The Curse of Guided Search*

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Abstract

We consider the effects of provision of information by a third party about seller qualities in a frictional search market with heterogeneous sellers and asymmetrc information between buyers and sellers. We show that this 'guided search' hurts buyers if they end up competing more intensively over high quality sellers and subsequently experience an increase in equilibrium prices that is greater than the increase in the equilibrium probability of trade with high quality sellers. We show that this will occur if the additional information provided about sellers is sufficiently small, or if the market has many more buyers than sellers. We also show that guided search always raises welfare, and provide numerical results quantifying the welfare gains and redistributional effects.

1 Introduction

At the heart of every economic theory is a description of how people exchange. Economists generally choose between one of two possible extremes: Walrasian general equilibrium or random matching. The general equilibrium extreme assumes the cost of communication is zero. Agents in a general equilibrium model need only report their characteristics to a central mechanism designer and a set of transfers are then carried out using this information.[1] At the opposite extreme

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[1] This central figure is often referred to as the Walrasian auctioneer.

is random matching. In random matching models, the cost of communication is assumed to be almost insurmountable. Instead, people are brought together by an exogenous matching technology and only then can they communicate with their potential trading partner.

Recent research has developed a class of trading models that assume trade is neither Walrasian nor random matching. These so-called directed search models allow types of communication that fall within the extremes of Walrasian and random matching environments. Such models derive trading structures in which mixed strategies characterize the equilibrium actions of buyers. For example, in a directed search model, we might assume that sellers can easily communicate characteristics of their product to buyers, but we might also assume that buyers cannot communicate with each other regarding which seller to visit. Alternatively, as we assume in the present paper, buyers might be distrustful about the claims of sellers and instead seek alternative sources of information to guide them in their search. The goal of this paper is to assess the potential costs and benefits of guided search.

There are many instances where buyers are distrustful of the claims of sellers. The most obvious case is where buyers are better informed about their valuation of a seller's good than is the seller herself. In this case, an auction is a useful method by which the seller can ensure she always gets the highest price without sacrificing probability of trade, which would occur if she set a posted price. Moreover, even if sellers know the value buyers place on their goods, there is little reason to expect in search equilibrium that sellers with low value goods will truthfully advertise themselves as such. This is because doing so will reduce the probability of a successful transaction, since search effort is costly and buyers exert less search effort in pursuit of sellers advertising low quality.

In this paper, to keep things simple, we assume that sellers with goods of different qualities all sell their goods by identical auctions without reserve prices.[2] We first characterise the trading equilibrium when buyers are unaware of any given seller's quality. We then ask what happens if a third party makes buyers better informed about the quality of sellers. We show that third party information given to buyers changes both the price and probability of trade with high quality sellers. Buyers are hurt by third party information about sellers

[2] This can be partially justified by the fact that competition among sellers reduces the reserve prices of competing auctions to zero (Julien, Kennes and King 2000). Posted prices do no better (Kultti 1999).

if the amount of additional information is sufficiently small or if sellers are sufficiently scarce. In this case, the equilibrium prices at high quality sellers' auctions increase more than the probability of trade with such sellers. Thus additional information about sellers may actually be a curse to buyers.

We find it somewhat surprising that most of the gains of third party information are concentrated on the sellers' side of the market. However, in looking at the stylized evidence, we find that this prediction does sit well with a number of stylized facts about third party information services. In particular, our model explains why many markets are administered by professional associations that are set up by the concerted actions of sellers. Medical and law associations are but two examples. Likewise, our model predicts why consumer action can be seen as counterproductive to their interests. Thus, our model explains the observed absence of industry watchdogs that are administered by the concerted action of buyers.

The rest of this paper is organized as follows. In section 2 we introduce our basic frictional search model and some standard equilibrium results when there is asymmetric information and search is unguided. In section 3 we introduce third party information and derive the equilibrium response of buyers to this information and the consequent welfare effects. Then in section 4 we present some numerical results to quantify these effects. Section 5 offers concluding remarks.

2 The model

In this section we describe the stochastic environment in which buyers and sellers trade. We consider a single market and let M denote the set and the number of buyers and let N denote the set and the number of sellers in the market. The overall ratio of buyers to sellers is denoted by $\Phi \equiv M/N$.

