On the aggregation of health status measures

Jens Leth Hougaard, Hans Keiding

Institute of Economics, University of Copenhagen, Studiestraede 6, DK-1455 Copenhagen K, Denmark

Received 1 March 2004; received in revised form 1 December 2004; accepted 1 April 2005
Available online 28 June 2005

Abstract

In the present paper, we address the problem of finding conditions under which aggregation of individual health status measurements (e.g. QALY’s) is meaningful in the sense that there is a universal unit of measurement for health. The problem is studied in a model where different aspects of health take the form of Lancasterian characteristics to be produced by the individuals using commodities obtained in the market. For a meaningful unit of measurement to exist, marginal rates of substitution between different aspects of health should not differ among individuals, and for this to happen in an equilibrium of the economy considered, certain assumptions of separability (of technology and/or preferences) must be satisfied. This means that universal measures of health will be meaningful only if there are not too many spillovers in achieving different aspects of health.

© 2005 Elsevier B.V. All rights reserved.

JEL classification: D6; I0

Keywords: Health characteristics; Marginal rates of substitution; Health status measures; General equilibrium

1. Introduction

In recent years, the use of measures of health state or health related quality of life has become widespread as well in theoretical as in practical investigations; among these, the
most important one is the quality adjusted life years (QALY) index which is widely discussed in the literature (cf. e.g. Torrance, 1986; Torrance and Feeny, 1989; Loomes and McKenzie, 1989; Bleichrodt et al., 1997; Bleichrodt and Quiggin, 1997; see also the survey in Dolan, 2000).

While the foundations of individual health status measurement has received considerable attention, not much has been said about measuring health status, and in particular changes in health status, for populations consisting of several individuals, which indeed is where the measurement is put to practical use. This would of course be unnecessary if all individuals were identical, so that they evaluated health states in exactly the same way. However, this seems an unusual assumption to make in economic analysis, not to mention that even casual evidence suggests that it is not true.

If individuals have different opinions about health states, then they would also have different utility representations of their preferences, and adding utility gains becomes a much less simple matter. Indeed, if one treatment gives a utility gain of 0.1 to individual A and 0.3 to individual B, while another treatment yields 0.2 to A and 0.1 to B, should we then prefer the first treatment which gives a total gain of 0.4 to the second with total gain only 0.3? This seems to presuppose that the utility gains of the two individuals are in some sense "the same", so that the gain of one individual compares meaningfully to that of another individual. For practical use, we will of course need to know the distribution of types A and B in the population, information that is hardly ever available in practice.

What is at stake here is that we use the health status measure not only as a measure of individual health gains, but also as a common unit of measurement, valid for all individuals, so that we can measure the total health gain for society. This is parallel to what happens when economic variables are aggregated; we can speak meaningfully of total consumption in society, even though it involves many individuals and many commodities, since these can all be put on a common scale using money values as unit of measurement. Classical aggregation theory will tell us that this works because commodities can be evaluated in money terms using their prices which are the same for all individuals. Technically, what is needed is that marginal rates of substitution between any pair of commodities are the same for all individuals; then there is a meaningful common unit of measurement.

The counterpart of this in health status measurement is that if all improvements in health are to be measured on the same scale, then the trade-off between, say, improved ability to see and improved ability to hear must be independent of the individuals involved, see e.g. Baldwin et al. (1990). As above, this corresponds to saying that the marginal rates of substitution between different aspects of health should be identical across individuals, even though their individual utilities (and society’s weighing of these individuals) may differ.

Now, such equality of marginal rates of substitution is the result of the market mechanism when we speak about ordinary commodities, since people exchange commodities until equality obtains. Concerning health, on the other hand, matters are quite different since individual health cannot be traded. Hence, there is no direct way of achieving equality of marginal rates of return. The question therefore arises whether it can be brought about by some indirect exchanges, in the sense that people devote more or less commodities to the production of individual health, thereby being able to make additional exchanges of commodities.
In the present paper, these notions are made precise using a general equilibrium model with Lancasterian characteristics, thus following the tradition in health economics set by Grossman (1972). With this model we investigate the conditions under which the consumers will exhibit identical marginal rates of substitutions between health characteristics. The crucial property turns out to be one of separability, either of the technology producing health characteristics or of the utility function evaluating the characteristics produced.

To provide an illustration of the problem, assume that there are two health characteristics $h_1$ and $h_2$, where $h_1$ measures the ability to move around, and $h_2$ the ability to hear; both are measured on a scale from 0 to 1. There are two types of individuals; the first type weighs the two characteristics equally whereas the second type considers $h_2$ to be twice as important as $h_1$. It is easy to construct meaningful individual health indices $Q_1$ and $Q_2$ for individual 1 and 2, respectively, for example:

$$Q_1(h_1, h_2) = \frac{1}{2}h_1 + \frac{1}{2}h_2, \quad Q_2(h_1, h_2) = \frac{1}{3}h_1 + \frac{2}{3}h_2$$

Consider now a treatment which gives rise to an improvement by 1/10 (of a unit) in characteristic $h_1$ but has a side effect of reducing characteristic $h_2$ by 1/15; the changes in health indices are $\Delta Q_1 = 1/60$ and $\Delta Q_2 = -1/90$, so the two individuals have a very different assessment of the treatment, and none of the individual indices would be useful as an index for health improvement in society as a whole.

In practice, values of health indices (such as the EQ-5D) are found as averages of individual health indices. Taking averages in a society with three individuals of type 1 and four individuals of type 2, we get an index of the form:

$$Q(h_1, h_2) = \frac{17}{42}h_1 + \frac{25}{42}h_2$$

for which our treatment results in a gain of 1/1260; however, a majority (namely all type 2 individuals) will become worse off from this treatment. To the conceptual problem of using weighted averages must be added the statistical problems of obtaining samples which reflect the true distribution of preferences in the population, problems which are usually circumvented by assuming that preferences are identical or differ only by random noise.

