Economics of Banking Lecture 17

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Lenders of last resort

- The Martin model
- The strategic determination of reserves
- Emergence of lenders of last resort

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A case where deposit insurance is suboptimal

As usually, there are three dates, t = 0, 1, 2.

Investments:

1. A storage technology,

2. A risky technology, one unit at t = 0 yields \tilde{R} at t = 2, with

$$\widetilde{R} = egin{cases} rac{1}{q}R & ext{ with probability } q \ 0 & ext{ with probability } 1-q \end{cases}$$

where q can be chosen by the bank (private information). Expected payoff is R and variance is

$$\sigma^2\left(\tilde{R}\right) = q\left(R - \frac{1}{q}R\right)^2 + (1-q)R^2 = \frac{1-q}{q}R^2,$$

decreasing in q, 0 when q = 1.

Depositors' choices

Households are impatient with probability π .

If the contracts satisfy standard constraints

$$\pi c_{1} = 1 - I,$$

$$(1 - \pi)\tilde{c}_{2} = I\widetilde{R},$$
(1)

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then date 2 expected utility is

$$qu\left(\frac{l}{1-\pi}\frac{1}{q}R\right) \leq u\left(q\frac{l}{1-\pi}\frac{1}{q}R\right) = u\left(\frac{l}{1-\pi}R\right)$$

by concavity of u, so q = 1 is optimal.

Socially optimal investment is found by maximizing expected utility subject to (1).

Optimal contracts

Assume that banks maximize the utility of the representative consumer.

The contract will specify withdrawal c_E at date 1 and withdrawal c_L at date 2, outcome risk must be absorbed by the bank.

There is a bank run if depositors observe a signal (a "sunspot") at t = 1, probability of this signal is p.

If the fraction of depositors observing the signal is μ , then we maximize

$$(1 - \mu p) [\pi u(c_E) + (1 - \pi)u(c_L)] + \mu p \hat{\pi} u(c_E).$$

with $\hat{\pi}$ is the probability that bank can satisfy the fraction μ of depositors,

$$\hat{\pi} = \frac{1 - I(1 - L)}{c_E}.$$
 (2)

Banks will have no incentive to choose risky projects, they compete for depositors who are risk averse.

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Introducing deposit insurance in this model

Deposit insurance promises D to depositors in bankrupt banks.

Then bank runs are prevented if D is large enough, but:

Bankruptcy may occur also if the bank has a risky project which fails.

The insurance scheme must be financed, assume that this is done by taxing depositors.

The tax T collected should satisfy

$$TR = (1 - \pi)\mu D.$$

If no or few banks fail, surplus B will be distributed among depositors in surviving banks.

A deviating bank

Consider a case where all banks have chosen the safe project with q = 1.

If a bank chooses q < 1, then the deposit contract should be suitably revised.

Assume that the bank proposes a contract with a markup 1/q on the repayment at t = 2.

The expected utility of a patient depositor is

$$qu\left(\frac{1}{q}\left[c_L-\frac{TR}{1-\pi}\right]+\frac{B}{1-\pi}\right)+(1-q)u(D).$$

Since we consider a deviation from q = 1, bankruptcy will occur only due to failed projects, so probability probability 1 - q.

Also, since only one bank deviates, we have that B = TR.

Deviation may be attractive

The partial derivative of expected utility wrt. q is

$$\left.\frac{\partial U}{\partial q}\right|_{q=1} = -u'(c_L)\left[c_L - \mu D\right] - u(D) + u(c_L),$$

which is < 0 when $\mu < 1$ and D is close to c_L .

Thus q = 1 is not an equilibrium choice when there is deposit insurance.

Alternative: Liquidity provision

Banks can borrow specially issued money with their assets as collateral.

In next period, banks can change back to their original assets, if not able to pay back, it defaults.

Side effects of the lender of last resort?

We have seen that deposit insurance prevents bank runs but has side effects.

