Directed Search without Price Directions

May 2001

by

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Abstract

This paper considers a dynamic directed search equilibrium in which sellers are restricted to use selling mechanisms without ex ante price announcements. The restriction is in keeping with the empirical observation that prices are rarely included in labor market advertisements. We show that a selling mechanism without ex ante price announcements – an auction without a reserve price - is sufficient to encourage efficient job entry. Therefore, this ex ante selling mechanism is both optimal and invariant to changes in the economic environment.

JEL: E24, J31, J41, J64, D44

Keywords: auctions, posted prices, directed search, price advertisements

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Introduction

"Directed search" is a new development in the theory of unemployment. Models of directed search address a wide range of economic questions related to the problems of search and recruiting (Montgomery (1991), Acemoglu and Shimer (2000), `Julien, Kennes, and King (2000), and Burdett, Shi and Wright (2001)). These models have these same essential ingredients:

1) A multitude of spatially separated capacity constrained buyers and sellers.
2) Each seller chooses an ex ante selling mechanism such as an auction, posted price, etc.
3) Each buyer commits an offer to buy from (usually) one seller conditional on 2).
4) Unemployment of sellers exists if some sellers do not receive acceptable offers while unemployment of buyers exists if some offers are rejected.

Unemployment in directed search models exists even if there are equal numbers of buyers and sellers. The basic idea is that it is possible, and indeed likely, that two buyers inadvertently visit one seller and consequently leave another seller without a buyer. This is easily seen by an example. Suppose there are three buyers {Buyer 1, Buyer 2, Buyer 3} and three sellers {Seller 1, Seller 2, Seller 3}

Buyer 1	visits	Seller 1
Buyer 2	visits	Seller 1
Buyer 3	visits	Seller 3

Here unemployment exists because seller 2 does not receive an offer and because only one offer to buy from buyer 1 or buyer 2 can be accepted. This reasoning implies that the amount of unemployment is closely connected to there being a large number of sellers receiving multiple offers. This type of unemployment occurs because difficulties in coordination force buyers to randomise in their decision about which seller to approach.

The importance of multiple offers in directed search theory is quite different from the usual view of multiple offers in conventional Mortensen-Pissarides search models (Pissarides 2000). In these models, the probability of multiple job offers is zero. This is because unemployment is modelled by assuming random search in which agents "bump" into each infrequently. Howitt and McAfee (1987) give the following 'fishing' description.

Fish want to be caught and swim at rate s. Fishermen cast nets of size r. The instantaneous probability that a fish is caught is given by a function p(s,r). The probability that two fish are caught at exactly the same time is zero. (Not an exact quote.)

Both directed and random search theory presume that matching is time consuming. Both also imply a matching technology between unemployed workers and firms. The main difference is that only directed search theory assumes that buyers know where sellers are located.

Models of directed search typically assume that ex ante price announcements direct trade between buyers and sellers. However, a fundamental problem with this basic assumption is that real-life labor market advertisements do not typically convey ex ante price information. Casual observation reveals instead that workers rarely include salary expectations in their resumes and firms rarely include starting salaries in their job advertisements. This observation raises the following questions. Does the absence of ex ante prices imply that unemployment is the outcome of random search? Is a labor market likely to be inefficient if ex ante price advertisements are absent? By what mechanism are equilibrium prices determined if ex ante prices are excluded from labor market advertisements?

In this paper we consider a model of directed search in which sellers of labor are restricted to use selling mechanisms that *exclude* ex ante prices. The restriction rules out selling mechanisms such as posted prices (Montgomery 1991 and Burdet, Shi and Wright 2001) and also auctions with advertised reserve prices (Julien, King and Kennes 2000). However, the restriction does permit a wide variety of selling mechanisms that do not convey price information to buyers. We believe that these restricted selling mechanisms are of important theoretical interest because they appear to be so widely used in the real world. We also believe that we can give some basic insight into why selling mechanisms without ex ante prices are potentially superior to their seemingly unrestricted cousins.

