

Implementing the Mortensen Rule

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Abstract

We show that, in settings where meetings can be multilateral, the allocation rule proposed by Mortensen (1982) can be relatively straightforward to implement: as a local auction conducted by sellers. The implications of using this mechanism in a simple model of the labour market are then explored. We characterize the equilibrium properties of this model, which include wage dispersion, and examine its implied Beveridge curve. A dynamic version of the model is calibrated to the US labour market, and we show that the model can account for observed vacancy rates, given parameters that are chosen to match the average wages and the natural rate of unemployment, although the implied wage dispersion is quite small. Finally, in the limit, as the time between offer rounds in the model approaches zero, the equilibrium approaches the Walrasian competitive equilibrium.

Key words: search, matching, unemployment theory

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INTRODUCTION

In his landmark paper, among other things, Mortensen (1982) proposed a system of property rights, or an allocation rule, in matching games that would ensure efficiency:

“The agent responsible for each event [i.e., match] receives its total capital value less a compensation paid to every other agent whose play is terminated by the event equal to that agent’s value of continuing to play.”

According to this rule, the entire surplus of the match should be allocated to the initiator of the match, or the “matchmaker”. Although this rule was originally conceived in quite general settings, the matching literature that has followed, for the most part, has focussed on settings in which agents meet pairwise, and engage in bilateral bargaining.¹ This has led to the view that the Mortensen rule can be implemented only by somehow circumventing the bargaining process. Hosios (1990), for example, argues:

“Implementing Mortensen’s scheme may thus be difficult; it requires ex ante commitment by all players coupled with the ability to accurately and publicly identify the matchmaker among any pair of traders.”

In the context of these models, Hosios proposes an alternative rule that is more consistent with the bilateral bargaining structure: in the presence of constant returns to scale in the matching technology, each agent’s share of the surplus should be set equal to their marginal contribution to matches. In the case of Cobb-Douglas matching, for example, each agent’s exponent in the generalized Nash bargaining problem should be set equal to that agent’s exponent in the matching function. This is, of course, the famous “Hosios rule”.

¹ See Pissarides (2000) for an authoritative survey.

In this paper we argue that, in settings where meetings can be multilateral, the Mortensen rule can be relatively straightforward to implement: as a local auction conducted by sellers (i.e., workers).² We demonstrate this point in a simple setting with complete information, but the point is quite general.³ We then explore the implications of using this mechanism in a simple model of the labour market. We characterize the equilibrium properties of this model, which include wage dispersion, and examine its implied Beveridge curve. A dynamic version of the model is calibrated to the US labour market, and we show that the model can account for observed vacancy rates, given parameters that are chosen to match the average wages and the natural rate of unemployment. Finally, in the limit, as the time between offer rounds in the model approaches zero, the equilibrium approaches the Walrasian competitive equilibrium.

The remainder of this paper is structured as follows. In Section 1, we demonstrate the equivalence of the Mortensen rule and local auctions in frictional setting. In Section 2, the static model of the labour market is presented, and its equilibrium properties are analysed. In Section 3 we consider a discrete time dynamic model, and draw out the implications for the equilibrium Beveridge curve implied by the theory, and the Walrasian limit. We also present the results from two numerical simulations of the stationary equilibrium of the model, calibrated to the US economy. Section 4 then concludes and provides some suggestions for future work.

² Hosios (1990) recognized that “auction markets” would implement the Mortensen rule (see p. 291 in his paper) but, for him, that meant aggregate markets with a Walrasian auctioneer, rather than local auctions.

³ Moreover, as demonstrated by McAfee (1993), this mechanism is an equilibrium when sellers can choose mechanisms.

1. THE MORTENSEN RULE AND LOCAL AUCTIONS

Consider a market that consists of a number $M > 0$ of identical buyers and a number $N > 0$ of identical sellers. Each seller has one identical good for sale. The seller's reservation value of the good is zero and each buyer's valuation of the good is $y > 0$. Buyers are randomly allocated to sellers in the following way: the probability that a buyer visits any particular seller is given by $p = 1/N$. The expected number of matches (or the “matching function”) is given by $H(N, M) = N(1 - (1 - 1/N)^M)$

For any finite M and N , this matching function has decreasing returns to scale but, in the limit where M and N are large, has constant returns to scale. In this paper, except where indicated, we will restrict attention to this limit. In this limit: $p = 0$ and $H(M, N) = N(1 - e^{-\phi})$, where $\phi = M/N$.

The random assignment of buyers to sellers yields a set of local markets, each with one seller and a stochastic number of buyers. Let $\theta(m)$ denote the probability that the local market of a seller has m buyers. It is easy to show that $\theta(0) = e^{-\phi}$ and $\theta(1) = \phi e^{-\phi}$. Let $\xi(m)$ denote the conditional probability that any particular *buyer* is one of m buyers in a local market. The probability the buyer is alone at his or her chosen local market is given by $\xi(1) = \theta(0) = e^{-\phi}$.