2.1 Sellers and products

Each seller is one of two quality types: *good* or *bad*. Let η denote the fraction of sellers that are of the good type. Each bad seller has a single unit of a low quality product with quality level $\underline{q} \geq 0$ and each good seller has a single unit of a high quality product with quality level $\overline{q} > \underline{q}$. The average quality of the products of all sellers is denoted by

$$\widetilde{q} \equiv \eta \overline{q} + (1 - \eta) \underline{q}.$$

Every seller sells their product using an ascending bid auction. For simplicity, the reserve price at every auction is assumed to be zero.

2.2 Buyers and bidding

Each buyer $i \in M$ seeks to buy one unit of the product. Buyers are identical in their willingness to pay for quality and the net utility function of buyers over all outcomes of bidding at an auction of seller j is given by

$$u_i (q_j, p_{ij}) = \begin{cases} q_j - p_{ij} & \text{if } p_{ij} \text{ is the winning bid} \\ 0 & \text{otherwise} \end{cases},$$

where q_j is the quality of seller j's product and p_{ij} is the bid of buyer i at seller j's auction.

A buyer can purchase the product only by going to a seller's location and participating in that seller's auction. Upon visiting a seller, the buyer becomes perfectly informed of the good's quality, before bidding commences. The bidding at seller j's auction depends on the number of buyers visiting seller j. Under an ascending bid auction, if m_j is the number of buyers choosing to visit seller j, buyer i does best by using the bidding strategy

$$p_{ij}^* = \begin{cases} 0 & \text{if } m_j = 1 \\ q_j & \text{if } m_j > 1 \end{cases}.$$

Thus, a seller receives a non-zero price for his or her product if and only if more than one buyer turns up to the auction.

We make these relatively simplistic assumptions about the auctions in the model so as to focus on the effects of provision of information about seller qualities by a third party.

2.3 Search frictions and submarkets

A search friction exists because buyers are uncoordinated and can choose to visit the location of only one seller. This decision is interpreted broadly as representing the buyer's maximum sphere of attention.[3]

[3] The assumption that buyers can visit only one seller is analytically convenient, but our results would not change qualitatively if buyers could visit more than one seller. What really matters is that buyers cannot visit *all* sellers.

In the absence of any additional information, all sellers appear identical to all buyers. When search is guided, we assume that buyers are provided with (possibly imperfect) information about the qualities of sellers. That is, under guided search, buyers can distinguish seller qualities to some extent, and what we call *submarkets* are created.

Wolinsky (1988) shows (also in our appendix) that in a large market and in a mixed strategy equilibrium where buyers randomize over seller locations, the probability distribution that exactly b buyers turn up to a seller's auction in a (sub)market with buyer-seller ratio ϕ is given by

$$\Pr(x = b) = \begin{cases} e^{-\phi} & \text{for } b = 0 \\ \phi e^{-\phi} & \text{for } b = 1 \\ 1 - e^{-\phi} - \phi e^{-\phi} & \text{for } b > 1 \end{cases} .$$

In our model, the most important of these probabilities for the seller is that for $b > 1$, since only in this case does the seller receives a non-zero price from his or her auction. Accordingly, we define the function $p(\phi) = 1 - e^{-\phi} - \phi e^{-\phi}$. The expected profit of a seller with product quality q_j in a submarket of tightness ϕ is then given by

$$V_j = p(\phi)\, q_j. \tag{1}$$

From a buyer's point of view, what matters is whether or not they are alone at a seller's auction, since if they are alone they will get a strictly positive surplus from the auction while if they are not alone their surplus will be zero. The probability that a buyer is alone at any given seller in a (sub)market with market tightness ϕ is given by $e^{-\phi}$ and the expected utility of buyer i of visiting a seller in a (sub)market with tightness ϕ is given by

$$U_i = e^{-\phi} q_z, \tag{2}$$

where q_z is the expected quality of products in this (sub)market.

2.4 Unguided equilibrium

In an unguided equilibrium, all sellers appear identical to all buyers. In a symmetric mixed strategy equilibrium, buyers visit each seller with probability $1/N$. In such an equilibrium, total welfare is

$$W_0 = N\left(1 - e^{-\Phi}\right) \widetilde{q} \tag{3}$$

and the welfare of buyers is

$$U_0 = N\Phi e^{-\Phi}\widetilde{q} \tag{4}$$

The total welfare levels of bad and good sellers are given by $X_0 = (1-\eta)\, Np\,(\Phi)\, \underline{q}$ and $Y_0 = \eta Np\,(\Phi)\, \overline{q}$ respectively.