Auctions can take on many forms. According to McAfee and McMillan (1987), “an auction is a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from market participants." In this paper, we show that auctions without reserve prices solve the coordination problem of directed search just as effectively as ex ante pricing mechanisms. Our 'ex post' pricing mechanism works as follows. On the one hand, a seller is able to claim the entire surplus of a match if he has multiple offers. This is simply a case of Bertrand competition between buyers. On the other hand, when faced with a single buyer, the seller relinquishes the entire surplus of a match. In this case, under the rules of the auction, the buyer acts as a monopoly. The two different bargaining situations imply that the pricing of labor follows Bulow and Klemperer's (1996) basic insight that "no amount of bargaining power is as valuable as attracting one extra bona fide bidder".

A directed search equilibrium is similar to that of Pissarides (2000). The basic problem is to identify the surplus of a match that is implied by the matching technology and then to allocate it to the participants according to a sharing rule. In Pissarides' model, the sharing rule is an exogenous Nash Bargaining solution. In a directed search model, the sharing rule is typically asumed to be endogenous - the solution of a non-cooperative game (ref: Burdett, Shi and Wright 2001 and Julien, Kennes and King 2000). This approach is necessary because optimal ex ante pricing is not invariant to changes in the economic environment. For example, if productivity increases, posted prices must also increase. An alternative approach is to specify a ex post selling mechanism which is always optimal for any parameterisation. This mechanism turns out to an auction without a reserve price. Not surprisingly this exogenous pricing rule yields a much simpler exposition of directed search theory because the complicated game theoretic elements are removed.

The remainder of the paper is structured as follows. The next sction introduces a dynamic equilibrium model of directed search model without price advertisements. The following section analyses the equilibrium. This section is then followed by an analysis of the social planning problem and a discussion of the advantage of ex post pricing as a method of economising on menu costs. The final section offers concluding remarks.

The Model

Consider a simple economy in which N similarly skilled, risk neutral workers face an infinite horizon, perfect capital markets, and a common discount factor β. At the start of each period $t = 0,1,2,3,...$, there are $N - E_t$ unemployed workers of productivity $y_0 = 0$ and E_t workers in jobs of productivity $y_1 = y > 0$. There are also $M_t = \phi(N - E_t)$ vacant jobs that carry a cost k per job per period. Assume one worker can work at most one job and one job can employ at most one worker. Assume also that any match in the current period may dissolve in the subsequent period with probability ρ. The labour market is decentralised with each worker advertising a second price auction for their labour services. Finally, in order to motivate the existence of equilibrium unemployment, assume that a vacant job can make an offer to only one worker each period

Let Λ_i denote the total expected discounted surplus of a match between an unemployed worker and a job of productivity y_i at the start of any period (where a job of productivity $y_0 = 0$ is of course the unemployed state – i.e. home production). A second price auction implies that the workers share W_i^j of the total surplus Λ_i is equal to the surplus Λ_j of the worker's second best available job offer. Thus

(1) $$W_i^j = \Lambda_j$$

The ex ante selling mechanisms are identical and consequently indistinguishable to the owners of vacant jobs. Thus vacancies are randomly distributed over searchers in equilibrium. Therefore, the net addition of H_t workers is given by the following endogenous matching technology

(2) $$H_t = (N - E_t)(1 - e^{-\phi})$$

where $1 - e^{-\phi}$ is the probability that a worker obtains at least one offer. The rate of exogenous separations ρ implies that the supply of employed workers in the next period is given by

(3) $$E_{t+1} = (1 - \rho)(E_t + H_t)$$

Equation (1) determines the fraction of the surplus of a match that goes to an unemployed worker. The randomness of the number of job offers to each work implies that the expected present value of an unmatched worker is given by

(4) $$V = (e^{-\phi} + \phi e^{-\phi})\Lambda_0 + (1 - e^{-\phi} - \phi e^{-\phi})\Lambda_1$$

where $e^{-\phi} + \phi e^{-\phi}$ is the probability that an unemployed worker obtains either one or no job offers and $1 - e^{-\phi} - \phi e^{-\phi}$ is the probability of multiple job offers.