The *surplus* of a match between a buyer and a seller in a local market is equal to the value of the match minus the disagreement points of the seller and the buyer. Each local market contains one seller S , and a set of identical buyers $B = (B_1, B_2, \dots, B_m)$ if $m \geq 1$ and no buyers if $m = 0$. The surplus of a match between the seller and any particular buyer B_b is given by:

$$\Lambda_b(S, B) = V(S, B_b) - d_s(B) - d_b(B) \quad (1.1)$$

where $V(S, B_b)$ is the total value of the match, $d_s(B)$ is the disagreement point of the seller, and $d_b(B)$ is the disagreement point of the buyer.⁴ The total valuation of a match is:

$$V(S, B_b) = y \quad (1.2)$$

Once inside the local market, the buyer can trade only with the seller, hence, the disagreement point of the buyer is:

$$d_b(B) = 0 \quad (1.3)$$

The disagreement point of the seller is given by $\max V(S, B_{-b})$ - the maximum total value of the good to the seller and the set of other buyers. In this example, with identical buyers, the disagreement point of the seller can take two possible values:

$$d_s(B) = \begin{cases} 0 & \text{if } m = 1 \\ y & \text{if } m \geq 2 \end{cases} \quad (1.4)$$

Applying the Mortensen Rule

In this setting, applying the Mortensen rule, where the buyer is the initiator of the match, using equations (1.1) – (1.4) we have the *ex post* payoff to the buyer:

$$W_b^M(B) = \begin{cases} y & \text{if } m = 1 \\ 0 & \text{if } m \geq 2 \end{cases} \quad (1.5a)$$

And, hence, the *ex post* payoff to the seller:

$$W_s^M(B) = \begin{cases} 0 & \text{if } m = 1 \\ y & \text{if } m \geq 2 \end{cases} \quad (1.5b)$$

⁴ See Binmore, Rubinstein and Wolinsky (1986) for a definition of the surplus of a match in a static game.

Accordingly, the *ex ante* payoffs, under the Mortensen rule, to each buyer and seller are, respectively:

$$EW_b^M = \xi(1)y = e^{-\phi}y \quad (1.6a)$$

$$EW_s^M = [1 - \theta(0) - \theta(1)]y = [1 - e^{-\phi} - \phi e^{-\phi}]y \quad (1.6b)$$

Applying the Auction Mechanism

Consider now, alternatively, the allocations generated by, for example, an ascending-bid auction, where the reserve price is set at the outside option of the seller: zero in this case. For a buyer, if the random assignment of buyers to sellers determines that he is alone in the seller's local market, then the seller is obliged to sell the good at the reserve price: zero. In this case, the payoff to the buyer is the full value of y . If, alternatively, any other buyer is allocated to that local market, then the price of the good is bid up to its full valuation from the point of view of buyers: y . In this case, the payoff to the buyer is zero (whether or not the buyer actually purchases the good). Hence, the *ex post* payoff to the buyer is:

$$W_b^A(B) = \begin{cases} y & \text{if } m = 1 \\ 0 & \text{if } m \geq 2 \end{cases} \quad (1.7a)$$

For a seller, if the random assignment of buyers to sellers determines that one buyer appears in her local market, then she is obliged to sell the good at the reserve price: zero. If, alternatively, two or more buyers appear in her local market, then the price of the good is bid up to its full valuation from the point of view of buyers: y . Hence, the *ex post* payoff to the seller is:

$$W_s^A(B) = \begin{cases} 0 & \text{if } m = 1 \\ y & \text{if } m \geq 2 \end{cases} \quad (1.7b)$$

Moreover, accordingly, the *ex ante* payoffs, under the auction rule, to each buyer and seller are, respectively:

$$EW_b^A = \xi(1)y = e^{-\phi}y \quad (1.8a)$$

$$EW_s^A = [1 - \theta(0) - \theta(1)]y = [1 - e^{-\phi} - \phi e^{-\phi}]y \quad (1.8b)$$

Comparing (1.5a,b) with (1.7a,b) and (1.6a,b) with (1.8a,b), clearly, both the *ex ante* and *ex post* payoffs to buyers and sellers under the auction are identical to those under the Mortensen rule.

We now turn to analyse a frictional model of the labour market which uses this payoff structure to determine wages in equilibrium.

2. THE STATIC MODEL

Consider a labour market with a large number N of identical, risk neutral, job candidates where each candidate has one indivisible unit of labor to sell. There are $M = \phi N$ vacancies, where $\phi \geq 0$, and is determined by free entry. The output of a worker is $y_0 = 0$ if unemployed and $y_1 = y > 0$ if employed. It costs an amount k to create a vacancy, where $0 < k < y$. Each vacancy can approach only one candidate.