3 Guided search

We now consider the possibility that the search of buyers is guided by provision of information about seller qualities by a third party. We assume that the third party assesses, potentially imperfectly, the quality of every seller. The third party then divides the sellers into two groups with different expected quality levels, and buyers are informed of this partition. That is, buyers are informed of the expected quality of each group, and the fraction of sellers that have been allocated to each group.

To analyze the effect of the provision of this information partition on our frictional search market, we first derive the equilibrium responses of buyers to an arbitrary partition of sellers into two quality differentiated submarkets. We then consider the effect on buyers' welfare and overall welfare of such a partition of sellers.

Suppose that sellers are separated by some mechanism into two quality differentiated submarkets. Let q_l and q_h denote the expected quality levels of sellers in the two submarkets, and let α denote the fraction of sellers that are allocated to the submarket with expected quality q_h. Without loss of generality we assume $q_h > q_l$. Buyers are informed of q_l, q_h and α.

If sellers are separated in such a manner, the average quality of sellers across submarkets cannot change, thus

$$\widetilde{q} = \alpha q_h + (1-\alpha)\, q_l. \tag{5}$$

In addition, the number of buyers is fixed, so market tightness for each submarket is related to overall market tightness as follows:

$$\Phi = \alpha\phi_h + (1-\alpha)\,\phi_l, \tag{6}$$

where ϕ_l and ϕ_h denote the buyer-seller ratios of the two submarkets.

The division of sellers into submarkets leads to two basic types of equilibrium distribution of buyers across the submarkets. These distributions depend upon the average quality of sellers in each submarket, their relative numbers, and the overall ratio of buyers to sellers. These conditions are summarized by what we call the *exclusion constraint*:

$$q_h e^{-\Phi/\alpha} \geq q_l. \tag{EC}$$

The left hand side of the exclusion constraint is the expected utility of a buyer if all buyers locate in the high quality submarket. The right hand side of this constraint is the expected quality of sellers' products in the low quality submarket. If (EC) is satisfied, a buyer is better off to locate in the high quality submarket even though if he located in the low quality submarket he would not have to compete with any other buyers and could obtain a payoff of q_l with certainty. Thus if the partition of sellers into submarkets satisfies (EC), all buyers locate in the high quality submarket so that $\phi_h = \Phi/\alpha$ and $\phi_l = 0$.

If the partition of sellers into submarkets does not satisfy (EC), buyers locate in both the high quality and low quality submarkets. In a mixed strategy equilibrium where (EC) is not satisfied, we must have

$$q_h e^{-\phi_h} = q_l e^{-\phi_l}. \tag{7}$$

That is, the expected utility to buyers must be the same from locating in either submarket.

The behavior of buyers can therefore be expressed as a function of the distribution of sellers over the submarkets. This function depends crucially on (EC). From (6) and (7), market tightness in the high quality submarket is given by

$$\phi_h = \begin{cases} \Phi/\alpha & \text{if EC} \\ \Phi + (1-\alpha)\ln(q_h/q_l) & \text{otherwise} \end{cases}. \tag{8}$$

Similarly, market tightness in the low quality submarket is given by

$$\phi_l = \begin{cases} 0 & \text{if EC} \\ \Phi - \alpha\ln(q_h/q_l) & \text{otherwise} \end{cases}. \tag{9}$$

If (EC) holds, sellers in the low quality submarket are excluded, and from (2), a buyer's utility in any period is simply $U = e^{-\Phi/\alpha} q_h$. If (EC) does not hold, buyers visit both submarkets with strictly positive probability and from (2) and (7), a buyer's utility in any period is $U = e^{-\phi_h} q_h$. Substituting in (8), for any distribution of sellers across submarkets, the expected payoff of a buyer when search is guided is given by

$$U = \begin{cases} e^{-\Phi/\alpha} q_h & \text{if EC} \\ e^{-\Phi} q_h^{\alpha} q_l^{1-\alpha} & \text{otherwise} \end{cases} . \tag{10}$$

The utility of buyers is a linear function of the expected quality of sellers in the high submarket if sellers in the low submarket are excluded. If no sellers are excluded, the utility of buyers is a Cobb-Douglas function of the expected qualities of the sellers' products in each submarket with the weights being the fraction of sellers in each submarket.

