Solutions to Exercises in
Economics of Banking
Chapter 7

1. The possible losses have the sizes $k\ell$, where k is a nonnegative integer ranging from 0 to $m\ell$, and the probability of losing $k\ell$ follows the binomial distribution

$$\mathsf{P}\{k\ell\} = \binom{m}{k} p^k (1-p)^{m-k},\ k = 0, 1, \dots, m.$$

When default depends on a random variable $\tilde{z}$, we may write number of defaults as

$$\widetilde{N}_m = \sum_{i=1}^{m} \widetilde{x}_i,$$

and its expected value is $\sum_{i=1}^{m} \mathsf{E}\tilde{x}_i = m\overline{p}$, since all the $\widetilde{x}_i$ are identically distributed with mean $\overline{x}$. The variance is found as

$$\mathrm{Var}[\widetilde{N}_m] = \sum_{i=1}^{m} \mathrm{Var}[\widetilde{x}_i] + \sum_{i=1}^{m} \sum_{j=1}^{m} \mathrm{Cov}[\widetilde{x}_i, \widetilde{x}_j].$$

Here $\mathrm{Var}[\widetilde{x}_i] = \overline{p}(1-\overline{p})$ and

$$\mathrm{Cov}[\widetilde{x}_i, \widetilde{x}_j] = \mathsf{E}\left[\widetilde{x}_i \widetilde{x}_j\right] - \mathsf{E}\widetilde{x}_j \mathsf{E}\widetilde{x}_j.$$

In the computation of $\mathsf{E}\left[\widetilde{x}_i \widetilde{x}_j\right]$, only the case where $x_i = x_j = 1$ gives a contribution different from 0, and the probability of this case is $p(\tilde{z})^2$, so that we get the expresssion

$$\mathrm{Var}(\widetilde{N}_m) = m\overline{p}(1-\overline{p}) + m(m-1)\left(\mathsf{E}\left[p(\widetilde{z})^2\right] - \overline{p}^2\right).$$

Since

$$\mathrm{Var}\left[\frac{1}{m}\widetilde{N}_m\right] = \frac{1}{m^2}\mathrm{Var}\left[\widetilde{N}_m\right] = \frac{1}{m}\overline{p}(1-\overline{p}) + \frac{m-1}{m}\left(\mathsf{E}\left[p(\widetilde{z})^2\right] - \overline{p}^2\right)$$

which clearly converges to $\mathsf{E}\left[p(\widetilde{z})^2\right] - \overline{p}^2$ for $m \to \infty$.[1] Since this last quantity can be recognized as the variance of $p(\tilde{z})$, we see that in the situation where default probability is random,

[1] typo in text – $\widetilde{p}^2$ should be $\overline{p}^2$.

the variance per engagement, and through this the riskiness of the portfolio, is not reduced when the number of engagements increases.

2. We choose $T = 1$, so that expected present value of a loan of size 1 is $e^{-\lambda}$, so that the capital charge on the loan will be $0.08e^{-\lambda}$.

The loss distribution for the security is such that

$$\tilde{L} = \begin{cases} 1 & \text{with probability } 1 - e^{-\lambda} \\ 0 & \text{with probability } e^{-\lambda}. \end{cases}$$

Thus, the probability of a loss, which always is 1, is $e^{-\lambda}$, and

$$\mathrm{VaR}_{0.99} = \begin{cases} 1 & \text{if } e^{-\lambda} > 0.01 \\ 0 & \text{otherwise.} \end{cases}$$

The two capital charges will be equal if

$$0.08e^{-\lambda} = 0.01$$

giving the value of 2.08 of λ.

3. The expected return can be found as

$$0.95 * 0.10 + 0.05 * (-0.5) = 0.07$$

so that expected return amounts to 7%.

4. The capital charges from the loan depends on the method chosen by the bank for computing risk-weighted assets. If the bank uses the standard approach, the percentages are unchanged, and so are the capital charges. If it uses the internal ratings based approach, it must take into account that the probability of default of the borrower may have increased, which eventually will lead to a higher capital charge.

5. The transition probabilities between ratings are as follows:

	A	B	D
A	0.92	0.07	0.01
B	0.03	0.90	0.07

(i) The mean value of bond A is found as

$$0.92 \cdot 112 + 0.07 \cdot 109 + 0.01 \cdot 51 = 111.18,$$

and the mean value of bond B is similarly found as

$$0.03 \cdot 110 + 0.90 \cdot 108 + 0.07 \cdot 51 = 104.07.$$

(ii) The variance of bond A is found as

$$0.92 \cdot (112 - 111.18)^2 + 0.07 \cdot (109 - 111.18)^2 + 0.01 \cdot (51 - 111.18)^2 = 37.17$$

and the standard deviation is $\sqrt{37.17} = 6.10$. The variance of B is similarly found to be 212.11, and the standard deviation is 14.56.

(iii) The two-bond portfolio consists of one bond rated A and one bond rated B, and the future states of the portfolio, its value in these states, and the probability that the portfolio will be in the relevant state are:

(A,A)	(A,B)	(B,A)	(B,B)	(A,D)	(B,D)	(D,A)	(D,B)	(D,D)
222	220	219	217	163	160	161	159	102
0.027	0.828	0.002	0.063	0.064	0.005	0.0003	0.009	0.0007

(iv) The mean of the portfolio can now be found by computation following the approach in (i) as 215.25, and alternatively as the sum of the two means found in (i). The variance of the portfolio is 249.27, which also can be found as in (ii) or more simply as the sum of the variances of the two bonds (which are independent). The standard deviation is 15.79.

(v) The marginal risk can be interpreted as the increase in standard deviation caused by adding the second bond to the portfolio, 15.79 – 6.10 = 9.69.