In the example, the indices were all found as weighted averages of the health characteristics $h_1$ and $h_2$, although with weights depending on the individual. As noted by several authors (see e.g. Dolan, 1997; Busschbach et al., 1999), health status indices need not have this simple form. This opens up for a possible way out of the interpersonal comparison problem – one which will be studied at length in the sequel – which could be contemplated in the case where preferences over health states are not so simple as in the example above, but would depend on the actual health state in the same way as consumer preferences depend on actual consumption; suppose for example that the indices are given by

$$Q_1(h_1, h_2) = h_1h_2, \quad Q_2(h_1, h_2) = h_1^{1/3}h_2^{2/3}$$
then there are certain health states of the individuals, namely all \((h^1_1, h^1_2)\) and \((h^2_1, h^2_2)\) with

\[ \frac{h^2_1}{h^1_1} = \frac{2h^2_2}{3h^1_1} \]

such that marginal rate of substitution between the two characteristics are the same (both are willing to exchange the two characteristics in the same proportion). If individual health states satisfy this condition then indeed it makes sense to construct a health index which weighs the characteristics according to the common marginal rate of substitution.

This is not a particularly new insight since most macroeconomics is based on the same reasoning, only in this case concerning commodities rather than health characteristics, so that the market secures that marginal rates of substitution are the same. The question which arises, and which will be addressed in the sequel, is whether there is any inherent mechanism in the economy which will perform the role of the market, at least indirectly.

The paper is structured as follows: Section 2, introduces our basic general equilibrium model with health characteristics. Once the model is there, we may use it to exhibit examples of what can go wrong in aggregation of health status and changes in health; the examples also give a hint as to the general conditions for meaningful aggregation. The formal part of the discussion is then resumed in Section 4, where we show that suitable separability conditions may save the day. Indeed, it is shown that equality of marginal rates of substitution obtains if and only if the separability condition is satisfied. In Section 5, the impact of the results for aggregation of health measures is discussed. The proofs of the theorems in the text have been collected in an Appendix A.

2. A general equilibrium model with health characteristics

In this section, we introduce the formal model which will be used for studying aggregation of individual health states. We consider a general equilibrium model with consumption characteristics in the sense of Lancaster (1971). In our model, consumers are concerned about their bundle of characteristics (related to health as well as to other aspects of life), and commodities are only important insofar as they can be used as inputs in the individual’s own (“household”) production of characteristics. This approach to health consumption using Lancaster’s consumer characteristics is not new in health economics, see e.g. the intertemporal health consumption model of Grossman (1972). Moreover, the connection between QALYs and Lancaster’s characteristics was also noticed by Williams (1985), Culyer (1990).

Thus, consider an economy \(E\) with \(l\) commodities. Consumers \(i \in \{1, \ldots, m\}\) buy commodity bundles \(x_i \in \mathbb{R}^l_+\) in the market and transform these commodity bundles into bundles \(\xi_i\) of \(L\) different consumers’ characteristics which are used for final consumption. Each consumer \(i\) has a utility function \(u_i\) defined on bundles of characteristics, that is on \(\mathbb{R}^L_+\).

In our interpretation, the characteristics bundle \(\xi_i = (\xi_{i1}, \ldots, \xi_{iL})\) of individual \(i\) specifies the situation of the individual with respect to the different aspects of health as well as possibly other identifiable aspects of well-being, such as ordinary consumption activities. For simplicity of notation, we assume that utility is defined only on characteristics and not
on the commodity part of the bundle; this is no restriction as we may always consider each ordinary commodity as another characteristic (since characteristics in our model include other aspects of satisfaction than health, we have used the general notation $\xi_j$ here rather than the notation $h_1, h_2$ used in the introductory section).

To obtain the characteristics bundle $\xi_i$, the consumer uses a household technology $T_i \subset \mathbb{R}_+^l \times \mathbb{R}_+^{L_i}$ transforming commodity bundles into characteristics bundles; a household production is a pair $(x_i, \xi_i)$ belonging to $T_i$. Since $T_i$ reflects the individual’s perceptions of how to transform commodities (for example food and beverages) and services (prevention or treatments) into (different types of) health, we would rather expect the household technologies to differ between agents. However, we may also think of the household technology as a set of ‘blueprints’ for health-seeking behavior, specifying which commodities (food, beverages, medical services) to buy in order to achieve a desired enjoyment of health.

At this point we notice that although the framework of our analysis is static, the time dimension is not necessarily absent, since in our interpretation, commodities as well as characteristics may be distinguished according to their date of availability, so the only limitation is that there is a common finite horizon. In this sense, our use of health characteristics corresponds with that of Grossman (1972), and this interpretation of characteristics as health states at particular dates allows for comparison of a particular health state in one period with another health state experienced in several periods, which is what lies behind the time tradeoff method known from the literature on QALYs.

The production of ordinary commodities in the model is introduced in a very summary way, since this is not a matter of interest in the present context; we assume that there is a given set $Y \subset \mathbb{R}_+^l$ of feasible aggregate commodity bundles in the economy $E$.

In the analysis to follow, we shall assume that economies satisfy standard assumptions of general equilibrium theory, namely the following.

(a) For each consumer $i$, $u_i$ is continuous, weakly monotonic (i.e. $\xi'_i > \xi_i$ implies that $u_i(\xi'_i) > u_i(\xi_i)$; here and in the sequel, inequality signs $>$ and $\geq$ for vectors mean that the inequalities hold in each coordinate), and quasi-concave.

(b) The household technologies are well-behaved in the following sense: for each $i$, $T_i$ is closed, convex, monotonic (i.e. for all $(x_i, \xi_i) \in T_i$, if $x'_i \geq x_i$, $x'_i \neq x_i$, then there is $\xi'_i \geq \xi_i$, $\xi'_i \neq \xi_i$ with $(x'_i, \xi'_i) \in T_i$) and relevant (i.e. if $(x_i, \xi_i) \in T_i$ and $\xi_i \geq 0$, $\xi_i \neq 0$, then $x_i \geq 0$, $x_i \neq 0$). Moreover, $T_i$ satisfies free disposability (i.e. if $(x_i, \xi_i) \in T_i$, and if $x'_i \geq x_i$, $\xi'_i \leq \xi_i$, then $(x'_i, \xi'_i) \in T_i$).

(c) Commodity production satisfies usual assumptions: the set $Y$ is convex, satisfies free disposability and contains 0 in its interior.