Proposition 1 *A partition of sellers into two quality differentiated submarkets deceases the welfare of buyers if (i) the partition does not satisfy (EC) or (ii) the market is sufficiently 'thin'. The welfare of buyers increases otherwise.*

Proof. If (EC) does not hold,

$$\begin{aligned} U - U_0 &= e^{-\Phi} q_h^{\alpha} q_l^{1-\alpha} - e^{-\Phi} \widetilde{q} \\ &= e^{-\Phi} \left[q_h^{\alpha} q_l^{1-\alpha} - (\alpha q_h + (1-\alpha) q_l) \right] \\ &< 0 \text{ for all } q_h \neq q_l \text{ and } \alpha \in (0,1). \end{aligned}$$

To see the inequality, note that in general, $x^{\alpha} y^{1-\alpha} < \alpha x + (1-\alpha) y$ for $\alpha \in (0,1)$ and $x \neq y$. Taking the log of the left-hand side, $\log\left(x^{\alpha} y^{1-\alpha}\right) = \alpha \log x + (1-\alpha) \log y < \log(\alpha x + (1-\alpha) y)$ since log is a concave function, and log monotone implies $x^{\alpha} y^{1-\alpha} < \alpha x + (1-\alpha) y$.

If (EC) holds,

$$\begin{aligned} U - U_0 = e^{-\Phi/\alpha} q_h - e^{-\Phi} \widetilde{q} \\ < 0 \text{ if } \Phi > [\alpha/(1-\alpha)] \ln(q_h/\widetilde{q}) \\ > 0 \text{ otherwise.} \end{aligned}$$

■

Thus provision of information about sellers can hurt buyers since it forces them to compete more intensely for high quality products. Proposition 1 demonstrates that buyers are always hurt by third party information provision if the creation of the low quality submarket is not so informative as to warrant exclusion of these sellers. In the case where low quality sellers are excluded, proposition 1 demonstrates that buyers are always hurt by third party information provision if the market is sufficiently thin – if it has only a few sellers. In both cases, buyers are hurt because they pay much higher prices in equilibrium even though they obtain high quality products with greater frequency. In a thick market, buyers are benefited by third party information provision because the associated price increases for high quality products are much smaller.

If there are two submarkets, total welfare is given by

$$W = \begin{cases} \alpha N \left(1 - e^{-\Phi/\alpha}\right) q_h & \text{if EC} \\ \alpha N \left(1 - e^{-\phi_h}\right) q_h + (1-\alpha) N \left(1 - e^{-\phi_l}\right) q_l & \text{otherwise} \end{cases} . \quad (11)$$

The decentralized actions of buyers in response to the creation of quality differentiated submarkets raises a question of whether submarket creation is socially efficient. For example, if (EC) is satisfied, then there is increased competition between buyers for the remaining high quality sellers, and low quality sellers do not get to trade at all. Likewise, if low quality sellers are included, then it is not clear that the distribution of buyers over submarkets will be optimal. However, it is possible to show that the creation of submarkets always raises social welfare.

Proposition 2 *A partition of sellers into two quality differentiated submarkets always increases social welfare.*

Proof. If (EC) does not hold,

$$\begin{aligned} W - W_0 &= \alpha N \left(1 - e^{-\phi_h}\right) q_h + (1-\alpha) N \left(1 - e^{-\phi_l}\right) q_l \\ &\quad - N \left(1 - e^{-\Phi}\right) \widetilde{q} \\ &= N e^{-\Phi} \left[\alpha q_h + (1-\alpha) q_l - q_h^{\alpha} q_l^{(1-\alpha)}\right] \text{ by eqs (5), (8) and (9)} \\ &> 0 \text{ for all } q_h \neq q_l \text{ and } \alpha \in (0,1) . \end{aligned}$$

If (EC) holds,

$$W - W_0 = \alpha N \left(1 - e^{-\Phi/\alpha}\right) q_h - N \left(1 - e^{-\Phi}\right) \widetilde{q}.$$

Note that from (5), $W - W_0$ is strictly decreasing in q_l, hence the worst case is when q_l is as large as possible that satisfies (EC), that is when $q_l = q_h e^{-\Phi/\alpha}$. Thus we only need to show that $W - W_0 \geq 0$ for (EC) satisfied with equality. In this case,

$$\begin{aligned} W - W_0 &= \alpha N \left(1 - e^{-\Phi/\alpha}\right) q_h - N \left(1 - e^{-\Phi}\right) \left(\alpha q_h + (1-\alpha) q_h e^{-\Phi/\alpha}\right) \\ &= N q_h \left[\alpha e^{-\Phi} - e^{-\Phi/\alpha} + (1-\alpha) e^{-\Phi \frac{1+\alpha}{\alpha}}\right] \end{aligned}$$

