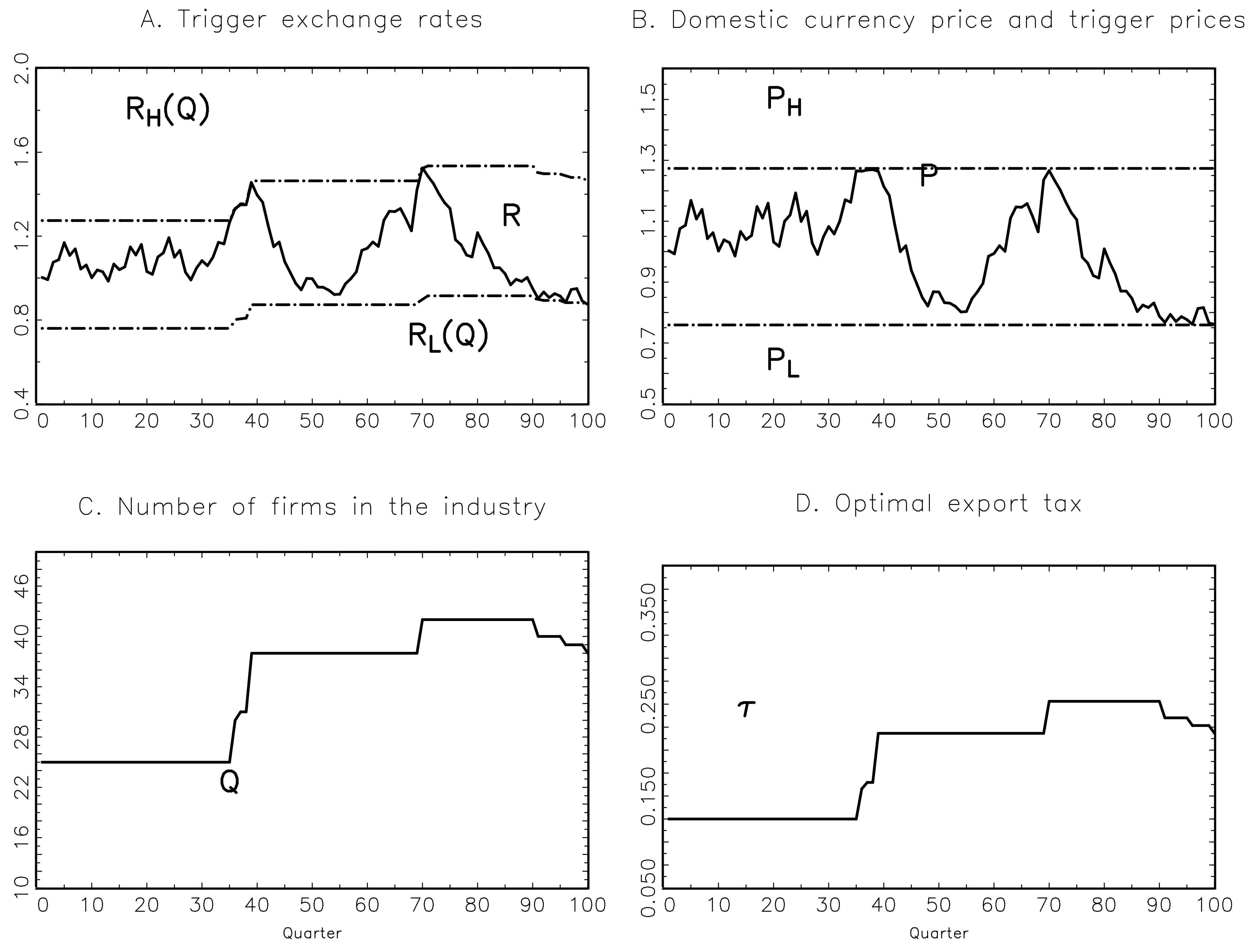
Figure 2: Sample paths from the numerical model: LDC case

Figure 3: Sample paths from the numerical model: DC case



R
Q
0
1
2
3
4
5
6
0
10
20
30
40
50
60
70
80
90
100
R_H(Q)
R(Q_f)
R_L(Q)


A. Trigger exchange rates
B. Domestic currency price and trigger prices
C. Number of firms in the industry
D. Optimal export tax
$R_H(Q)$
$R_L(Q)$
R
P_H
P_L
P
Q
τ
Quarter
Quarter


A. Trigger exchange rates
R_H(Q)
R
R_L(Q)
2.0
1.6
1.2
0.8
0.4
0 10 20 30 40 50 60 70 80 90 100
B. Domestic currency price and trigger prices
P_H
P
P_L
1.5
1.3
1.1
0.9
0.7
0.5
0 10 20 30 40 50 60 70 80 90 100
C. Number of firms in the industry
Q
46
40
34
28
22
16
10
0 10 20 30 40 50 60 70 80 90 100
Quarter
D. Optimal export tax
τ
0.350
0.250
0.150
0.050
0 10 20 30 40 50 60 70 80 90 100
Quarter