The order of play is as follows. Given N job candidates, M vacancies enter the market. Once the number of entrants has been established, vacancies choose which candidate to approach. Once vacancies have been assigned to candidates, wages are determined. We solve the model using backwards induction.

Ex Post Wage Determination

Here, we take as given the number of entrants, and the assignment of vacancies to candidates. Let w_i^j denote the wage earned by a worker who is employed in a job with output y_i $i \in \{0,1\}$, whose outside option is employment in a job with output y_j . Notice that, for notational convenience, we have classified an unemployed worker as earning a wage w_0^0 . Through the ex post auction, equilibrium wages have the following structure:

$$w_i^j = y_j \tag{2.1}$$

Thus, if a worker is approached by exactly one firm, his outside option is $y_0 = 0$, and so his wage is $w_1^0 = y_0 = 0$. If a worker is approached by two or more firms then his outside option, when negotiating with any of these firms, is the output from any of the other firms: $y_1 = y$. In this case $w_1^1 = y_1 = y$. Notice that *ex post*, a vacancy will receive the payoff y if she is alone when approaching a candidate, and zero otherwise.

The Assignment of Vacancies to Workers

Here, we consider the choices made by the different vacancies about particular candidates to approach. This strategic decision is modeled, with some care, in Julien, Kennes, and King (2000), and we refer the reader to section 2.2 in that paper for a detailed analysis. In this paper, we follow the tradition of restricting attention to the unique symmetric mixed strategy equilibrium, in which vacancies randomize over candidates with equal probability. Let p_i^j denote the probability that a candidate earns wage w_i^j . Consequently, in a large market such as this, the probability distribution of wages facing candidates (for any ϕ , and using (2.1)) is:

$$w_i^j, p_i^j = \begin{cases} w_0^0 = 0 & p_0^0 = e^{-\phi} \\ w_1^0 = 0 & p_1^0 = \phi e^{-\phi} \\ w_1^1 = y & p_1^1 = 1 - e^{-\phi} - \phi e^{-\phi} \end{cases} \quad (2.2)$$

Here, as in all the static models of this type, the unemployment rate is given by the probability that any particular candidate will have no vacancies approach him. As in Section 1, above, this is given by:

$$u = e^{-\phi} \quad (2.3)$$

and the number of matches or hires (the matching function) is given by:

$$H(M, N) = N(1 - e^{-M/N}) \quad (2.4)$$

We now turn the determination of the value of M , given N , through entry.

Vacancy Entry

If a vacancy is able to hire a candidate then the profit from creating a vacancy is equal to its output (y) minus the cost from creating it (k) and minus the wage paid to the worker (w). The probability of hiring a candidate, and the wage paid to the candidate depends on the assignment of other vacancies, and firms create vacancies as long as the expected profit from doing so is positive. Let q denote the probability that the firm is alone when approaching the candidate, so that $1 - q$ is the probability that at least one other firm approaches the candidate. As in section 1, above:

$$q = e^{-\phi} \quad (2.5)$$

By (2.2), whenever a vacancy is assigned to a worker with at least one other firm present, the worker is paid the full output from the match: $w_1^1 = y$. Hence, the only

situation in which the vacancy stands to make a positive profit is when the firm is alone when approaching a candidate. Thus, expected profits from a vacancy are: $\pi = q(y - w_1^0) - k$. Using (2.2) and (2.5), we then have:

$$\pi = e^{-\phi} y - k \quad (2.6)$$

With competitive entry, we have the additional condition:

$$\pi = 0 \quad (2.7)$$

Using (2.6) and (2.7), we can now determine the equilibrium value of ϕ from the equation:

$$e^{-\phi} = k / y \quad (2.8)$$

or:

$$\phi = \ln y - \ln k$$

With ϕ determined in equation (2.8) all of the endogenous variables are determined. Equation (2.8) also tells us the equilibrium unemployment rate in this model: simply the ratio of the cost to the output from a vacancy:

$$u = k / y \quad (2.9)$$

Similarly, equilibrium unfilled vacancies can be found:

$$v = \frac{M - N(1 - e^{-\phi})}{N} = \phi - 1 + e^{-\phi}$$

and, using (2.8):

$$v = k / y + \ln y - \ln k - 1 \quad (2.10)$$

Using (2.9) and (2.10), we can also derive the Beveridge curve:

