Lecture Note 5

Comments to Malthus and Ricardo

1. On the possibility of a general over-supply

The comments of Malthus on possible imbalance between aggregate production and aggregate demand has been rediscovered in the 20th century as an early approach to the contemporary theory of employment. Clearly, the standard model of Keynesian economics cannot be applied as it is to the world of Malthus, so we must consider what was actually the message of Malthus, who argued that there could be situations where the use of the incomes generated from production would not automatically ensure that the product is demanded.

Here is a version of Malthus' theory as formulation by Eagly (1974). We consider an aggregate model of an economy with a work force $\overline{N}$ of given size, consisting of productive labour N_1 producing material goods and unproductive labour N_2 which produces services, so that

$$\overline{N} = N_1 + N_2. \tag{1}$$

To produce the material goods, we assume that one will need both labour and machines in a ratio α, so that the capital needed to employ a worker is $w + \dfrac{p}{\alpha}$ (where p is the price of the (aggregated) good and w is the wage rate), so with a capital of the fixed size $\overline{K}$, the need for productive labour is

$$N_1 = \frac{\overline{K}}{w + \dfrac{p}{\alpha}}. \tag{2}$$

There is no need for capital outlays in the production of services. Net output of the good in the economy Z is proportional to N_1,

$$Z = aN_1,$$

and it can be used either for consumption C, which takes the form of services, or for investment, that is increase in the capital stock. The supply of commodities going to this inter-sectoral exchange is assumed to be a given fraction of output,

$$C = cZ,$$

and for this to fit with the number of workers in the service sector, we must have that $wN_2 = cZ = caN_1$ or

$$N_2 = \frac{ca}{w}N_1. \tag{3}$$

Finally, we have that the produced quantity Z, or equivalently aN_1, is fully used up for investment I and consumption,

$$aN_1 = I + wN_2. \tag{4}$$

If investment is given (or "exogenous", as one would say today), then we have three variables N_1, N_2 and w, but we have four equations, namely (1) – (4), so the system does not necessarily have a solution. This might be considered as what Malthus had in mind: in order to have an equilibrium, the level of investment cannot be exogenously given but has to take a specific value – technically I should be a variable and not a constant.

In economic terms it means that there is no automatic adjustment which would establish an equilibrium as suggested by Say. To get there one would have to add some mechanism for dividing income into investment and consumption. This could be obtained by adding a theory about loans and interest rates, introducing a new variable i (the rate of interest) and letting I depend on i, making the system determined.

Malthus is very explicit about the problems of determining the right level of investment in society. If investment is too low, the growth of society's wealth is endangered, and if it is too big, and consumption is too small, then the incentives to invest are destroyed. This problem, that "the principle of saving, pushed to excess, would destroy the motive to production", a formulation close to the "paradox of saving" in macroeconomics textbooks. Malthus suggests that a political decision may be necessary to determine the correct size of investment.

Here is a model of Lange (1938) which captures the idea of adding a market for loanable funds. We begin with a simple Keynesian model,

$$\begin{aligned} Y &= C + I && (5)\\ I &= F(i, C) && (6)\\ M &= L(i, Y), && (7) \end{aligned}$$

where (as usual) Y is aggregate income, C consumption, I investment, i the interest rate, and M the (given) quantity of money held by individuals. The problem of the optimum propensity to consume is to find a level of C which maximizes I given that the equations of the model should hold, so that i and Y can be determined suitably.

We may characterize the optimum as follows: If I^* be the maximal value of I, then inserting it into (6) we obtain C as an implicit function of i with derivative $\frac{\mathrm{d}C}{\mathrm{d}i} = -\frac{F'_i}{F'_C}$. Similarly, (7) gives us Y as implicit function of i with derivative $\frac{\mathrm{d}Y}{\mathrm{d}i} = -\frac{L'_i}{L'_Y}$. Now differentiating (5) w.r.t. i, using that $\frac{\mathrm{d}I}{\mathrm{d}i} = 0$ in optimum and inserting the other derivatives, we get that

$$\frac{F'_i}{F'_C} = \frac{L'_i}{L'_Y} \text{ or, equivalently, } \frac{\mathrm{d}i}{\mathrm{d}C} = -\frac{F'_C}{F'_i} = -\frac{L'_Y}{L'_i} = \frac{\mathrm{d}i}{\mathrm{d}Y}.$$

Using the last expression, we see that an increase in I caused by a higher level of consumption will induce an increase in the rate of interest which in its turn will reduce investment. In this way, we have found a balance between consumption and investment at I^*.

