Chapter 8

Search and Money

Traditionally, money has been viewed as having three functions: it is a *medium of exchange,* a *store of value,* and a *unit of account.* Money is a medium of exchange in that it is an object which has a high velocity of circulation; its value is not derived solely from its intrinsic worth, but from its wide acceptability in transactions. It is hard to conceive of money serving its role as a medium of exchange without being a store of value, i.e. money is an asset. Finally, money is a unit of account in that virtually all contracts are denominated in terms of it.

Jevons (1875) provided an early account of a friction which gives rise to the medium-of-exchange role of money. The key elements of Jevons's story are that economic agents are specialized in terms of what they produce and what they consume, and that it is costly to seek out would-be trading partners. For example, suppose a world in which there is a finite number of different goods, and each person produces only one good and wishes to consume some other good. Also suppose that all trade in this economy involves barter, i.e. trades of goods for goods. In order to directly obtain the good she wishes, it is necessary for a particular agent to find someone else who has what she wants, which is a single coincidence of wants. A trade can only take place if that other person also wants what she has, i.e. there is a double coincidence of wants. In the worst possible scenario, there is an *absence of double coincidence of wants,* and no trades of this type can take place. At best, trading will be a random and time-consuming process, and agents will search, on average, a long time for trading partners.

Suppose now that we introduce money into this economy. This money could be a commodity money, which is valued as a consumption good, or it could be fiat money, which is intrinsically useless but difficult or impossible for private agents to produce. If money is accepted by everyone, then trade can be speeded up considerably. Rather than having to satisfy the double coincidence of wants, an agent now only needs to find someone who wants what she has, selling her production for money, and then find an agent who has what she wants, purchasing their consumption good with money. When there is a large number of goods in the economy, two single coincidences on average occur much sooner than one double coincidence.

The above story has elements of search in it, so it is not surprising that the search structure used by labor economists and others would be applied in monetary economics. One of the first models of money and search is that of Jones (1976), but the more recent monetary search literature begins with Kiyotaki and Wright (1989). Kiyotaki and Wright's model involves three types of agents and three types of goods (the simplest possible kind of absence of double coincidence model), and is useful for studying commodity monies, but is not a very tractable model of fiat money. The model we study in this chapter is a simplification of Kiyotaki and Wright (1993), where symmetry is exploited to obtain a framework where it is convenient to study the welfare effects of introducing fiat money.

8.1 The Model

There is a continuum of agents with unit mass, each having preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $0 < \beta < 1$, c_t denotes consumption, and $u(\cdot)$ is an increasing function. There is a continuum of goods, and a given agent can produce only one of the goods in the continuum. An agent gets zero utility from consuming her production good. Each period, agents meet pairwise and at random. For a given agent, the probability that her would-be trading partner produces a good that she likes to consume is x, and the probability that she produces what her would-be trading partner wants is also x. There is a good called money, and a fraction M of the population is endowed with one unit each of this stuff in period 0. All goods are indivisible, being produced and stored in one-unit quantities. An agent can store at most one unit of any good (including money), and all goods are stored at zero cost. Free disposal is assumed, so it is possible to throw money (or anything else) away. For convenience, let $u^* = u(1)$ denote the utility from consuming a good that the agent likes, and assume that the utility from consuming a good one does not like is zero.

Any intertemporal trades or gift-giving equilibria are ruled out by virtue of the fact that no two agents meet more than once, and because agents have no knowledge of each others' trading histories.

8.2 Analysis

We confine the analysis here to stationary equilibria, i.e. equilibria where agents' trading strategies and the distribution of goods across the population are constant for all t. In a steady state, all agents are holding one unit of some good (ignoring uninteresting cases where some agents hold nothing). Given symmetry, it is as if there are were only two goods, and we let V_g denote the value of holding a commodity, and V_m the value of holding money at the end of the period. The fraction of agents holding money is μ, and the fraction holding commodities is $1-\mu$. If two agents with commodities meet, they will trade only if there is a double coincidence of wants, which occurs with probability x^2. If two agents with money meet, they may trade or not, since both are indifferent, but in either case they each end the period holding money. If two agents meet and one has money while the other has a commodity, the agent with money will want to trade if the other agent has a good she consumes, but the agent with the commodity may or may not want to accept money.