The profit of a firm is equal to the expected output of match minus its capital cost and the expected discounted wages it pays to the worker. The supply of vacant jobs is determined by free entry. Therefore, the expected profit Π of a job of productivity $y_i = y$ making an offer to an unemployed worker is equal to zero. Thus equation (1) and the randomness of job offers implies

(5) $$\Pi = e^{-\phi}(\Lambda_1 - \Lambda_0) - k = 0$$

where $e^{-\phi}$ is the probability that the vacant job is alone in its offer to an unemployed worker. The value of a unmatched worker in the next period determines the outside option of an unmatched worker in the current period, Thus

(6) $$\Lambda_0 = \beta V$$

The total surplus of a high productivity job is equal to the output of a high productivity job plus the discounted future flow of income from such a job weighted by the probability of an exogenous job separation into unemployment. Thus

(7) $$\Lambda_1 = y + \beta[\rho V + (1-\rho)y] + \beta^2(1-\rho)[\rho V + (1-\rho)y] + ... = \frac{y + \beta\rho V}{1 - \beta(1-\rho)}$$

The future profits of a vacant job do not appear in this equation because profits are always equal to zero. Another implication of this free entry condition is that the vacancy: unemployment ratio is stationary.

Equilibrium

The number of workers employed in each period is $E_t + H_t$. Equations (2) and (3) imply that, in a stationary equilibrium, the unemployment rate is given by

(8) $$\bar{u} = \frac{\rho e^{-\bar{\phi}}}{1-(1-\rho)e^{-\bar{\phi}}}$$

Equations (4), (5), (6) and (7) determine the values of V, Λ_0, Λ_1 and ϕ. From these equations it is easy to show (appendix) that the equilibrium value of labour market tightness is given by

(9) $$k = ye^{-\bar{\phi}} + k(1-\rho)\beta(e^{-\bar{\phi}} + \bar{\phi}e^{-\bar{\phi}})$$

The left side of equation (9) is the cost of a vacancy while the right side of this expression is the benefit. The benefit of a vacancy has two components. The first component is simply the probability a worker is not matched (which is equal to the probability that a vacancy is alone in its offer to a worker) times the output of a match in the current period. In a static model, this component completely summarizes the returns to posting a vacancy. The second component summarizes the future benefits of a vacancy. It is equal to the cost of a vacancy, times the 'effective' discount factor β(1-ρ), times the probability that a worker does not obtain multiple offers. The probability of no multiple offers in the future is crucial to determing the share of the surplus of a match that goes to the firm because it determines the outside option of a worker. The higher this probability is, the greater the share of a match that goes to the firm, on average.

The workers share of surplus determines their wages. The equations for wages in each possible bargaining state are given by

(10) $$w_0^0 = 0$$

(11) $$\frac{w_1^j + \beta\rho V}{(1-\beta(1-\rho))} = \Lambda_j \qquad j \in \{0,1\}$$

where w_i^j denotes the wage of a worker in wage state W_i^j. In equilibrium, $n_1^0 = (1-\bar{u})\phi e^{\phi} / (1-e^{\phi})$ workers earn the low wage, w_1^0, while $n_1^1 = (1-\bar{u})(1-\phi e^{\phi} - e^{\phi})/(1-e^{\phi})$ workers earn the high wage, w_1^1. The average wage is given by

$$\bar{w} = [w_1^0 \phi e^{\phi} + w_1^1 (1-\phi e^{\phi} - e^{\phi})]/(1-e^{\phi}) \tag{12}$$

The average wage is equal to the outcome of 'posted price' model if equilibrium job entry is the same as with auctions (ref: Cao and Shi 2000 and Julien, King and Kennes 2001). In this case, posted prices are obviously dependent on such things as productivity, vacancy costs, exogenous rates of separations, etc. The auction is by contrast independent of these considerations.