Whether this was what Malthus had in mind, is open to doubt and has indeed been debated. Malthus probably did not think in Keynesian terms, but he definitely had an impact on the thinking of Keynes.

2. Malthus' criticism of Adam Smith

The emphasis on division of labor which is a main theme with Adam Smith, had the natural consequence that he saw the development of manufacture or industry as the way towards increasing wealth in society. Malthus, on the other hand, considered agriculture and its development as the key factor in promoting the wellbeing of society's inhabitants.

Behind this difference of opinion lies not only different assessments of the contribution of the two sectors, but also a different view of what should be the objective of society's economic activity. While Adam Smith and some of his contemporaries made a big leap forwards by the identification of society's wealth as the annual product of its activities (rather than the amount of gold which it has collected), one can see Malthus' argumentation for the importance of agriculture as a consequence of an even more sophisticated approach to what constitutes the happiness of society.

The following simple formalism, due to Hisamatsu (2015), illustrates the arguments of Malthus. We consider a society which produces two goods, namely (1) "food" and (2) "luxuries", using only labor inputs and with fixed coefficients,

$$X_1(t) = a_1 N_1(t),\ X_2(t) = a_2 N_2(t)$$

The input $N_i(t)$ of labor in the sector i is assumed to change at a rate n_i, $i = 1, 2$which in its turn depends on rates of capital accumulation, considered as given (we are in a classical world where capital accumulation takes the form of a wage fund, the annual outlays to labor).

Following Adam Smith, the real wealth of society at date t is determined by $(X_1(t), X_2(t))$, and without entering into a discussion of the exact way of weighing the two quantities together (which would demand a theory of value which Malthus largely avoided), we notice that if

$$X_i(t+1) \geq X_i(t),\ i = 1, 2,\ \text{and}\ (X_1(t+1), X_2(t+1)) \neq (X_1(t), X_2(t)),$$

then society's wealth has increased.

But wealth may not be (and with Malthus, is not) the same as overall wellbeing, or as Malthus would put it, happiness. According to his early writings, this happiness depends crucially on health and the command of the necessaries and conveniences of life. With respect to the health, Malthus repeatedly notices the unhealthiness of living conditions in the cities, where people are crowded together in overfilled rooms, and encouragement of agriculture would therefore increase the inflow to the market of goods produced under healthy conditions. If we let

$$b(t) = \frac{N_1(t)}{N_2(t)},$$

then a rise in $b(t)$ would mean that the relative social level of health is increasing. For the second aspect of happiness aspect, Malthus states that food is the most important part of the necessities of life, so we may abstract from the products of industry which goes largely to the owners of land and capital. Assuming that the latter also consume a fraction c of the agricultural products, the average worker's command over food is

$$\omega(t) = \frac{(1-c)X_1(t)}{N(t)}. \tag{8}$$

Taking logarithms and differentiating, we get that

$$\frac{\omega'(t)}{\omega(t)} = \frac{X_1'(t)}{X_1(t)} - n,$$

where $n = n_1 + n_2$ is the growth rate of labour, and we conclude that the growth rate in ω is positive or negative depending on whether the growth rate in output of food exceeds or falls short of the growth rate of labor.

Phrased in terms of this simple model, the argument of Malthus against Adam Smith goes as follows: Assume that the available surplus of capital is used only to increase manufacturing capital and not to agriculture. This means that the $n_2 > 0$ whereas $n_1 = 0$. With an unchanged labor force in sector 1 and more labor in sector 2, we get that $X_1(t+1) = X_1(t)$ and $X_2(t+1) > X_2(t)$. Thus, wealth has increased.

But what about happiness? Clearly, $b(t)$ must be falling over time, and rewriting (8) as

$$\omega(t) = \frac{(1-c)a_1 N_1(t)}{N(t)} = \frac{(1-c)a_1}{1+\frac{N_2}{N_1}} = \frac{(1-c)a_1}{1+\frac{1}{b(t)}}$$

we see that also $\omega(t)$ decreases. But then both components of happiness have become smaller, so wellbeing has deteriorated.

3. The Ricardian labour theory of value

While several authors propose a theory of prices based on labour *and* land, reducing labour to land using the basket of goods feeding a worker raised on a certain amount of land had already been proposed by Cantillon. Ricardo's method of reducing everything to labour involves the idea that the soil is available in different qualities, and that the labour value of land should be found on the land of poorest quality.