The abstract model is a device for studying allocations, these are the ways of producing and distributing commodities and health. An allocation in the economy $E$ is a pair $(x, \xi)$, where $x = (x_i)_{i=1}^m$ is a commodity allocation and $\xi = (\xi_i)_{i=1}^m$ a characteristics allocation. It is individually feasible if

$$x_i \in \mathbb{R}_+^l, \quad (x_i, \xi_i) \in T_i$$

for each $i$, so that the specified characteristics bundles can be obtained in the household technologies with the specified commodity bundles, and aggregate feasible if $\sum_{i=1}^m x_i \in Y$; it is feasible
if it is both individually and aggregate feasible. An allocation \((x, \xi)\) in \(E\) is \textit{Pareto optimal} if it is feasible and if there is no other feasible allocation \((x', \xi')\) such that \(u_i(\xi') \geq u_i(\xi)\) for all \(i\) with at least one strict inequality.

This completes the definition of the formal economy which constitutes the basic framework for our study. As it was said above, we are interested in situations where the market behaviour of consumers and producers result in allocations where all consumers have the same marginal rates of substitution between any pair of characteristics. It will therefore be convenient that these marginal utilities or individual shadow prices appear explicitly in the equilibrium concept of the model. Formally, we define a \textit{market equilibrium with characteristics prices} (shorthand: an equilibrium) as an array \((x, \xi, p, q)\), where \((x, \xi)\) is a feasible allocation, \(p\) a price system on commodities, and \(q = (q_i)_{i=1}^m\) a family of (individual) price vectors \(q_i \in \mathbb{R}^{L+}\) on characteristics, such that

(i) for each consumer \(i\), \(\xi_i\) maximizes \(u_i\) on

\[\{\xi'_i | \exists (x'_i, \xi'_i) \in T_i, q_i \cdot \xi'_{ij} \leq p \cdot x_i + (q_i \cdot \xi'_i - p \cdot x'_i)\},\]

(ii) the vector \(y = \sum_{i=1}^m x_i\) maximizes \(p \cdot y'\) over all \(y' \in Y\).

In the market equilibrium with characteristics prices, the consumer may (at least formally) buy both commodities and characteristics at the given prices, and in optimum the best bundles in the consumption set are chosen. The constraint of the consumer may look strange at a first sight, but it is a rather straightforward generalization of standard conditions to our context: it says that no preferred characteristics bundle can be bought at the commodity prices \(p\) and the characteristics prices \(q_i\), even if the consumer has access to the household technology \(T_i\) and can use the “profits” earned here for buying health characteristics. Note that due to the feasibility constraint, no actual transfers of characteristics take place; the characteristics prices are only used as constraints to the individual optimization of the consumer.

The following theorem shows that the first and second fundamental theorems of welfare economics hold in our model. A proof of the theorem is given in Appendix A.

\textbf{Theorem 1.} Let \(E\) be an economy satisfying standard assumptions. If \((x, \xi, p, q)\) is a market equilibrium with characteristics prices, then the allocation \((x_i, \xi_i)\) is Pareto optimal. Conversely, let \((x, \xi)\) be a Pareto optimal allocation in \(E\), where \(\xi_i \in \mathbb{R}^{L+}\) for all \(i\). Then there is price system \(p\) on commodities and a system of individual characteristics prices \(q = (q_i)_{i=1}^m\) such that \((x, \xi, p, q)\) is a market equilibrium with characteristics prices.

The result of Theorem 1 provides the basis for using characteristics or shadow prices in welfare considerations since changes in overall welfare may be evaluated by computing the aggregate value of the characteristics consumption. It can be noticed, however, that such a welfare measure would use individual shadow prices, which might very well differ among individuals. The basic problem is therefore to find conditions on the economy making the characteristics prices \(q_{ij}, i = 1, \ldots, m\), of any characteristic \(j = 1, \ldots, L\), identical.
3. Examples

In the present section, we demonstrate by examples that equality of characteristics prices does not prevail in general, even in simple two person economies. The examples will, however, help to illustrate the conditions under which equality indeed occurs.

In the first example, consumers have identical household technologies but differ in preferences.

Example 1. Suppose that the economy $E$ has two consumers and one (marketed) commodity $x$, (i.e. $l = 1$). Consumers transform arbitrary amounts of this commodity into bundles of two different characteristics $(\xi_1, \xi_2)$ (i.e. $L = 2$) using a common household technology $T$. The household technology is assumed to be given by $(x, (\xi_1, \xi_2)) \in T$ if

$$(\xi_1^2 + \xi_2^2)^{1/2} \leq x$$

Thus, the characteristics are made under conditions of joint production and constant returns to scale. Consumers have the utility functions:

$$u_1(\xi_{11}, \xi_{12}) = \xi_{11}^{\xi_{12}}$$
$$u_2(\xi_{21}, \xi_{22}) = \xi_{21}^{\xi_{22}}$$

Finally, there is an initial endowment of 1 unit of the commodity but no commodity production, i.e. $Y = \{y \in \mathbb{R} \mid y \leq 1\}$.

We obtain all Pareto optimal allocations $(x, \xi)$ by dividing the initial endowment between consumers and letting them produce characteristics from this endowment such as to maximize their utility. Thus, let $x_1 \in [0, 1]$; then $(\xi_{11}, \xi_{12})$ and $(\xi_{21}, \xi_{22})$ satisfy:

$$\xi_{11}^2 + \xi_{12}^2 = x_1^2$$
$$\xi_{21}^2 + \xi_{22}^2 = (1 - x_1)^2$$

Adding the individual maximization conditions, we get:

$$(\xi_{11}, \xi_{12}) = \left(\frac{x_1}{\sqrt{5}}, \frac{2x_1}{\sqrt{5}}\right), \quad (\xi_{21}, \xi_{22}) = \left(\frac{2(1 - x_1)}{\sqrt{5}}, \frac{1 - x_1}{\sqrt{5}}\right),$$

Therefore, the marginal rate of substitution between characteristics for consumer 1 is given by

$$\text{MRS}_1 = \frac{\partial u_1/\partial \xi_1}{\partial u_1/\partial \xi_2} = \frac{1}{4}\frac{\xi_{12}}{\xi_{11}} = \frac{1}{2}$$

and similarly, the marginal rate of substitution for consumer 2 is $\text{MRS}_2 = 2$. As such, the marginal rates of substitution are independent of the parameter $x_1$. Moreover, $\text{MRS}_1$ and $\text{MRS}_2$ remain the same in all the Pareto optimal allocations (except for $x_1 = 0$ or $x_1 = 1$, where any individual price vector is a support for the preferred set of the individual getting nothing). We conclude that the marginal rates of substitutions differ for all Pareto optimal allocations.
To avoid that marginal rates of substitution differ, even when the household technology is the same for all, we would need that the isoquants in Fig. 1 were straight lines; this means that there is no joint production, or, otherwise put, that the common household technology is separable. In the next section, we show that this geometric intuition indeed fits with the general result; separability of household production is a necessary and sufficient condition for equality of marginal rates of substitution.