$$v = u - 1 - \ln u \quad (2.11)$$

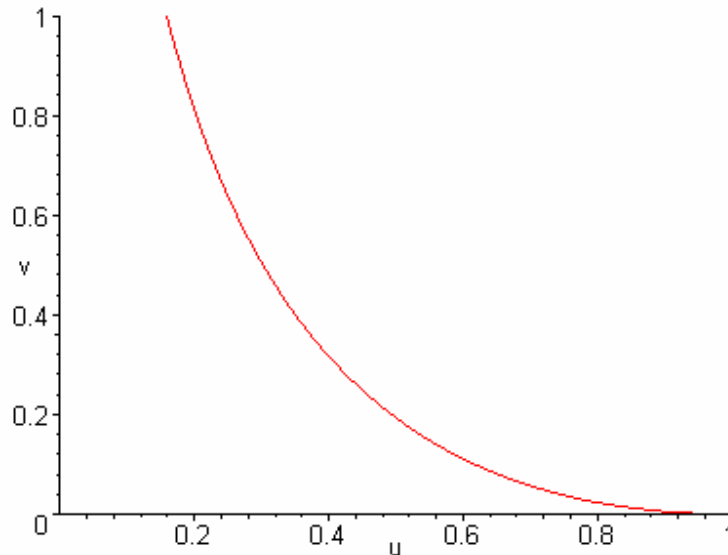


Figure 1: The Beveridge Curve in the Static Model

It is worthwhile to note some of the features of this equilibrium. First, from (2.8) and (2.11), we can see that the unemployment rate and the vacancy rate are determined purely by the *ratio* k/y . For example, if productivity and vacancy costs increased proportionately (as is typically assumed in balanced growth models) then unemployment and vacancies would be constant along this path. Also, in this simple static model, nothing shifts the Beveridge curve: changes in either of the two parameters (y and k) simply move the economy along the curve. Quite naturally, in the limit as $k \rightarrow y$, the ratio of vacancies to candidates $\phi \rightarrow 0$, the unemployment rate $u \rightarrow 1$, and the unfilled vacancy rate $v \rightarrow 0$. The equilibrium also has some wage dispersion – the exact amount of which could now be computed from (2.2).

A Numerical Example

If $y=1$ and $k=1/2$ then the equilibrium unemployment rate is 50%, the ratio of vacancies to candidates is 0.6931, and the unfilled vacancy rate is 19.31%. (Clearly, these unemployment and unfilled vacancy rates are very high, reflecting the fact that vacancies have only one chance to hire workers in this static model. More realistic rates occur in the dynamic model, which we analyze in Section 3 below.) The percentage of candidates that find employment, but are paid only their outside option (zero), is 34.655%, leaving 15.345% earning the top wage (1). This implies an average wage, among employed workers, of 0.307. The standard deviation of the wage distribution is 0.213. The wage distribution is also skewed to the right, as in most empirical studies.⁵

Directed and Undirected Search

This model could also be interpreted as one of *undirected* search. If for, example, we interpret the random allocation of vacancies over candidates, according to the matching function (2.3), as a matching *technology* in the usual sense (for example, Pissarides (2000)), rather than as the outcome of a mixed strategy equilibrium, then this model fits into the category of undirected search. There are, however, two key differences between this interpretation of the model and the standard Pissarides model. First, the matching technology here allows for multilateral matches, according to an urn-ball process, while the standard model restricts matches to be pair-wise. Secondly, while both wage determination mechanisms (here and in the standard model) are *ex post*, here we use an auction or, equivalently, the Mortensen rule.

To assess the empirical relevance of this model, we now present a dynamic version, and examine its steady state equilibrium.

⁵ The equilibrium wage distribution in this model is skewed to the right for any value of $y/k < 3.513$, and skewed to the left for any higher value of y/k .

3. THE DYNAMIC MODEL

There is large number, N , of identical risk neutral workers facing an infinite horizon, perfect capital markets, and a common discount factor $\beta \in (0,1)$. In each time period, each worker has one indivisible unit of labor to sell. At the start of each period $t = 0,1,2,3,\dots$, there are E_t employed workers, with productivity $y_1 = y$, and $(N - E_t)$ unemployed workers, with productivity $y_0 = 0$. Hence $(N - E_t)$ is the number of agents in the labor force that are actively searching for employment. Also, at the beginning of each period, there are $M_t = \phi_t(N - E_t)$ vacancies. In each period a vacant job has a capital cost of k . Any match in any period may dissolve in the subsequent period with fixed probability $\rho \in (0,1)$. In each period, any vacant job can enter negotiations with at most one worker. However, unemployed workers apply to all firms.

Within each period, the order of play is the same as in the static model: at the beginning of the period, given the state, new vacancies enter. Once the number of entrants has been established, vacancies choose which workers to approach. Once new vacancies have been assigned to candidates, wages are determined through the auction (or, the Mortensen rule). We now consider the determination of the equilibrium, using backward induction, within a representative period t .