The term in the square brackets can be written as

$$e^{-\Phi\frac{1+\alpha}{\alpha}}(1-\alpha)-e^{-\frac{\Phi}{\alpha}}\left(1-\alpha e^{\Phi\frac{1-\alpha}{\alpha}}\right)$$

This is positive since $e^{-\Phi\frac{1+\alpha}{\alpha}} > e^{-\frac{\Phi}{\alpha}}$ and $\alpha < \alpha e^{\Phi\frac{1-\alpha}{\alpha}}$ for all $\Phi > 0$ and $\alpha \in (0,1)$. ■

Welfare is improved by submarket creation because buyers' search is directed more accurately and thus the number of quality adjusted matches increases.

4 Numerical results

In this section we present results of a numerical implementation of our model so as to quantify the magnitudes of the welfare effects discussed above. In this section we normalize $N = 1$ and assume that sellers are equally likely to be good or bad (i.e. $\eta = \frac{1}{2}$). We also fix $\overline{q} = 1$ so that we are left with the buyer-seller ratio, Φ, and the bad quality level, $0 \leq \underline{q} < 1$, as the main parameters of interest.

As above, we assume that a third-party separates sellers into two groups, and these groups are of equal size (i.e. $\alpha = \frac{1}{2}$). The third-party labels sellers as either 'good' or 'bad', but it does so imperfectly. In particular, a seller will be correctly labelled with probability $\rho \geq \frac{1}{2}$. We assume that ρ is common knowledge, thus the expected quality of a seller labelled as good is

$$q_h = \underline{q} + \rho\left(1-\underline{q}\right)$$

and the expected quality of a seller labelled as bad is

$$q_l = 1 - \rho\left(1-\underline{q}\right).$$

We then present various welfare effects as a function of ρ. In particular, we calculate the percentage changes in total welfare, welfare of buyers, and welfare of good and bad sellers relative to their levels in the unguided equilibrium. We also calculate the ratio of the changes in welfare of buyers, and good and bad sellers to the change in total welfare, which we call 'relative impacts'.

First, figures 1 and 2 show the effects of guided search as a function of ρ when $\Phi = 1$ and $\underline{q} = \frac{1}{2}$. In this case, (EC) does not hold for the entire range of ρ. Accordingly, all of the welfare gains accrue to good sellers, while buyers and bad sellers are made worse off. At most, the gain in total welfare is 3.28%.

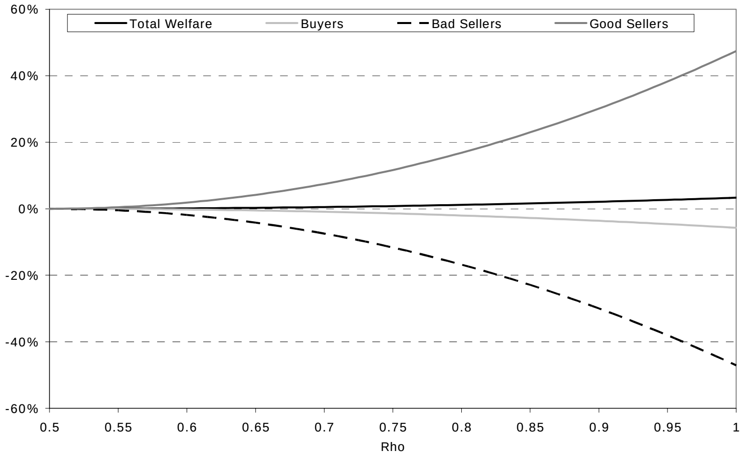

Figure 1: Welfare changes when $\Phi = 1$ and $\underline{q} = \frac{1}{2}$.