<u>Social Planning Problem</u>

Suppose that a social planner faces the same basic constraints of the decentralised economy but is free to choose the supply of vacancies each period. The social planning objective is then to maximize the expected output of workers minus the expected cost of vacancies subject to the exogenous matching techology and the exogenous rate of job separations. The social planning problem can be stated as follows:

$$S = \max_{\{E_t, E_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \{ y(E_t + H_t) - kM_t \} \tag{13}$$

such that equations (2) and (3) are satisfied. Note that equations (2) and (3) imply

$$\phi_t = M_t / (N - E_t) = -(\ln(N - E_{t+1}/(1-\rho)) - \ln(N - E_t)) \tag{14}$$

In order to derive the Euler equation of the solution of the social planning problem, it is convenient to substitute equation (14) into the objective function and write the maximisation problem in the form of a dynamic programming problem.

$$V(t) = \max_{E_t, E_{t+1}} \{ \frac{y_g}{(1-\rho)} (E_{t+1}) + k(N - E_t)(\ln(N - E_{t+1}/(1-\rho)) - \ln(N - E_t)) + \beta V(t+1) \tag{15}$$

The first order conditions of the social planning problem are as follows

$$<E_t> \qquad V'(E_t) = k(1+\phi_t)$$

$$<E_{t+1}> \qquad 0 = y/(1-\rho) + ke^{\phi_t} + \beta V'(E_{t+1})$$

In a steady state

$$(16) \qquad k = ye^{-\bar{\phi}} + k(1-\rho)\beta(e^{-\bar{\phi}} + \bar{\phi}e^{-\bar{\phi}})$$

The solution of the social planning problem is the same as the decentralised equilibrium. Thus the equilibrium is always efficient. This result stands in stark contrast to earlier work on random matching (Hosios 1990). It is closely related to the efficiency result of Moen (1987) who shows that competing posted prices efficiently coordinate trade bewteen local labor markets. We have shown that these competing ex ante price advertisements are not necessary for a general claim of efficient job entry in directed search equilibrium.

Conclusions

Why do sellers not advertise prices in real life? The simple answer provided by the present paper is that they do not have to advertise prices in order for the market to behave efficiently. A related general equilibrium game theoretic answer is found in Julien, King and Kennes (2000). That paper shows that competition forces the equibrium reserve prices to equal the workers' outside options. Thus competition implies that ex ante prices do not play any allocative role.

The use of selling mechanisms without ex ante prices may have important advantages over selling mechanisms with ex ante prices. In particular, they may play a role in markets where unemployment is of sufficient duration and where the use of posted prices is hindered by the existence of frequent changes in money, technology, regulations etc. It is possible to construct a wide range of models in which costly posted prices changes are needed in the face of such shocks (Akerlof and Yellen (1985), Parkin (1986)). A model of ex ante prices would of course be subject to these shocks. A model without ex ante prices by contrast economises on menu costs because the selling mechanism does not need to be changed in response to changes in the economic environment.

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Appendix

The decentralised equilibrium is characterised by four equations and four unknowns.

(4) $$V = (e^{-\phi} + \phi e^{-\phi})\Lambda_0 + (1 - e^{-\phi} - \phi e^{-\phi})\Lambda_1$$

(5) $$\Lambda_0 = \beta V$$

(6) $$\Lambda_1 = \frac{y + \beta\rho V}{1 - \beta(1-\rho)}$$

(8) $$\Pi = e^{-\phi}(\Lambda_1 - \Lambda_0) - k = 0$$

Equations (5) and (6) imply

(i) $$\Lambda_1 - \Lambda_0 = \frac{y - (1-\beta)\beta(1-\rho)V}{1 - \beta(1-\rho)}$$

Equation (4) can be rewritten as

(ii) $$V = \Lambda_1 - (\Lambda_1 - \Lambda_0)(1+\phi)e^{-\phi}$$

Substitute equation (8) into equation (ii) to get

(iii) $$V = \Lambda_1 - k(1+\phi)$$

Substitute equation (6) into equation (iii) to get

(iv) $$V = \frac{y + \beta\rho V}{1 - \beta(1-\rho)} - k(1+\phi)$$

Rewrite equation (iv) as follows

(iv') $$V = [y - k(1+\phi)(1 - \beta(1-\rho)]/(1-\beta)$$

Substitute equation (iv') into equation (i) to get

(v) $$\Lambda_1 - \Lambda_0 = y + \beta(1-\rho)k(1+\phi)$$

Substitute equation (v) into equation (8) to get

(10) $$k = ye^{-\bar{\phi}} + k(1-\rho)\beta(e^{-\bar{\phi}} + \bar{\phi}e^{-\bar{\phi}}) \quad ///$$