From an individual agent's point of view, let π denote the probability that other agents accept money, where $0 \leq \pi \leq 1$, and let π' denote the probability with which the agent accepts money. Then, we can write the Bellman equations as

$$V_g = \beta \left\{ \begin{array}{c} \mu x \max_{\pi' \in [0,1]} \left[\pi' V_m + (1-\pi') V_g \right] + \mu (1-x) V_g \\ +(1-\mu) \left[x^2 (u^* + V_g) + (1-x^2) V_g \right] \end{array} \right\}, \quad (8.1)$$

$$V_m = \beta \left\{ \mu V_m + (1-\mu) \left[x\pi (u^* + V_g) + (1 - x\pi) V_m \right] \right\}. \quad (8.2)$$

In (8.1), an agent with a commodity at the end of the current period meets an agent with money next period with probability μ. The money-holder will want to trade with probability x, and if the money-holder wishes to trade, the agent chooses the trading probability π' to maximize end-of-period value. With probability $1-\mu$ the agent meets another commodity-holder, and trade takes with probability x^2. If the agent trades, she consumes and then immediately produces again.

Similarly, in (8.2), an agent holding money meets another agent holding money with probability μ, and meets a commodity-holder with probability $1-\mu$. Trade with a commodity-holder occurs with probability $x\pi$, as there is a single coincidence with probability x, and the commodity-holder accepts money with probability π.

It is convenient to simplify the Bellman equations, as we did in the previous chapter, defining the discount rate r by $\beta = \frac{1}{1+r}$, and manipulating (8.1) and (8.2) to get

$$rV_g = \mu x \max_{\pi' \in [0,1]} \pi' (V_m - V_g) + (1-\mu) x^2 u^*, \quad (8.3)$$

$$rV_m = (1-\mu) x \pi (u^* + V_g - V_m). \quad (8.4)$$

Now, we will ignore equilibria where agents accept commodities in exchange that are not their consumption goods, i.e. commodity equilibria. In these equilibria, if two commodity-holders meet and there is a single coincidence, they trade, even though one agent is indifferent between trading and not trading. It is easy to rule these equilibria out, as in Kiyotaki and Wright (1993) by assuming that there is a small trading cost, $\varepsilon > 0$, and thus a commodity holder would strictly prefer not to trade for a commodity she does not consume. Provided ε is very small, the analysis does not change.

Now, there are potentially three types stationary equilibria. One type has $\pi = 0$, one has $0 < \pi < 1$, and one has $\pi = 1$. The first we can think of as a non-monetary equilibrium (money is not accepted by anyone), and the latter two are monetary equilibria. Suppose first that $\pi = 0$. Then, an agent holding money would never get to consume, and anyone holding money at the first date would throw it away and produce a commodity, so we have $\mu = 0$. Then, from (8.3), the expected utility of each agent in equilibrium is

$$V_g = \frac{x^2 u^*}{r}. \tag{8.5}$$

Next, consider the mixed strategy equilibrium where $0 < \pi < 1$. In equilibrium we must have $\pi' = \pi$, so for the mixed strategy to be optimal, from (8.3) we must have $V_m = V_g$. From (8.3) and (8.4), we then must have $\pi = x$. This then gives expected utility for all agents in the stationary equilibrium

$$V_m = V_g = \frac{(1-\mu)x^2 u^*}{r}. \tag{8.6}$$

Now, note that all money-holders are indifferent between throwing money away and producing, and holding their money endowment. Thus, there is a continuum of equilibria of this type, indexed by $\mu \in (0, M]$. Further, note, from (8.5) and (8.6), that all agents are worse off in the mixed strategy monetary equilibrium than in the non-monetary equilibrium, and that expected utility is decreasing in μ. This is due to the fact that, in the mixed strategy monetary equilibrium, money is no more acceptable in exchange than are commodities ($\pi = x$), so introducing money in this case does nothing to improve trade. In addition, the fact that some agents are holding money, in conjunction with the assumptions about the inventory technology, implies that less consumption takes place in the aggregate when money is introduced.