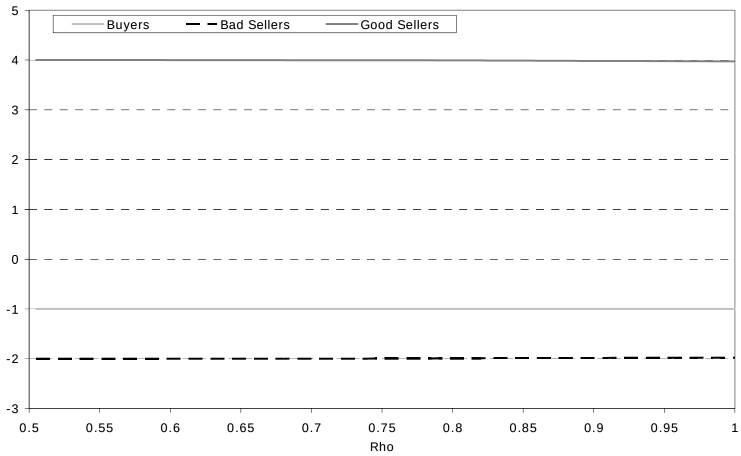

Figure 2: Relative impacts when $\Phi = 1$ and $\underline{q} = \frac{1}{2}$.

Next, figures 3 and 4 show the effects when $\Phi = \frac{1}{2}$ and $\underline{q} = \frac{1}{10}$. In this case buyers are made better off if ρ is sufficiently large. Nevertheless, most of the welfare gains continue to accrue to good sellers, and bad sellers are always made worse off. Due to the fall in the quality level of bad sellers' products, the maximum potential welfare gain has risen to around 46%.

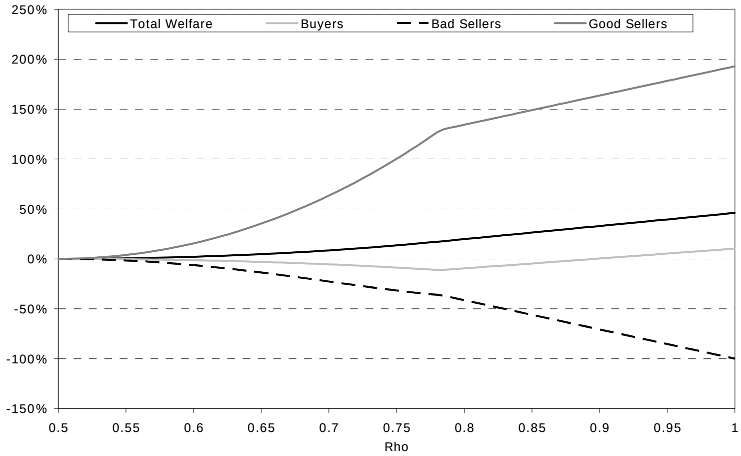

Figure 3: Welfare changes when $\Phi = \frac{1}{2}$ and $\underline{q} = \frac{1}{10}$.

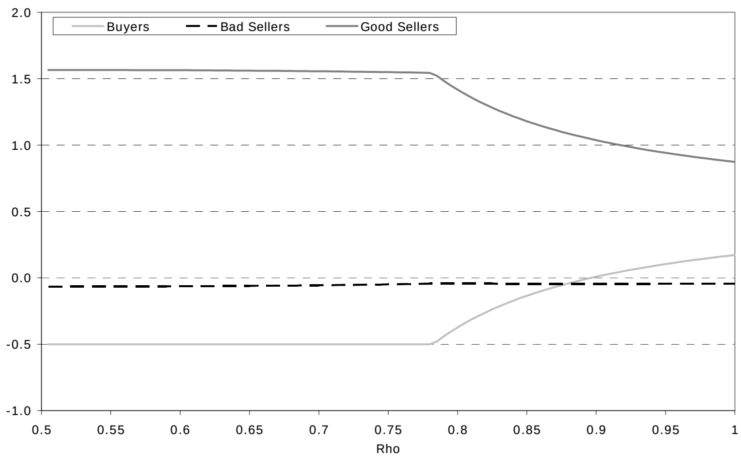

Figure 4: Relative impacts when $\Phi = \frac{1}{2}$ and $\underline{q} = \frac{1}{10}$.

Finally, figures 5 and 6 show the effects when $\Phi = \frac{1}{10}$ and $\underline{q} = \frac{1}{10}$. In this case, both good and bad sellers are made better off for low values of ρ while buyers are made worse off. As ρ increases, eventually (EC) holds and the welfare of bad sellers falls while the welfare of buyers starts to rise and the welfare of good sellers continues to rise. For high enough values of ρ, most of the welfare gains accrue to buyers.