(Excessive risk-taking by banks)

What about bailout?

Possible effect on liquid reserves of banks

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The Ratnovski model

Two banks, three periods of time, t = 0, 1, 2.

Each bank takes 1 as deposits at t = 0, with return at t = 2.

Return is R_H or R_L , each with probability 1/2.

At t = 1: Liquidity shock with probability p: Additional investment L needed, $R_L > L > R_L - 1$.

New feature (moral hazard): Bank may change the investment project:

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$$(1-p)R_L - 1 > E$$
 (used only if liquidity shock)

Return to bank

If liquidity shock and low return:

After additional investment, net worth is negative: $R_L < 1 + L$, so bank managers prefer the moral hazard option

If high return, $(1-p)R_H - (1+L) > E$ or

$$R_H > \frac{1+L}{1-p} + E$$

so net worth remains positive after the additional investment.

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No lender of last resort

Expected profits to the bank with no reserve

$$\mathbb{E}\pi_1=(1-\rho)\frac{R_H+R_L}{2}-1.$$

If reserve of size L:

Collect ordinary deposits 1 and additional deposits L at t = 0: no bankruptcy at t = 1.

If it observed R_L , it will choose moral hazard and get E

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Reserve better than no reserve

Depositors can see that banks collect 1 + L instead of 1, demand repayment of

$$\frac{1}{1-\frac{p}{2}}$$

so that expected profit is

$$\mathbb{E}\pi_2 = (1-p)\frac{R_H + R_L}{2} + \frac{p}{2}(R_H - L) - \frac{p}{2}(L - E) - 1.$$

If L is big enough, then

$$\mathbb{E}\pi_2 \geq \mathbb{E}\pi_1$$

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banks will be liquid.

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With possibility of bailout

Rules:

Central bank intervenes only if *both* have liquidity shock. Cannot observe R_H, R_L .

Central bank cannot risk that the supported bank engages in moral hazard:

Value of bank must be at least E after the support.

Consequence: Bank with high returns will receive a rent of $E + (R_H - R_L)$ if supported.

The choice of the other bank matters

If the other bank has liquid reserve: Bailout only if the other bank has R_L .

$$\mathbb{E}\pi_1^2 = (1-p)\frac{R_H + R_L}{2} + \frac{p}{4}\left(E + \frac{R_H - R_L}{2}\right) - 1$$

If the other bank has no reserve:

$$\mathbb{E}\pi_1^1 = (1-p)\frac{R_H + R_L}{2} + \frac{p}{2}\left(E + \frac{R_H - R_L}{2}\right) - 1.$$

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Equilibrium choices

For suitably small values of L one gets

$$\mathbb{E}\pi_2 \ge \mathbb{E}\pi_1^2$$

(reserves is best response when the other bank has reserves) and for large enough L one gets

$$\mathbb{E}\pi_1^1 \ge \mathbb{E}\pi^2$$

(no reserves is best response when the other bank has no reserves)

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Where did the lender of last resort come from?

- We have assumed that central bank has the role of lender of last resort.
- What if there is no central bank? (or the central bank doesn't what to assume this role?)
- Case: USA before 1913
- (Morale: What matters is not who but how)

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The model

Banks take deposits and invest in a project with random payoff rate.

Three periods of time: At t = 0 decisions on investment and reserve are taken by the banks.

All have initial capital E and receive 1, decide upon reserves M.

Investment payoff comes at t = 2 as $\tilde{\pi} + \tilde{r}$, with

- Here $\tilde{\pi}$ is common for all banks, uniformly distributed in an interval $[\pi_L, \pi_H]$,
- \tilde{r} individual, \tilde{r} is uniformly distributed in $[0, 2\bar{r}]$.

At t = 1, the value π of π can be observed by all

The individual signal \tilde{r} can be observed but is not verifiable at t = 1.

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Bank runs

The depositors can demand their deposits back at t = 1.

The bank can liquidate the investment at t = 1 at the rate L.