Wage Determination

Let Λ_{it} denote the expected discounted value of a match between a worker and a job of productivity y_i once vacancies have been assigned in period t , for $i = 0,1$. Thus, Λ_{0t} represents the value of a “match” between a worker and the unemployment state. Here, through the bidding mechanism, the value of a worker’s contract W_{it}^j is equal to the expected discounted value Λ_{jt} of a match between the worker and the worker’s second best available alternative:

$$W_{it}^j = \Lambda_{jt} \tag{3.1}$$

The Assignment of Vacancies to Workers

As in the static model, we restrict attention to the unique symmetric mixed strategy equilibrium in which each vacancy randomises over all candidates with equal probability. Let p_{it}^j denote the probability that a candidate receives a contract worth W_{it}^j . Consequently, the probability distribution of contracts facing candidates (for any ϕ_t , and using (3.1)) is:

$$W_{it}^j, p_{it}^j = \begin{cases} W_{0t}^0 = \Lambda_{0t} & p_{0t}^0 = e^{-\phi_t} \\ W_{1t}^0 = \Lambda_{0t} & p_{1t}^0 = \phi_t e^{-\phi_t} \\ W_{1t}^1 = \Lambda_{1t} & p_{1t}^1 = 1 - e^{-\phi_t} - \phi_t e^{-\phi_t} \end{cases} \quad (3.2)$$

Whereas, in the static model, the expression $e^{-\phi}$ represents the unemployment rate; here, in the dynamic model, $e^{-\phi_t}$ represents, in any period t , the fraction of candidates that remain unemployed at the end of the period. In discrete time models such as this, the beginning and end of period unemployment rates (and vacancy rates) differ. Researchers have traditionally analysed beginning-of-period rates (see, for example, Shi (2001)) however, end-of-period rates are most directly comparable with their static counterparts. For this reason, we consider both in this paper. Since the number of candidates at the beginning of a period is $(N - E_t)$, the unemployment rate, at the *beginning* of any period t is given by

$$u^* = (N - E_t) / N \quad (3.3a)$$

whereas unemployment at the *end* of the period, which is given by:

$$u = u^* e^{-\phi_t} = \frac{N - E_t}{N} e^{-\phi_t} \quad (3.3b)$$

The number of new hires H_t is given by the matching function:

$$H_t = (N - E_t)(1 - e^{-\phi_t}) \quad (3.4)$$

Using (3.2), we can now express V_t , the value of being a candidate at the beginning of the period, in the following way:

$$V_t = (e^{-\phi_t} + \phi_t e^{-\phi_t})\Lambda_{0t} + (1 - e^{-\phi_t} - \phi_t e^{-\phi_t})\Lambda_{1t} \quad (3.5)$$

Moreover, a candidate's outside option Λ_{0t} can now be expressed as simply the value of being a candidate in the subsequent period:

$$\Lambda_{0t} = \beta V_{t+1} \quad (3.6)$$

Vacancy Entry

If a vacancy is able to hire a candidate then the profit from creating the vacancy is equal to the value of the match (Λ_{1t}) minus the cost from creating the vacancy (k) and minus the value of the contract paid to the worker (W_t). The total expected value of the match is equal to the current output y plus the expected discounted future flows of output from the match, and the outside options for the worker if the match separates:⁶

$$\Lambda_{1t} = y + \beta[\rho V_{t+1} + (1 - \rho)y] + \beta^2(1 - \rho)[\rho V_{t+2} + (1 - \rho)y] + \dots \quad (3.7)$$

The value of the contract paid to the worker depends on the assignment of vacancies to workers. Let q_t denote the probability, in period t , that a vacancy is alone when approaching a worker. This probability is:

⁶ Expected profits for vacancies are driven down to zero, in equilibrium, so future profits for the firm do not appear in (3.7).

$$q_t = e^{-\phi_t} \quad (3.8)$$

By equation (3.2), whenever a candidate is approached by more than one vacancy, the value of the candidate's contract is bid up to the entire value of the match: $W_{1t}^1 = \Lambda_{1t}$. Hence, as in the static model, firms can earn positive profits only when they are alone when approaching candidates. Thus, expected profits are: $\pi_t = q_t (\Lambda_{1t} - W_{1t}^0) - k$. Using (3.2) and (3.8) we get:

$$\pi_t = e^{-\phi_t} (\Lambda_{1t} - \Lambda_{0t}) - k \quad (3.9)$$

Competitive entry implies:

$$\pi_t = 0 \quad (3.10)$$

Equations (3.9) and (3.10), together, imply:

$$e^{-\phi_t} (\Lambda_{1t} - \Lambda_{0t}) = k \quad (3.11)$$

Employment Dynamics

New matches are created according to the matching function (3.4) and broken apart at rate ρ . This leads to employment dynamics:

$$E_{t+1} = (1 - \rho)(E_t + H_t) \quad (3.12)$$

where H_t is given in (3.4).

In this paper, we focus on the stationary equilibrium, where all flows are constant over time.

