

**Solutions to Exercises in
Economics of Banking
Chapter 4**

1. Since all individual portfolios are proportional, the market portfolio contains the assets in the same proportions as the individual portfolios. The market portfolio is efficient, in particular it minimizes the variance $\sigma^2(x)$ for given payoff, since otherwise security prices (and with them the payoffs) would change. If μ^0 is the mean payoff, the market portfolio solves

$$\begin{aligned} \min_x \sigma^2(x), \\ \mu(x) - \mu^0 \geq 0, \end{aligned}$$

with Lagrangian $\sigma^2(x) + \lambda(\mu(x) - \mu^0)$. and the first order conditions are

$$\lambda \rho + \Sigma x^0 = 0.$$

As always, the Lagrangian multipliers have an interpretation as the marginal value of the objective function, and since this is a minimization problem, we get that $\lambda = -\sigma^2(x^0)$, the variance of the market portfolio.

Summing up, we get that

$$\rho = \frac{1}{\sigma^2(x^0)} \Sigma x^0,$$

and since Σx^0 consists has coordinates $\sum_{j=1}^n \sigma_{ij} x_j^0 = \text{cov}(\tilde{r}_i, \tilde{m})$, which is the covariance between i th security and the market portfolio, here considered as a random variable and written as \tilde{m} , we get that

$$\beta_i = \frac{\text{cov}(\tilde{r}_i, \tilde{m})}{\text{var}(\tilde{m})}.$$

2. Using the formula in Problem 1, we get that the expected return r_A of security A is

$$r_A = 1.2 \cdot (12 - 4) + 4 = 12.8,$$

and similarly, we find

$$r_B = 0.9 \cdot (12 - 4) + 4 = 11.2,$$

$$r_C = 1.1 \cdot (12 - 4) + 4 = 12.8,$$

so that the expected return on the proposed portfolio is

$$12.8 \cdot 0.3 + 12.8 \cdot 0.4 + 12.8 \cdot 0.3 = 12.4$$

(the information about the variance of the market portfolio is not needed for computation of expected return).

3. Since the distribution is symmetric and we are interested only in one tail, we first fix k such that $1/k^2 = 0.01/2$, so that $k^2 = \sqrt{200} \sim 14$. For $\mu = 15,000$ and $\sigma = 30,000$ we get that the probability that $x - 15,000$ exceeds $14 \cdot 30,000$ is ≤ 0.01 , so that $\text{VaR}_{0.99}$ can be estimated as $420,000 + 15,000 = 435,000$.

If the distribution is normal, then we would have

$$\text{VaR}_{0.99} = 15,000 + 2.57 \cdot 30,000 = 92,100;$$

here 2.35 is the value of a standardized normal variable corresponding to 99.5%. It is seen that this estimated value is rather high for the given mean and standard variation, showing that Chebyshev's inequality gives a rather coarse approximation, a price to be paid for its general applicability.

4. If losses are *independent* and *identically distributed* over the days, and if VaR is computed using an approximation to a normal distribution, then the standard deviation of the distribution for 10 days of losses has a standard deviation which is $\sqrt{10}$ times that of the daily loss distribution. Denoting the latter by σ , and letting μ be the expected daily loss, we have that

$$20 = \mu + k\sigma,$$

where k is the fractile of the standardized normal distribution corresponding to the level at which VaR is computed (typically 0.99), from which we get that

$$k = \frac{20 - \mu}{\sigma},$$

and the 10-day VaR is then

$$\mu + \frac{20 - \mu}{\sigma} \sqrt{10} \sigma = \mu + \sqrt{10}(20 - \mu).$$

In the simple case of $\mu = 0$ we thus get that the 10-day VaR is $\sqrt{10}$ times the 1-day VaR.

5. The covariance between the two assets is found from

$$0.3 = \frac{\text{cov}(A, B)}{\sigma_A \sigma_B},$$

where $\sigma_A = \sigma_B = 1$ are the standard deviations of returns. We get that $\text{cov}(A, B) = 0.3$, so that the variance of returns on the portfolio consisting of equal shares of A and B is

$$\text{var}\left(\frac{1}{2}A + \frac{1}{2}B\right) = \frac{1}{4} \cdot 1 + \frac{1}{4} \cdot 1 + \frac{1}{4} \cdot 0.3 = 0.575,$$

and its standard deviation is $\sigma = 0.758$. Using the result of Problem 4, we get that the 5-day standard deviation is $0.758 \cdot \sqrt{5} = 1.7$. Assuming normality we find that with an expected

return of 1%, the percentage loss is no bigger than -2.955 with probability 0.99, and this gives us a the 5-day 99% VaR as

$$200,000 \cdot 2.955\% = 5,910.$$

6. From the observed data, it is seen that out of 300 observations, only three showed a loss of more than 5.7% of the value \$1,200,000, so that in only 1% of the cases, the loss exceeded $\$1,200,000 \cdot 0.057 = \$68,400$, which therefore is our best estimate of $\text{VaR}_{0.99}$.

7. Since the fund borrowed \$10,000,000 at 2% interest, it needs to pay back \$10,200,000. We must therefore find the probability that losses on a market portfolio of size \$11,000,000 exceeds \$800,000. Since the expected return is 5% and the standard deviation of this return is 8%, we find assuming normality that the probability of a standardized normal variable to be smaller than

$$\frac{-\frac{0.8}{11} - 5}{8} = -0.72$$

equals 24%.