Reason for bank runs: Banks may commit fraud:

Bank can secure a fraction f < 1 of the investment payoff $(\pi + r)(1 + E - M)$ to itself, leaving nothing for others.

We assume that crime pays (in some cases):

$$(1+E)(\pi_L+\overline{r})-1 < f(1+E)(\pi_L+\overline{r}).$$

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Independent (small) bank

Depositors are risk averse. They observe π and infer the banker will commit fraud if

$$\pi f(1 + E - M) \ge \pi (1 + E - M) + E - 1,$$

If π is below

$$\hat{\pi}(M) = \frac{1-M}{(1-f)(1+E-M)},$$

then the depositors run the bank.

Bank then chooses M so as to maximize

$$\int_{\pi_L}^{\hat{\pi}(M)} (M + (1 + E - M)L - 1)dF(\pi) + \int_{\hat{\pi}(M)}^{\pi_H} [M + (1 + E - M)(\pi + \bar{r}) - 1]dF(\pi).$$

and find the optimal value M_{IB}^* .

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Big bank

The big bank can close down fraction x of branches:

Let x be a fraction of banks to be closed down when the signal r is below x2M

Payoff on the investment of projects carried out becomes

$$\pi + \frac{x2\overline{r} + 2\overline{r}}{2} = \pi + (1+x)\overline{r},$$

and no fraud is possible if

$$egin{aligned} M+(1+E-M)xL+(1+E-M)(1-x)(\pi+(1+x)\overline{r})-1\ &\geq (1-x)f(\pi+(1+x)\overline{r})(1+E-M). \end{aligned}$$

Let $x(M, \pi)$ be the value of x for which there is =.

Optimal reserve in big bank

The big bank then finds the optimal M_{BB}^* maximizing

$$egin{aligned} &\int_{\pi_L}^{\hat{\pi}_{BB}(M)} [M+(1+E-M)x(M,\pi)L \ &+(1+E-M)(1-x(M,\pi))(\pi+(1+x)ar{r})-1]dF(\pi) \ &+\int_{\hat{\pi}_{BB}(M)}^{\pi_H} [M+(1+E-M)(\pi+M)-1]dF(\pi). \end{aligned}$$

It can be shown that

$$M_{BB}^* < M_{IB}^*$$

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Coalition of independent banks

Coalition should mimic the behavior of the big bank.

But the members are independent, compensation should be considered for

- closing down
- keeping open but not committing fraud

Coalition specifies a *liquidation rule*

$$\mathcal{L}(M,\pi,r) = egin{cases} 1 & ext{if } r ext{ is small} \ 0 & ext{otherwise}, \end{cases}$$

and a *debt restructuring rule* $D(M, \pi, r)$ (to a non-liquidated bank with signal r at date t = 2 instead of the original debt to the depositors.

How it works

Assume that π has been observed. If on their own, then $\pi < \hat{\pi}(M)$ means that depositors run all the banks.

But if

$$r^*(M,\pi) = rac{1-M}{(1-f)(1+E-M)} - \pi,$$

then $r^*(M, \pi)$ is the critical level where fraud pays for the small bank.

Now let x^* be the fraction of banks to be liquidated.

The coalition assigns M + (1 + E - M)L - 1 to banks with $r \le x^*2M$, and to the others

$$f(1+E-M)(\pi+r) = M + (1+E-M)(\pi+r) - 1 + T(r),$$

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Finding closure ratio and reserve

Here T(r) is a transfer which makes the bank indifferent between fraud and no fraud, that is

$$T(r) = 1 - M + (f - 1)(1 + E - M)(\pi + r).$$

Transfers should be balanced, this defines the fraction $x^*(M, \pi)$ to be closed down.

Now (as before), one may finde the optimal reserve M_C^* maximizing expected profits (of all banks in the coalition).

It can be shown (as was to be expected) that

$$M_{IB}^* > M_C^* > M_{BB}^*$$

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