Returning to our example, it easy to obtain more general results stating that if all consumers behave according to standard assumptions (i.e. they all have the same household technology with constant returns to scale as well as homothetic preferences which are sufficiently different) then there are no equilibria with identical characteristics prices. In fact, as the following example will show, even with identical household technology and identical preferences we will not automatically obtain identical characteristics prices if the assumption of constant returns to scale is abandoned.

**Example 2.** Suppose that the common household technology $T$ is such that $(x, \xi) \in T$ if

$$(t \xi^2_1 + \xi^2_2)^{1/2} \leq t, \quad t \leq x$$

Fig. 1. Isoquants of household technology; consumers with different tastes choose in such a way that marginal rates of substitution differ.
and that both consumers have the utility function $u$ with

$$u(\xi_1, \xi_2) = \xi_1 \xi_2$$

As before, we find all Pareto optimal allocations by choosing $x_1 \in [0, 1]$ and maximizing $u$ under each of the constraints:

$$x_1 \xi_{11}^2 + \xi_{12}^2 = x_1^2, \quad (1 - x_1) \xi_{21}^2 + \xi_{22}^2 = (1 - x_1)^2$$

This yields characteristics bundles:

$$(\xi_{11}, \xi_{12}) = \left( \frac{x_1}{2}, \frac{x_1}{\sqrt{2}} \right), \quad (\xi_{21}, \xi_{22}) = \left( \frac{(1 - x_1)}{2}, \frac{1 - x_1}{\sqrt{2}} \right)$$

Once more we have inequality of marginal rates of substitution, which with the particular utility function are \(\text{MRS}_1 = \sqrt{x_1}\) and \(\text{MRS}_2 = \sqrt{1 - x_1}\), respectively, differing except in the case $x_1 = (1 - x_1)$ where the identical consumers are treated equally.

What has happened here is that due to differences in consumption levels, the consumers effectively have different household technologies (Fig. 2). So even if preferences over characteristics bundles are the same, the consumers end up with different marginal rates of substitution. These aggregation problems rooted in wealth differences has been pointed out repeatedly in the literature on foundations of cost-effectiveness analysis, see e.g. Garber.

![Fig. 2](image-url)
and Phelps (1997). To obtain equal marginal rates of substitution when isoquants may differ among consumers, we would need that indifference curves were straight lines with the same slope, meaning that consumers would have affinely separable utility functions. We return to this situation below.

4. Equality of individual characteristics prices

Since, in general, there is no reason to expect equality of individual characteristics prices, we shall subsequently concentrate on finding conditions under which equality actually does occur. There are four cases to be distinguished.

(1) All consumers have different household technologies and different utility functions. In this case equality of marginal rates of substitution for characteristics would obtain only exceptionally.

(2) All consumers have identical household technologies and identical utility functions. Then there is essentially only one consumer, so the problem of equality becomes trivial. It may be remarked, however, that even here the situation is not entirely trivial, as it was shown in Example 2 of the previous section.

(3) All consumers have a common household technology but arbitrary individual utility functions. This case will be considered in Section 4.1.

(4) All consumers have a common utility function but arbitrary individual household technologies. This case is treated in Section 4.2.

4.1. The case of a common household technology

In this section, we consider the case where every individual has access to the same technology for transforming commodities into characteristics. Dealing with health-related characteristics, this means that the household technology contains all procedures by which services and goods available in the market can be used to improve individual health in one or several of its aspects. Needless to say that this is a very restrictive assumption, since some methods of obtaining health, both by treatment and by the way of conducting your life, may not be known, or indeed possible, for everybody. Assuming identical household technologies therefore amounts to making the case for equality of marginal rates of substitution as favorable as possible; we shall see that even so we need further restrictions.

Different individual characteristics prices may be caused by a transformation curve in characteristics space where the slope depends on the chosen output bundle. In this case, assigning suitable preferences over characteristics to individuals, it is always possible find market equilibria for which one individual has a marginal valuation of some characteristic which differs from that of some other individual. If the transformation curve is a straight line, however, this cannot occur. Indeed, in optimum the slope of a linear transformation curve will define characteristics prices which are obviously the same for all. In the following we shall hence consider a general form of such a linear-transformation-curve condition.
A household technology $T \subset \mathbb{R}_+^l \times \mathbb{R}_+^L$ is said to be separable if $T = \sum_{j=1}^L T_j$ with

$$T_j \subset \mathbb{R}_+^l \times \{\xi \in \mathbb{R}_+^L \mid \xi_k = 0, k \neq j\}$$

for $j = 1, \ldots, L$. In other words, a technology is separable if each of the characteristics are produced in a separate technology; producing a characteristics bundle $\xi = (\xi_1, \ldots, \xi_L)$ means that $\xi_1$ is produced in the technology $T^1$ having the first characteristic as output, $\xi_2$ similarly in the specific technology $T^2$ for producing characteristics number 2, etc. In such a technology, what can be produced with a given input of commodities is obviously the convex hull of each of the productions obtainable by putting everything into the technology $T^j$ producing the $j$th characteristic, for $j = 1, \ldots, L$. This clearly means that transformation surfaces are hyperplanes in $\mathbb{R}_+^L$.

Intuitively, separability of the household technology means that what the individual does in order to produce some characteristic $j$ has no side-effects or spin-offs on the production of any other characteristic. Thinking of characteristics as representing enjoyment of life in its various forms, including both consumption activities and different aspects of health, this is not a particularly attractive property. In real life, many activities which enhance one kind of health will typically be beneficial also for other aspects of the individual’s health, and on the contrary some efforts to promote a particular form of health may have detrimental effects on other aspects of health. However, with separable household technology you can produce any of the characteristics with no effect on the production of the others. Indeed, the situation may be considered as one where you can choose from a “menu” of characteristics, each having its “price” in terms of commodities to be given up.