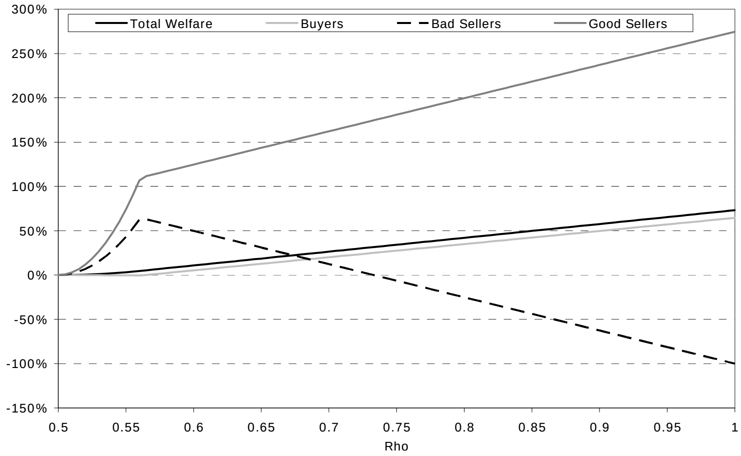

Figure 5: Welfare changes when $\Phi = \frac{1}{10}$ and $\underline{q} = \frac{1}{10}$.

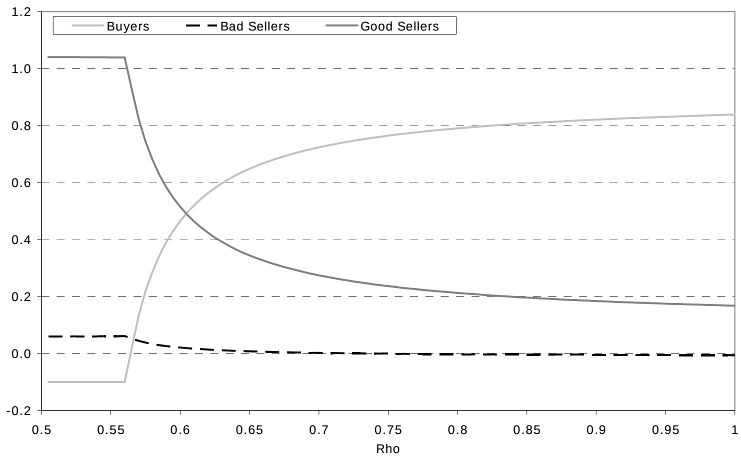

Figure 6: Relative impacts when $\Phi = \frac{1}{10}$ and $\underline{q} = \frac{1}{10}$.

5 Conclusion

This paper is a first attempt at modeling the effects of third party information in directed search equilibrium. The main finding is that the benefits of third party information are unlikely to be evenly distributed between buyers and high quality sellers, but are instead concentrated on the sellers' side of the market. Thus we have shown that buyers have very little interest in pursuing concerted action to set up such information sources while high quality sellers have such an interest. We have also shown that buyers are harmed by third party information if the additional information provided about sellers is sufficiently small, or if the market has many more buyers than sellers. Thus, we have derived a model that gives a range of parameters in which third party info is actually harmful to buyers.

Our model could be extended in a number of directions. For example, we assumed that third party information is not priced. This assumption reflects, perhaps, perfect competition between third party information providers, or information that is provided by a public entity. However, we believe that optimal pricing and distribution of third party information is potentially an important topic and that much could be learned by studying the behavior of a profit maximizing third party monopolist.

Another possible extension is signalling by sellers. In this paper, we can eliminate signalling simply by assuming that only buyers are informed about the seller quality. Relaxing this assumption will potentially introduce a tradeoff between mechanisms designed to signal type – possibly mechanisms that use price commitments and/or subsidies – and the provision of third party information.[4] Such a model could endogenize the choice of selling mechanism by decentralized sellers as a function of third party information. Thus a third party information service could potentially transform the methods by which sellers choose to sell. One could ask: Did eBay lead to the widespread use of auctions, or did the widespread use of auctions lead to eBay?[5] We leave these topics to future research.

[4] Experimental work on alternative selling mechanism is done by Cason *et al* (2003). Theoretical work is by Coles and Eeckhout (2004).

[5] eBay is a well known third party information provider that administers a set of decentralized auctions on the Internet

6 Appendix

For readers who are unfamiliar with our matching technology, in this appendix we briefly cover the basic results relevant for this paper. For further details, the reader is referred to Julien *et al* (2000). Frictional assignment is the outcome of buyers being uncoordinated over which seller to visit in a market. An equilibrium with frictional assignment is a mixed strategy equilibrium in the location of buyers over sellers (Julien *et al* 2000 and Burdett *et al* 2001). A mixed strategy of a buyer $i = 1, 2, \ldots, M$ is a vector $\sigma^i = \left(\sigma_1^i, \sigma_2^i, \ldots, \sigma_N^i\right)$ where σ_j^i is the probability that buyer i visits seller j's auction. We focus on symmetric equilibria in which $\sigma^i = \sigma^j$ for all i and j.