The Stationary Equilibrium

In the stationary equilibrium, equations (3.4), (3.5), (3.6), (3.7), (3.11), and (3.12) become:

$$H = (N - E)(1 - e^{-\phi}) \quad (3.13)$$

$$V = (e^{-\phi} + \phi e^{-\phi})\Lambda_0 + (1 - e^{-\phi} - \phi e^{-\phi})\Lambda_1 \quad (3.14)$$

$$\Lambda_0 = \beta V \quad (3.15)$$

$$\Lambda_1 = \frac{y + \beta \rho V}{1 - \beta(1 - \rho)} \quad (3.16)$$

$$e^{-\phi}(\Lambda_1 - \Lambda_0) = k \quad (3.17)$$

$$E = (1 - \rho)(E + H) \quad (3.18)$$

Equations (3.13)-(3.18) are six equations in six unknowns: H, E, ϕ, V, Λ_0 , and Λ_1 . This system is block recursive, with equations (3.14)-(3.17) determining ϕ, V, Λ_0 , and Λ_1 . Then, once ϕ is determined, equations (3.13) and (3.18) determine E and H . Once these variables are determined, then the solutions for all the other endogenous variables can be found in the stationary equilibrium.

Solving this set of equations, one obtains the following equation, which uniquely determines the value of market tightness, ϕ .

$$e^{-\phi} = [1 - (1 - \rho)\beta(1 + \phi)e^{-\phi}] \frac{k}{y} \quad (3.19)$$

With ϕ determined in (3.19), the following values are determined:

$$\Lambda_0 = \frac{y - (1 - \beta(1 - \rho))e^{\phi} k}{(1 - \beta)(1 - \rho)} \quad (3.20)$$

$$\Lambda_1 = \frac{y - \rho e^{\phi} k}{(1 - \beta)(1 - \rho)} \quad (3.21)$$

$$V = \frac{y - (1 - \beta(1 - \rho))e^{\phi} k}{\beta(1 - \beta)(1 - \rho)} \quad (3.22)$$

$$E = \frac{(1 - \rho)(1 - e^{-\phi})}{1 - (1 - \rho)e^{-\phi}} N \quad (3.23)$$

$$H = \frac{\rho(1 - e^{-\phi})}{1 - (1 - \rho)e^{-\phi}} N \quad (3.24)$$

Using (3.23) in (3.3a,b), we get the equilibrium unemployment rates at the beginning and end of the period, respectively:

$$u^* = \frac{\rho}{1 - (1 - \rho)e^{-\phi}} \quad (3.25a)$$

$$u = \frac{\rho e^{-\phi}}{1 - (1 - \rho)e^{-\phi}} \quad (3.25b)$$

The vacancy rate at the beginning of a period is given by $v^* = M / N = \phi(N - E) / N$.

Using (3.23) we get:

$$v^* = \frac{\rho\phi}{1 - (1 - \rho)e^{-\phi}} \quad (3.26a)$$

At the end of a period, the unfilled vacancy rate $v = (M - H) / N$ can be found, using $M = \phi(N - E)$, together with (3.23) and (3.24) to get:

$$v = \frac{\rho(e^{-\phi} - 1 + \phi)}{1 - (1 - \rho)e^{-\phi}} \quad (3.26b)$$

Using (3.25a,b) and (3.26a,b), we can now find an expression for the ratios v^*/u^* and v/u in terms of ϕ :

$$v^* / u^* = \phi \quad (3.27a)$$

$$v / u = (e^{-\phi} - 1 + \phi)e^{\phi} \quad (3.27b)$$

At this point, it is instructive to compare (3.27b) with the Beveridge curve in the static model (2.11). In both the static and dynamic models, $e^{-\phi}$ represents the fraction of candidates that go unmatched in equilibrium. In the static model, this is the unemployment rate. Thus, in the static model, (2.27b) reduces to (2.11). However, in the dynamic model, the number of candidates in a period is significantly smaller than the number of workers (the workforce) N , and the Beveridge curve is somewhat different.

Beveridge Curves in the Dynamic Model

Beveridge curves for both the beginning and end of periods can be found in the following way. From (3.25a) we have:

$$\phi = \ln(1 - \rho) - \ln(1 - \rho / u^*)$$

Using this in (3.27a), we have the *beginning-of-period Beveridge curve*:

$$v^* = u^* (\ln(1 - \rho) - \ln(1 - \rho/u^*)) \quad (3.28a)$$

Similarly, (3.25b), can be re-written as:

$$e^\phi = \rho/u + 1 - \rho$$

Or:

$$\phi = \ln(\rho/u + 1 - \rho)$$

Using these in (3.27b), and collecting terms, we have the *end-of-period Beveridge curve*:

$$v = u((\rho/u + 1 - \rho)(\ln(\rho/u + 1 - \rho) - 1) + 1) \quad (3.28b)$$

Notice that in the dynamic model, unlike the static model, a shift parameter (ρ) appears in the Beveridge curves. Figures 2a and 2b, below, illustrate the beginning-of-period and end-of-period Beveridge curves, respectively, where ρ has been set equal to 0.01 in each case.