However, relevant or not, it is a fact that the separability assumption is quite crucial if characteristics prices are to be identical in equilibrium, as demonstrated by the following result.

**Theorem 2.** Let $T \subset \mathbb{R}_+^l \times \mathbb{R}_+^L$ be a household technology satisfying constant returns to scale, and let $E_T$ be the class of economies satisfying standard assumptions such that all consumers have the household technology $T$. Then the following conditions are equivalent:

(i) for all economies $E \in E_T$, all equilibria $(x, \xi, p, q)$ of $E$ with $\xi_i \neq 0$ for all $i$ satisfy $q_1 = \cdots = q_m$.

(ii) the household technology $T$ is separable.

The proof of Theorem 2 is given in Appendix A. The result states that if we are to reckon with equality of individual characteristics prices in the sense that it is not achieved due to some coincidence of individual preferences, then the method of producing characteristics from commodities (i.e. the household technology) must satisfy separability. If not, there will be some constellation of individual utility functions such that individuals value small changes in health of a particular aspect differently, measured either in terms of other aspects of health or in monetary terms. If this occurs, then the weight of this aspect of health is ambiguous, and there is no simple answer to the question of how to set the weight of this particular aspect of health in a cost-effectiveness analysis.
4.2. The case of common preferences

We now turn to the case where individuals no longer have access to the same household technology. In our interpretation, this means that the same treatment may affect people differently, or that there may be medical treatments which are not open to all individuals, either because they are not admitted, or simply because they are not aware of their existence. As such, the assumption of individual household technologies may seem more natural than the assumption of a common household technology, as in the previous section.

As mentioned earlier, if both household technologies and utility functions differ, then equality of individual characteristics prices would obtain only as an exceptional case. Therefore, we examine the situation where all individuals have the same utility function.

Intuitively, identical preferences imply consensus among all individuals on the general way of assessing alternative characteristics bundles. On the other hand, due to differences in household technologies, consumers will typically end up with different characteristics bundles and this, in turn, will typically give rise to different marginal rates of substitution even though the underlying utility function is the same. As such, the interpretation follows the standard case of inequality in access to health – if some individuals cannot use the health improving methods that are available to others, the result will be an inferior state of health in certain respects. As a consequence, the marginal utility of an improvement in the relevant aspects of health would typically be greater for those people than for the rest of the population.

The case where such differences do not occur, making aggregation possible in a meaningful way, almost suggests itself; this is the case where marginal rates of substitution are independent of the bundle chosen, so that indifference surfaces are hyperplanes. As it can be seen from the statement of the result, it may be considered as a dual of Theorem 2, since the separability condition on preferences takes the place of the previous separability condition for household technologies. It is assumed that preferences are homothetic and thereby can be described by a utility function that is homogeneous (of degree one); in this way we exclude the possibility that differences in marginal rates of substitution might arise only due to differences in wealth.

**Theorem 3.** Let $u$ be a given homogeneous utility function on characteristics, let $\mathcal{T}$ be the set of all household technologies which satisfy constant returns to scale, and let $\mathcal{E}_u$ be the set of all economies satisfying standard assumptions such that each individual $i$ has utility function $u$ and has household technology $T_i \in \mathcal{T}$. Then the following are equivalent:

(i) for all economies $\mathcal{E} \in \mathcal{E}_u$, all equilibria $(x, \xi, p, q)$ of $\mathcal{E}$ with $\xi_i \neq 0$ for all $i$ satisfy $q_1 = \cdots = q_m$, 
(ii) the common utility function is affine, so that

$$u(\xi_1, \ldots, \xi_L) = \sum_{j=1}^{L} u_j \xi_j$$

where $u_j \geq 0$ are nonnegative numbers.
The proof of Theorem 3 is given in Appendix A. The result shows that if we are to expect that individual characteristics prices will be identical, then the utility function must be (additively) separable. The intuition behind this result is as follows: even though people evaluate health profiles in the same way, they may come up with very different bundles due to their differing access to health; when bundles differ, it might well happen that the marginal rates of substitution (the slopes of the indifference curves in the two-dimensional case) are different, and that would give us a contradiction. Therefore, the indifference curves must have the same slope everywhere, which means that utility functions must be separable. This implies preference independence of the various health characteristics in the sense that if a given combination of some characteristics is preferred to another combination of the same characteristics, given identical assignment in the remaining characteristics, then it will be preferred no matter what this assignment in the remaining characteristics looks like. Separability is usually considered a very strong assumption on preferences.

It might be noted that if (ii) of Theorem 3 holds, then the utility function may also be written in the form:

\[ u(\xi_1, \ldots, \xi_k, \xi_{k+1}, \ldots, \xi_L) = \sum_{j=1}^{k} u_j \xi_j + \sum_{j=k+1}^{L} u_j \xi_j \]

where the first \( k \) characteristics are those related to (non-health) consumption, whereas the remaining characteristics are health-based, so that it splits linearly into a consumption-related term together with a standard health index, which for example is the form assumed by Dolan and Edlin (2002).

Separability is often (but not always) assumed when methods for assigning values to health status indices are proposed. For instance, Sintonen (1981) determines linear weights assigned to each of the 11 basic aspects of health, and the final index is then found as the weighted average. Also in QALY measurement, separability enters as an essential assumption; the time tradeoff methodology assumes (multiplicative) separability in life expectation and quality-of-life, and the standard gamble relies on von Neumann–Morgenstern utility representation of lotteries over life and death, thereby implicitly assuming preference independence (cf. Dolan, 2000). On the other hand, in assigning values to the 243 possible states in EQ-5D, the approaches taken are not necessarily linear in the different health dimensions, thus violating the separability assumption.

By and large, the authors concerned with finding a logical basis for health status measurements have typically relied on separability or independence assumptions. What is added by our result is that this assumption is indeed necessary; if health status measurement is to make sense even in a world where all have identical preferences over health characteristics, these preferences must be representable by a separable utility function.

5. Final remarks

In this paper, we have investigated conditions for health status measurement to be meaningful in a context of interpersonal comparisons. We have argued that a minimal
demand would be that marginal rates of substitution between different health characteristics should be the same for all individuals, and we then moved on to check whether such equality of marginal rates of substitutions is achieved by the economic intercourse of individuals, as indeed it is in the case of ordinary consumption commodities. We did this in a context where characteristics (of health as well as other desiderata) are produced by agents in a household technology using commodities bought in the market.

