Solutions to Exercises in
Economics of Banking
Chapter 10

1. To obtain the efficient allocation agent I must buy from agent II who must buy from agent III who again should buy from agent I. This is easily achieved if agent I can issue an IOU for delivery of one unit at $t = 3$ to agent II against the good, and if II can use this IOU as payment for delivery of the good from III. What matters here is that the IOU of agent I could be reused by agent III who had no use for I's direct delivery.

If agents of type III cannot be trusted when they write an IOU (that is when they promise delivery at date 2), then multilateral agreements couldn't be used (since III would default on delivery). However, sequential trades using IOU are possible: agent III should pay at $t = 2$ using the paper obtained from the sale to I at $t = 1$, and then there is no need for trusting the delivery. Thus, the IOU of agent I works as money in this situation (not those of III as suggested in the text).

2. Assuming that each bank has the probability p of getting a request from one of the other banks, we get the following normal form game, where bank 1 chooses row, bank 2 column and bank 3 matrix:

morning

	morning	afternoon
morning	$(-(1-2p)C, -(1-2p)C, -(1-2p)C)$	$(-(1-p)C, -D+2pC, -(1-p)C)$
afternoon	$(-D+2pC, -C, -C)$	$(-D, -D, -D)$

afternoon

	morning	afternoon
morning	$(-(1-p)C, -(1-p)C, -D+2C)$	$(-(1-p)C, -D+pC, -D+pC)$
afternoon	$(-D+pC, -C, -D+pC)$	$(-D+pC, -D+pC, -D+pC)$

(as in the text, the term $C + qC$ which has been omitted from all the payoffs).

If bank 2 and 3 merge into a single one, the resulting liquidity game will be as in the text, however with an asymmetry, since bank 1 will receive requests from bank 2 + 3 with probability $2p$ in the morning and $2q$ in the evening, so whereas bank 1 receives requests with probability p and q as before.

3. Agent i pays $\sum_k x_{ij}^k$ to agent j, $j \neq i$, and the liquidity reserved needed if i must pay before receiving any payment is

$$\sum_{j \neq i} \sum_k x_{ij}^k.$$

In order to minimize the need for liquidity, one should choose a sequencing of the payments such that the sum of the liquidity needs of all agents to carry out payment k, in addition to the reserves remaining from previous payments, is as small as possible. If there are K payments, and Σ denotes the set of all permutations σ of K, then the liquidity need of agent i before and liquid fonds after the first payment are given by

$$L_i^{\sigma(1)} = \sum_{j \neq i} x_{ij}^{\sigma(1)} \text{ and } A_i^{\sigma(1)} = \max\left\{0, \sum_{j \neq i} x_{ij}^{\sigma(1)} + \sum_{j \neq i} x_{ji}^{\sigma(1)}\right\},$$

for the kth payment, $k > 1$, they are

$$L_i^{\sigma(k)} = \max\left\{0, \sum_{j \neq i} x_{ij}^{\sigma(k-1)} - A_i^{\sigma(k-1)}\right\} \text{ and } A_i^{\sigma(k)} = \max\left\{0, A_i^{\sigma(k-1)} + \sum_{j \neq i} x_{ji}^{\sigma(k-1)} - \sum_{j \neq i} x_{ij}^{\sigma(k)}\right\}.$$

Therefore, total liquidity needs of agent i in the permutation σ are $L_i^{\sigma} = \sum_k L_i^{\sigma(k)}$ and the liquidity needs are minimized if

$$\sigma^0 \in \operatorname{argmin}_{\sigma \in \Sigma} \sum_i L_i^{\sigma}.$$

4. If there are n merchants, competition for customers may be modeled using the "circular city" approach: We then assume that customers are spread evenly on a circle with unit perimeter, so that each merchant has a share $1/n$ of the buyers. Using standard formula (see e.g. the treatment of a circular city with banks in Chap.11) one gets that the equilibrium price when all accept cards is

$$p^* = c + \frac{t}{n},$$

where t is transportation cost, and c is the unit cost of delivering the good,

$$c = d + D(f^*(c_I - a))m^n(c),$$

consisting of production cost d and cost of payment cards (the 'n' in $m^n(a)$ stands for 'net' and is not related to the number of merchants). Each merchant has a profit of t/n^2.

If all accept cards but one merchant decides not to accept cards any more, then with new prices p_1 for this merchant and p_2 for the two neighboring merchants we have that

$$p_1 + tx_1 = p_2 + t\left(\frac{1}{n} - x_1\right) - b_B,$$

where x_1 is the distance from merchant 1 to the last of her customers and b_B the benefit of cards for the marginal customer, and demand is then found as

$$2x_1 = \frac{p_2 - p_1 - b_B}{t} + \frac{1}{n},$$

and the monopolistic price is then

$$p_1 = \frac{1}{2}d + \frac{1}{2}\left[p_2 - b_B + \frac{t}{n}\right].$$

From here on the situation will be more complex than with two merchants: The neighboring merchants are immediately affected, and they may win or lose customers from merchant 1. The remaining $n-3$ merchants are indirectly affected, and the general symmetry is lost, so that in a new equilibrium, we may have different prices for all merchants, since they will adapt to the changes in prices of merchant 1 to a different degree.

Assuming that the remaining merchants keep the original equilibrium price, $p_2 = p^*$, we get after inserting p^* and $b_B = \beta(f)D(f)$ that

$$p_1 = d + \frac{1}{2}[D(f)(m^n(a) - \beta(f))] + \frac{t}{n}.$$

If $a \le \overline{a}$, then $\beta f > f(c_I - a) \ge m^n(a)$, and consequently profit per unit has decreased. For a such that

$$D(f)(\beta(f) - m^n(a)) \ge \frac{2t}{n},$$

we get that unless the market share of merchant 1 increases by $1/n$ so as to eliminate the neighbors, we have that profit of the merchant is reduced, since

$$\frac{2}{n}\left[\frac{1}{2}D(f)(m^n(a) - \beta(f)) + \frac{t}{n}\right] \le -\frac{t}{n^2} + \frac{2t}{n^2} = 0.$$

5. The background literature is to be found in section 10.3 on payment cards. In the Tirole-Rochet model for payment cards and their use, one derives a condition for the welfare optimality of the system: the benefit of the marginal card user and the merchant should correspond to the resource cost of card transactions. The situation sketched in the problem suggests that the merchant fee is rather too high, and it is also to be expected, that not very many users stop using card if their fee will be raised somewhat. The solution is therefore to change the interchange fee between the issuer and the acquirer.