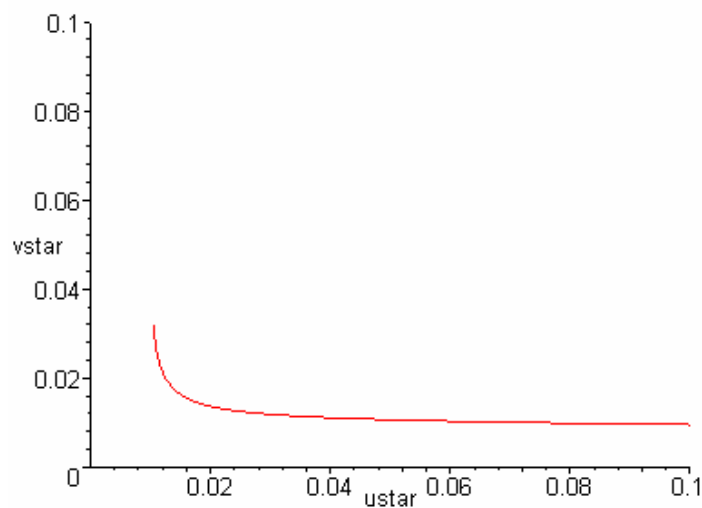


Figure 2a: The Beginning-of-Period Beveridge Curve in the Dynamic Model

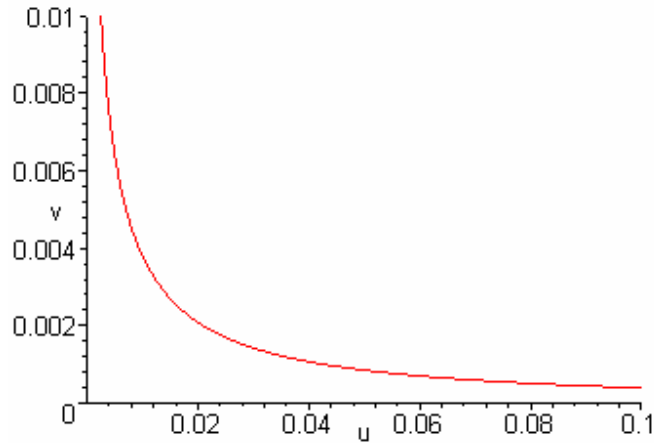


Figure 2b: The End-of-Period Beveridge Curve in the Dynamic Model

Two things are worth noting about these Beveridge curves. First, the beginning-of-period curve is undefined for values of the unemployment rate below the separation rate ρ (for reasons that are clear in equation (3.28a)). Second, in each case, the curves shift outward as ρ increases. Moreover, in the limit, as $\rho \rightarrow 0$, both unemployment and vacancies also approach zero.

Calibrated Examples

There are only four parameters in this model: (β, ρ, y, k) . In this paper we consider two different calibrations of the stationary equilibrium. In each case, parameters are chosen so that endogenous variables match data from Katz and Autor's (1999) study of the US labour market for 1995. The two calibrations differ only in the following sense: in the first, parameters are chosen so that the *beginning-of-period* unemployment rate matches the estimated natural rate of unemployment in that year (3.9%), whereas in the second, the *end-of-period* unemployment rate matches this figure. Since Katz and Autor consider weekly data, the discount factor is tied down to $\beta = 0.999$. Similarly, using Kuhn and Sweetman's (1998) estimate of a 4% monthly separation rate, we set $\rho = 0.01$ (as in the

graphical representations of the Beveridge curves, in Figures 7a,b above). (This also implies that the length of time that workers have to consider offers is one week.) We therefore choose values of y and k to match the weekly average real wage in 1982 dollars (\$255) and the natural rate of 3.9%. The model then gives us implied values of the vacancy rate and measures of wage dispersion, which can be compared with those in the data.

The following table summarizes the results from the first calibration, matching the beginning-of-period unemployment rate.

Parameter Values: $\beta = 0.999$, $\rho = 0.01$; $y = 339$, $k = 5800$
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Statistic	Model	Data
Mean Wage	255	255
Unemployment Rate	3.9	3.9
Vacancy Rate	1.1	1.5
Standard Deviation of Log Wage	0.01	0.616

Table 1: Matching Unemployment at the Beginning of the Period

The first thing to note from Table 1 is that the required cost of vacancies k is quite high, when considering weekly costs. The reason for this is that, in this model, these costs terminate once a vacancy is filled – and vacancies are filled quite quickly in equilibrium. In reality, there are fixed costs when creating jobs and these costs can be quite high. Following Pissarides (2000), though, to keep the state vector as small as possible, we model all costs as flow costs. As a consequence, to balance the expected present value of the infinite stream of the benefits from a job, the costs appear large.