In our analysis, it turned out that although we cannot hope for equality of MRS holding in broad generality, it may appear in special cases. Our results show that separability is not only a sufficient assumption for meaningfulness of health status measurement, but it is indeed necessary in the sense that if we do not assume separability (of either technology or preferences), then there will be equilibrium situations where health status measurement is not meaningful.

In obtaining these results, we assumed either that technologies were identical, so that individuals differed only in preferences, or that preferences were identical, individuals differing only in their household technologies. Such cases of identical preferences or technologies are of course special, and in a general approach to cost-effectiveness analysis (such as e.g. Hansen et al., 2004), one would avoid such assumptions. We have considered the special cases in order to obtain as strong results on necessity as possible, since if we allow for differences in both technologies and preferences, then it is straightforward that equality of marginal rates of substitution will obtain only as exception.

The separability assumption on preferences has been used widely in the literature, and what our treatment adds to the state of knowledge is mainly that generalization beyond separability is not possible. This makes the situation differ somewhat from what is known from other fields of economic science, where separability is used as a simplifying assumption; the difference is that in these cases, generalizations exist but are difficult to analyze, while in our case generalizations simply do not exist. Meaningful health state measures which are interpersonally comparable simply demand that utility functions must have this form.

The other case of separability, related to technology, seems not to have been given much – if any – attention in the literature. Separability in technology means that each health characteristic is produced in a way which is independent of the quantity produced of all the other characteristics. This assumption is indeed restrictive; it means that eating fast food cannot make you fat (unless you want it to), since fast food (or any food) can be obtained as a consumption characteristic in production which does not interfere with any of the health characteristics.

It may come as a surprise that this unrealistic separability condition in the way of getting health and other forms of satisfaction of wants from commodities bought in the market has not been investigated before, since the separability assumption on utility functions is so well-known. There may be several reasons for this; first of all, most – indeed, almost all – research in the foundations of health measurement has either considered a single individual or assumed that all individuals are identical with respect to preferences, in which case the problem does not arise. Secondly, only few studies have used the Lancasterian approach for modelling health characteristics, and without this the implicit assumptions on household technologies are not so easily identified.
As mentioned repeatedly, the separability assumptions are rather restrictive, and the fact that health status measurement cannot quite be relied upon if they are not fulfilled may give rise to some scepticism with respect to the widespread and increasing use of such measurements. It should of course be remembered that some rather successful fields of economics rely on exactly the same kind of assumptions; this is the case for the Heckscher–Ohlin model of international trade (see e.g. Krugman and Obstfeld, 2003), which has tradable commodities produced from nontradable production factors in a separable, constant returns to scale technology. One of the results in this model, the celebrated factor price equalization theorem, says that the common prices on tradables give rise to identical prices on nontradables, a result which looks much like what we were looking for, with ‘countries’ replaced by ‘individuals’ and ‘production factors’ replaced by ‘characteristics’. Thus, in some sense, Theorems 2 and 3 may be considered as a welfare theoretical basis for current measurement of health status by indices such as the EQ-5D.

Unfortunately, the analogy does not hold all the way through. In particular, while factor price equalization obtains also outside the restrictive framework of the original model, our results show that we cannot expect equality of marginal rates of substitution without separability which indeed were shown to be both necessary and sufficient conditions.

Summing up, our results point to severe weaknesses in the theoretical foundations of using health status measures when dealing with more than an single individual, weaknesses that give rise to some reservations concerning the meaningfulness of such aggregated health status measures in cost-effectiveness analysis.

If simple methods of aggregation fail, there is of course a need for other methods which do not use ad hoc aggregation, and the use of health status indices may indeed be justified by the fact that so far they are the best available. But this does not mean that evaluating medical treatments or new pharmaceutical products cannot be performed without aggregating health characteristics into a single number to be put in the denominator of a cost-effectiveness ratio. Indeed, the cost-effectiveness ratio itself is a means of avoiding an aggregation (namely of health and commodities), and in this perspective, it should not be beyond reach to avoid also some of the aggregations which are inherent in the approach to outcome measurement using health status indices. This, however, is beyond the scope of the present paper.

Appendix A. Proofs of theorems

Proof of Theorem 1. The proof that the allocation \((x, \xi)\) associated with an equilibrium \((x, \xi, p, q)\) is Pareto optimal (that is main Theorem 1 of economic welfare theory holds) is standard, and we leave it to the reader. For the converse, assume that \((x, \xi)\) is Pareto optimal. For each consumer \(i\), define the set:

\[
P_i^0 = \{\xi' \in \mathbb{R}_+^L \mid u_i(\xi'_i) \geq u_i(\xi_i)\}
\]

\(P_i\) is closed and convex since \(u_i\) is continuous and quasi-concave; we embed \(P_i\) in \(\mathbb{R}^{l+mL}\) putting 0 in all places except in the \(i\)th of the \(m\) segments of \(L\) coordinates. Denote the resulting set by \(\tilde{P}_i\).
For each \( i \), let \( \tilde{T}_i \) be the set \( T_i \) embedded in \( \mathbb{R}^{l+ml} \) by putting 0 in all coordinates except the first \( l \), where the sign is reversed, and the \( l \)th segment of \( L \) coordinates. Again, \( \tilde{T}_i \) is closed and convex. Finally, the closed and convex set \( Y \subset \mathbb{R}^l \) is embedded in \( \mathbb{R}^{l+ml} \) as \( \tilde{Y} \) by adding 0s in all but the first \( l \) coordinates.

Now consider the set:

\[
Z = \sum_{i=1}^{m} \tilde{P}_i - \sum_{i=1}^{n} \tilde{T}_i - \tilde{Y}
\]

We have that \( Z \) is closed and convex, and that it contains the 0 vector, which has the representation:

\[
0 = \sum_{i=1}^{n} \tilde{\xi}_i - \sum_{i=1}^{n} (\tilde{x}_i, \tilde{\xi}_i) - \tilde{y}
\]

for some \( y \in Y \). By separation of the convex sets \( Z \) and \( \text{Neg} = \emptyset \), where \( \text{Neg} \) is the cone of negative vectors in \( \mathbb{R}^{l+ml} \). Suppose to the contrary that \( u \in Z \cap \text{Neg} \), so that

\[
u = \sum_{i=1}^{n} \tilde{\xi}_i' - \sum_{i=1}^{n} (\tilde{x}_i', \tilde{\xi}_i') - \tilde{y}'
\]

for some \( y' \in Y \); but this means that the allocation \((x', \xi')\) is at least as good as \((x_i, \xi_i)\) and that there is a feasible allocation \((x''_i, \xi''_i)\) with \( x''_i > x'_i \) for all \( i \). By the monotonicity properties of \( T_i \) and \( u_i \), \( i = 1, \ldots, m \), there is then a feasible allocation \((x''_i, \xi''_i)\) with \( u_i(\xi''_i) \geq u_i(\xi_i) \), contradicting Pareto optimality.