Next, consider the equilibrium where $\pi = 1$. Here, it must be optimal for the commodity-holder to choose $\pi' = \pi = 1$, so we must have $V_m \geq V_g$. Conjecturing that this is so, we solve (8.3) and (8.4) for V_m and V_g to get

$$V_g = \frac{(1-\mu)x^2 u^*}{r(r+x)} \left[\mu(1-x) + r + x\right], \tag{8.7}$$

$$V_m = \frac{(1-\mu)xu^*}{r(r+x)}\left[-(1-\mu)x(1-x)+r+x\right], \tag{8.8}$$

and we have

$$V_m - V_g = \frac{(1-\mu)x(1-x)u^*}{r+x} > 0$$

for $\mu < 1$. Thus our conjecture that $\pi' = 1$ is a best response to $\pi = 1$ is correct, and we will have $\mu = M$, as all agents with a money endowment will strictly prefer holding money to throwing it away and producing.

Now, it is useful to consider what welfare is in the monetary equilibrium with $\pi = 1$ relative to the other equilibria. Here, we will use as a welfare measure

$$W = (1-M)V_g + MV_m,$$

i.e. the expected utilities of the agents at the first date, weighted by the population fractions. If money is allocated to agents at random at $t = 0$, this is the expected utility of each agent before the money allocations occur. Setting $\mu = M$ in (8.7) and (8.8), and calculating W, we get

$$W = \frac{(1-M)xu^*}{r}\left[x+M(1-x)\right]. \tag{8.9}$$

Note that, for $M = 0$, $W = \frac{x^2u^*}{r}$, which is identical to welfare in the non-monetary equilibrium, as should be the case.

Suppose that we imagine a policy experiment where the monetary authority can consider setting M at $t = 0$. This does not correspond to any real-world policy experiment (as money is not indivisible in any essential way in practice), but is useful for purposes of examining the welfare effects of money in the model. Differentiating W with respect to M, we obtain

$$\frac{dW}{dM} = \frac{xu^*}{r}[1-2x+2M(-1+x)],$$

$$\frac{d^2W}{dM^2} = 2(-1+x) < 0.$$

Thus, if $x \geq \frac{1}{2}$, then introducing any quantity of money reduces welfare, i.e. the optimal quantity of money is $M^* = 0$. That is, if the absence of double coincidence of wants problem is not too severe, then introducing money reduces welfare more by crowding out consumption than it increases welfare by improving trade. If $x < \frac{1}{2}$, then welfare is maximized for $M^* = \frac{1-2x}{2(1-x)}$. Thus, we need a sufficiently severe absence of double coincidence problem before welfare improves due to the introduction of money. Note that from (8.6) and (8.9), for a given μ, welfare is higher in the pure strategy monetary equilibrium than in the mixed strategy monetary equilibrium.

8.3 Discussion

This basic search model of money provides a nice formalization of the absence-of-double-coincidence friction discussed by Jevons. The model has been extended to allow for divisible commodities (Trejos and Wright 1995, Shi 1995), and a role for money arising from informational frictions (Williamson and Wright 1994). Further, it has been used to address historical questions (Wallace and Zhou 1997, Velde, Weber and Wright 1998). A remaining problem is that it is difficult to allow for divisible money, though this has been done in computational work (Molico 1997). If money is divisible, we need to track the whole distribution of money balances across the population, which is analytically messy. However, if money is not divisible, it is impossible to consider standard monetary experiments, such as changes in the money growth rate which would affect inflation. In indivisible-money search models, a change in M is essentially a meaningless experiment.

While credit is ruled out in the above model, it is possible to have credit-like arrangements, even if no two agents meet more than once, if there is some knowledge of a would-be trading partner's history. Kocherlakota and Wallace (1998) and Aiyagari and Williamson (1998) are two examples of search models with credit arrangements and "memory." Shi (1996) also studies a monetary search model with credit arrangements of a different sort.

8.4 References

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