The model generates a beginning-of-period vacancy rate that is quite close to the figure used (from the OECD Main Economic Indicators) – a slight underestimation of the vacancy rate: 1.1% versus 1.5% in the data. Also some wage dispersion is present in the model (the two weekly wages are \$254.26 and \$266.58) but much less than in the data: the model generates a standard deviation of the log wage of only 0.01, compared to the figure of 0.616 reported by Katz and Autor for US data. The model’s wage distribution is also skewed to the right with 86.5% of the employed earning the lower wage.

Table 2 summarizes the corresponding figures when matching the end-of-period unemployment rate.

Parameter Values: $\beta = 0.999$, $\rho = 0.01$; $y = 391$, $k = 10000$

Statistic	Model	Data
Mean Wage	255	255
Unemployment Rate	3.9	3.9
Vacancy Rate	0.1	1.5
Standard Deviation of Log Wage	0.0145	0.616

Table 2: Matching Unemployment at the End of the Period

Clearly, to achieve an unemployment rate of 3.9% at the end of the period, we must have a higher unemployment rate at the beginning of the period. Through the Beveridge curve relationships in Figures (7a,b), (which are stable as values of y and k vary) higher unemployment rates will reduce vacancy rates. In this case, the end-of-period vacancy rate is less than 10% of the figure in the data. The wage dispersion figure, as in Table 1, is also significantly below that from the Katz and Autor study.

Interpretations

When considering beginning-of-period unemployment and vacancy rates, this model generates a plausible number for the vacancy rate. However, this number falls significantly when end-of-period rates are used. This raises the question of which rates are more appropriate to be used. Estimated vacancy rates use measures of vacancy advertisements. It seems reasonable to interpret these as occurring at the beginning of a period – that is, we expect the number of *unfilled* vacancies within any time period to be less than the number of *advertised* vacancies. For the purposes of policy, though, it is the unfilled vacancies, and unsuccessful candidates that are most relevant.

While this theory generates some wage dispersion, even in the best case, the model can only explain 2% of the dispersion found in the data. If one were to try to explain more of this dispersion using this framework, some other ingredient would be required. For example, in Julien, Kennes and King (2006) we allow for two different types of jobs to be created by firms – one with higher productivity and costs than the other.

A Note on the Walrasian Limit

The key friction in this framework is the time it takes for a firm to approach a worker with a job offer. In the static model firms can approach only one worker, so the time it takes to approach another is effectively infinite. In the dynamic model, as we have calibrated it, this takes one week. Clearly, as this length of time shrinks, the friction disappears. This is reflected in the equilibrium unemployment and vacancy rates derived above. As the length of time shortens the relevant value of ρ falls. For any finite value of ϕ , this means that u , u^* , v , and v^* all approach zero as $\rho \rightarrow 0$ (as can be seen from equations 3.25a,b and 3.26a,b). One interpretation of the results, therefore, is that a week between offers is sufficient to explain the unemployment and vacancy figures we observe – but not the wage dispersion.

4. CONCLUSION

The Mortensen rule is relatively straightforward to implement in frictional models where buyers are allocated to sellers according to a standard urn-ball process. We have shown that this rule can be implemented through local auctions conducted by sellers. Applying this to the labour market, as a wage determination mechanism, generates some wage dispersion among ex ante identical workers, while the random assignment generates unemployment and unfilled vacancies. Introducing the Mortensen rule in this way leads to a very simple and tractable model, with both directed and undirected search interpretations. The calibration of the dynamic version of this model, which embeds the Mortensen rule, shows that the model can account for observed vacancy rates, given parameters that are chosen to match the average wages and the natural rate of unemployment. However, quantitatively, the wage dispersion implied by the Mortensen rule alone, in a model with homogenous agents such as this, is quite small: approximately only 2% of observed values. Although this framework implies some wage dispersion, other factors must come into play if observed levels are to be explained.

A key implication of the using the Mortensen rule in this way is that the equilibrium in the model is constrained-efficient in the usual sense – unless a planner can somehow eliminate the underlying friction, he or she cannot increase aggregate expected output or utility by influencing decisions made by agents. With risk-neutral agents, the wage dispersion implied by the rule is efficient – although this dispersion is not necessary for efficiency: as noted in Kultti (1999), a price posting model in this setting with large markets implies the same expected payoffs and efficient entry. In this paper we have restricted attention to the limit case where M and N are large. As noted above, in this case the auction mechanism used here, (where the reserve price is equal to the seller's outside option) can be justified as an equilibrium one (McAfee (1993)). With finite numbers of players, it can be shown that this type of auction still implements the Mortensen rule, and thereby efficiency (Julien, Kennes, and King (2005)) but it cannot be justified as an equilibrium choice (Julien, Kennes, and King (2002)). However, in the context of aggregate labour markets, the large market assumption is quite natural.

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