Now, by separation of the convex sets \( Z \) and \( \text{Neg} \), there exists a nonnegative vector \((p, q) = (p, (q_i)_{i=1}^{m}) \in \mathbb{R}^{l+ml} \) such that the scalar product \((p, q) \cdot z\) for \( z \in Z \) is minimized at \( z = 0 \). But this means that (a) \( y \) maximizes \( p \cdot y \) on \( Y \), which is condition (ii) of market equilibrium with characteristics prices; furthermore it means that (b) \( \xi \) minimizes \( q_i \cdot \xi_i \) on \( P_i \), which, by the interiority condition, implies that \( q \cdot \xi''_i > q \cdot \xi_i \) for all \( \xi''_i \) with \( u_i(\xi''_i) > u_i(\xi_i) \), and that (c) \((x_i, \xi_i)\) maximizes \( q_i \cdot \xi_i' - p \cdot x_i' \) on \( T_i \).

To check equilibrium condition (i), let \( i \in \{1, \ldots, m\} \) and assume now that \( \xi''_i \) satisfies the inequality

\[
q_i \cdot \xi''_i \leq p \cdot x_i + \max_{(x_i', \xi_i') \in T_i} [q_i \cdot \xi_i' - p' \cdot x_i']
\]

Then from (c) we get that \( q_i \cdot \xi''_i \leq p \cdot x_i + [q_i \cdot \xi_i - p \cdot x_i] = q_i \cdot \xi_i \), and (b) implies that \( u_i(\xi''_i) \geq u_i(\xi_i) \), which is (i). \( \square \)

**Proof of Theorem 2.** (ii) \( \Rightarrow \) (i): Let \( \mathcal{E} \in \mathbb{E_T} \), and let \( (x, \xi, p, q) \) be an equilibrium. Then by the condition (i) of an equilibrium, for each \( i \) the characteristics bundle \( \xi_i \) is maximal for \( u_i \).
on the set of all:

\[ \{ \xi'_i \mid q_i \cdot \xi''_i \leq p \cdot x_i + \max_{(x'_i, \xi'_i) \in T_i} [q_i \cdot \xi'_i - p \cdot x'_i] \} = \{ \xi''_i \mid q_i \cdot \xi''_i \leq q_i \cdot \xi_i \} \]

clearly, this set contains:

\[ T_i(p) = \{ \xi'_i \mid \exists x'_i \cdot p \cdot x'_i \leq p \cdot x_i, (x'_i, \xi'_i) \in T_i \} \]

and \( \xi_i \) belongs to the boundary of \( T_i(p) \). Now, by constant returns to scale of \( T \), we have that

\[ T_i(p) = \lambda_i T_i(p) \]

where \( \lambda_i = p \cdot x_i \), and using (ii) we get that

\[ T(p) = \lambda^{-1} \{ \xi''_i \mid q_i \cdot \xi''_i \leq q_i \cdot \xi_i \} \]

Since the left hand side is independent of \( i \), so is the right hand side, and we conclude that \( q_1 = \cdots = q_m \).

(i) \( \Rightarrow \) (ii): Let \( C = [c^1, \ldots, c^L] \subset \mathbb{R}^L_{++} \) be a countable dense subset of \( \mathbb{R}^L_{++} \), and let \( \hat{p} \in \mathbb{R}^L_{++} \) be arbitrary. For each natural number \( m \), we define an economy \( E_m \in E_T \) with \( m \) consumers \( i = 0, 1, \ldots, m-1 \) as follows: consumer 0 has preferences such that

\[ u_i(\xi'_i) \geq u_i(\xi_i) \quad \text{if and only if} \quad \sum_{j=1}^{L} \xi'_j \geq \sum_{j=1}^{L} \xi_j \]

For \( i = 1, \ldots, m-1 \), let the utility function of consumer \( i \) be given by

\[ u_i(\xi) = \min_{j=1, \ldots, L} \frac{\xi_j}{c^j_i} \]

The aggregate availability set \( Y \subset \mathbb{R}^l \) is defined by

\[ Y = \{ y \in \mathbb{R}^l \mid \hat{p} \cdot y \leq 0, y_1 \leq 1 \} \]

(so that commodity 1 is available in the magnitude 1 and can be used as input, whereas the other commodities can only be obtained as output.)

Let \( (x, \xi, p, q) \) be an extended equilibrium in \( E_m \). By assumption, \( q_0 = \cdots = q_{m-1} = \hat{q} \). We claim that \( \sum_{i=0}^{m-1} \xi_i > 0 \). Suppose not, then there is \( j \in \{1, \ldots, L\} \) such that \( \xi_{ij} = 0 \) for all \( i \) and consequently \( u_i(\xi'_i) = 0 \) for \( i \geq 1 \). If \( \xi_i \neq 0 \), then also \( x_i \neq 0 \) (here we use part (b) of the standard assumptions), and then by monotonicity of \( T \) (also in (b) of the standard assumptions) there is \( \xi'_i \) with \( u_i(\xi'_i) > u_i(\xi_0) \) with \( (x_0 + x_i, \xi'_0) \in T \), contradicting the Pareto optimality of \( (x, \xi) \) (if consumer \( i \) gets utility 0 anyway then her commodities could just as well be given to consumer 0, who would become better off). This proves our claim.