Suppose that the sellers are divided into a number of mutually exclusive and completely exhaustive groups $G = 2, 3, ..., N$ according to some characteristic(s) that buyers can observe. Let N_s denote the number of sellers in group $s = 1, 2, \ldots, G$. Assume that the payoff to a buyer of winning a seller's auction is the same for every seller within a given group and denote this payoff by u_s.

Without loss of generality we can decompose the buyers' decision into a two-step process by which they first choose the group of sellers to visit and then choose which seller in the group to visit according to a probability distribution over that group. Let m_s be the probability that a buyer visits a seller in group s and let $\sigma_{s,i}$ denote the probability that a buyer visits seller i who is a member of group s, given that the buyer has chosen to visit a seller in group s. Within any given group s, in a mixed strategy equilibrium where all sellers are visited with strictly positive probability, the expected payoff to a buyer from visiting any seller must the same. Since a buyer gets u_s if alone at a seller in group s and 0 otherwise, we must have

$$(1 - \sigma_{s,i})^{m_s M - 1} u_s = (1 - \sigma_{s,j})^{m_s M - 1} u_s \text{ for all } i, j \in s,\ i \neq j.$$

This implies that $\sigma_{s,i} = \sigma_{s,j} = \sigma_s$ for all $i, j \in s$. Since $\sum_{i \in s} \sigma_s = 1$, we have $\sigma_s = 1/N_s$.

Now let us derive the probability distribution of buyers over sellers in a given group s. First, the probability that some subset of buyers of size $\beta \leq M$ all choose to visit group s is given by $m_s^\beta (1 - m_s)^{M-\beta}$. Then, given that all members of the subset of size β visit group s, the probability that exactly x of them visits a given seller is given by a binomial distribution with probability

σ_s, that is,

$$\frac{\beta!}{x!\,(\beta-x)!}\cdot\sigma_s^x\,(1-\sigma_s)^{\beta-x}\,.$$

To find the probability that $x \geq 0$ buyers visit a particular seller, we need to consider all possible subsets of buyers. For each β, there are $M!/\left(\beta!\,(M-\beta)!\right)$ subsets of size β. Summing over all possible β, we thus get the probability that exactly x buyers visit a particular seller as

$$\Pr(x)=\sum_{\beta=0}^{M}\left[\left(\frac{M!}{\beta!\,(M-\beta)!}\right)m_s^{\beta}\,(1-m_s)^{M-\beta}\left(\frac{\beta!}{x!\,(\beta-x)!}\right)\sigma_s^x\,(1-\sigma_s)^{\beta-x}\right].$$

For auctions, we will be most interested in the probabilities that $x=0$, $x=1$, and $x>1$. Let us define $\phi_s \equiv m_s M/N_s$. Thus, $M=\phi_s N_s/m_s$ for all s, and recalling that $\sigma_s = 1/N_s$, we can derive the probability distribution of buyers over sellers as shown in table 1.

# Buyers	**Probability**
0	$\left(1-\frac{m_s}{N_s}\right)^{\frac{\phi_s N_s}{m_s}}$
1	$\phi_s\left(1-\frac{m_s}{N_s}\right)^{\frac{\phi_s N_s}{m_s}-1}$
>1	$1-\left(1-\frac{m_s}{N_s}\right)^{\frac{\phi_s N_s}{m_s}}-\phi_s\left(1-\frac{m_s}{N_s}\right)^{\frac{\phi_s N_s}{m_s}-1}$

Table 1: Probability distribution of buyers over sellers.

In this paper, for simplicity, we work exclusively in the 'large market' case. Thus we take the limits of the above probabilities in table 1 as $N_s \to \infty$ while holding the group s buyer-seller ratio, ϕ_s, constant. This gives the probability distribution of buyers over sellers shown in table 2.

# Buyers	**Probability**
0	$e^{-\phi_s}$
1	$\phi_s e^{-\phi_s}$
>1	$1-e^{-\phi_s}-\phi_s e^{-\phi_s}$

Table 2: Probability distribution of buyers over sellers in a large market.

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