By the standard assumption (b), we now conclude that \( \sum_{i=1}^{m} x_i > 0 \), and it follows from the equilibrium conditions that \( p = \hat{p} \). Arguing as above, we have that \( \xi_i \) maximizes \( \hat{q} \cdot \xi \) on

\[ \{ \xi \mid \exists x : \hat{p} \cdot x \leq \hat{p} \cdot x_i, (x, \xi) \in T_i \} \]
and by constant returns to scale, we have that \( \lambda_i^{-1} \xi_i \) maximizes \( \bar{q} \cdot \xi \) on \( T(\hat{p}) \), where \( \lambda_i = \hat{p} \cdot x_i \) for each \( i \). We conclude that conv(\( \{ \lambda_i^{-1} \xi_i \mid i = 1, \ldots, m - 1 \} \)) belongs to the intersection of bd \( T(\hat{p}) \) with a hyperplane \( \{ \xi \mid \bar{q} \cdot \xi = M \} \), where \( M = \lambda_i^{-1} \hat{q} \cdot \xi_i \) is independent of \( i \).

Next, we notice that \( \bar{q}_j > 0 \) for all \( h \) since otherwise consumer 0 would not satisfy the individual optimality constraint (\( i' \)) at \( \xi_0 \). But this means that for \( i \geq 1 \):

\[
\lambda_i^{-1} \xi_i = \frac{M}{\bar{q} \cdot c_i} c_i
\]

so that for \( m \) large enough, the set conv(\( \{ \lambda_i^{-1} \xi_i \mid i = 1, \ldots, m - 1 \} \)) gets as close as desired to \( \{ \xi \in \mathbb{R}^L_+ \mid \bar{q} \cdot \xi = M \} \). But then

\[
T(\hat{p}) = \{ \xi \in \mathbb{R}^L_+ \mid \bar{q} \leq M \},
\]

and since \( \hat{p} \) was arbitrary, we have shown that for all \( p \in \mathbb{R}^L_+ \), the set \( T(p) \) is the intersection with \( \mathbb{R}^L_+ \) of a half-space in \( \mathbb{R}^L \). The rest of the proof consists in showing that this latter property implies that \( T \) is separable.

For \( j = 1, \ldots, L \), define \( T_j \) by

\[
T_j = T \cap \{(x, \xi) \in \mathbb{R}^L_+ \times \mathbb{R}^L_+ \mid \xi_k = 0, k \neq j \}
\]

Then \( T_j \) is a convex cone contained in \( T \), each \( j \), and \( \sum_{j=1}^L T_j \subset T \).

Assume that \( (x, \xi) \in T \) but \( (x, \xi) \notin \sum_{j=1}^L T_j \). Then for every array \( (x^1, \ldots, x^L) \in (\mathbb{R}^L)^L \) such that \( (x^j, e^j) \in T_j \) for each \( j \) (where \( e^j \) is the \( j \)th unit vector in \( \mathbb{R}^L \)), we have that \( x \neq \sum_{j=1}^L x^j \). The set:

\[
C = \left\{ \sum_{j=1}^L \xi_j x^j \mid (x^j, e^j) \in T_j, \ j = 1, \ldots, L \right\}
\]

is closed and convex (by convexity of each of the sets \( T_j \)), and

\[
\{x' \in \mathbb{R}^L \mid x' \leq x \} \cap C = \emptyset
\]

(by the free disposal property of household technologies), so by separation of convex sets, there is \( p \in \mathbb{R}^L_+ \) with

\[
p \cdot x = 1, \quad p \cdot x' > 1 \quad \text{for} \ x' \in C.
\]
However, by the assumptions of the Theorem, we know that there are \((\hat{x}^j, \hat{\xi}^j) \in T^j\) with \(p \cdot \hat{x}^j = 1, j = 1, \ldots, L,\) such that

\[
\xi = \sum_{j=1}^{L} \lambda_j \hat{\xi}^j
\]

for some \(\lambda_1, \ldots, \lambda_L \geq 0\) with \(\sum_{j=1}^{L} \lambda_j = 1.\) Clearly, for each \(j\) we have that \(\lambda_j \hat{\xi}^j = \xi_j,\) meaning that \((\lambda_j \hat{x}^j, \xi_j) \in T,\) and

\[
\sum_{j=1}^{L} \lambda_j \hat{x}^j \in C.
\]

However, since \(p \cdot \hat{x}^j = 1\) for each \(j,\) we get that

\[
p \cdot \left( \sum_{j=1}^{L} \lambda_j \hat{x}^j \right) = \sum_{j=1}^{L} \lambda_j (p \cdot \hat{x}^j) = 1,
\]

contradicting that \(p \cdot x' > 1\) for each \(x' \in C.\)

\(\square\)

**Proof of Theorem 3.** The proof of the implication (ii) \(\Rightarrow\) (i) follows the same line of reasoning as in Theorem 2 and is left to the reader.

(i) \(\Rightarrow\) (ii): Consider the convex set \(C(u) = \{\xi \mid u(\xi) \geq 1\};\) we say that \(\pi \in \mathbb{R}^L_+\) is a support of \(C(u)\) at \(\xi_0 \in C(u)\) if \(\pi \cdot \xi' \geq \pi \cdot \xi_0\) for all \(\xi' \in C(u).\)

Let \(\{c_1, c_2, \ldots\}\) be a countable set of points in \(\mathbb{R}^L_+\) which are dense \(\{c \in \mathbb{R}^L_+ \mid c_1 + \cdots + c_L = 1\}.\) For each natural number \(m,\) consider the economy \(E_m\) with \(m\) consumers, where each consumer \(i\) has utility function \(u\) and household technology:

\[
T_i = \{(x_i, \xi_i) \mid \xi_i \leq \|x_i\| c_i\}
\]

\(i = 1, \ldots, m,\) and where \(Y = \{x \mid x \leq (1, \ldots, 1)\}.\)

For each \(m,\) let \((x, \xi, p, q)\) be a market equilibrium of \(E_m\) which by assumption satisfies \(q_1 = \cdots = q_m = q^0.\) Then \(u(\xi_i) = u(\|x_i\| c_i)\) for each \(i,\) and without loss of generality we may assume that \(\xi_i = \|x_i\| c_i\), each \(i.\) From this it follows that \(C(u)\) has the same support \(q^0\) at all points where rays given by \(c_i\) intersect \(C(u).\) As \(m\) tends to infinity, we conclude that \(C(u)\) is the intersection of \(\mathbb{R}^L_+\) with a halfspace, so that \(u\) is indeed affine. \(\square\)

**References**


Sintonen, H., 1981. An approach to measuring and valuing health states, Social Science and Medicine, 15C.