While introducing several types of land the argumentation does not explicitly introduce that there are also more than one commodity. Following Samuelson (1966), we discuss the Ricardian labour value in a model with *two* commodities. It is instructive to begin with a very simple world, where labour is available in a fixed amount L. If commodity 1 needs a_1 unit of labour and commodity 2 needs a_2, then society can achieve all combinations (q_1, q_2) on the (labour) budget line

$$a_1 q_1 + a_2 q_2 = L,$$

and relative prices are fully determined by the slope $\frac{a_2}{a_1}$ of this line, independent of demand, and we have a clear-cut labour theory of value. Alternatively, if land is given as S we would have a budget line $b_1q_1 + b_2q_2 = S$ and a land theory of value. However, if both are fixed so that we have two constraints, then the relative prices are not uniquely defined without reference to demand.

Ricardo's world is one where labour is not fixed but can be reproduced. In this case we would have prices determined as

$$p_1 = wa_1 + rb_1,\; p_2 = wa_2 + rb_2,$$

where a_i, b_i are labour and land content of the prices of commodity i, for $i = 1, 2$. Assuming that to reproduce labour we need c_1 and c_2 of the two commodities, so that

$$w = c_1p_1 + c_2p_2,$$

we get that

$$w = c_1(wa_1 + rb_1) + c_2(wa_2 + rb_2) = (c_1a_1 + c_2a_2)w + (c_1b_1 + c_2b_2)r$$

or

$$\frac{w}{r} = \frac{c_1b_1 + c_2b_2}{1 - (c_1a_1 + c_2a_2)}.$$

From this we get that

$$\frac{p_1}{r} = \frac{c_1b_1 + c_2b_2}{1 - (c_1a_1 + c_2a_2)}a_1 + b_1,\; \frac{p_2}{r} = \frac{c_1b_1 + c_2b_2}{1 - (c_1a_1 + c_2a_2)}a_2 + b_1,$$

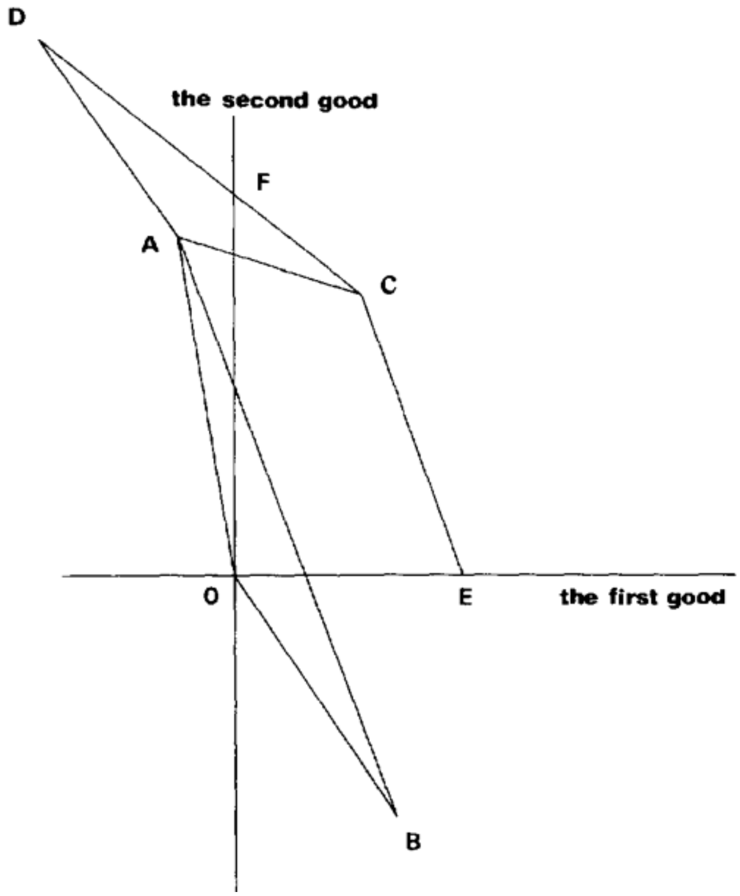
and we have expressed the prices using r, the price of land, as numéraire.

This situation, where land is fixed but labour is not, also gives uniquely determined relative prices, given as the slope of the budget line

$$\frac{p_1}{r}v_1 + \frac{p_2}{r}v_2 = S, \tag{9}$$

where S is available land and v_i is the net output (when the amount needed to reproduce labour is deducted) of commodity $i = 1, 2$. The situation is illustrated in the figure on the next page (taken from Negishi (1989),p.112), where A represents the (hypothetical) case where all land is used to produce commodity 2 (and some amount of commodity 1 must be used from outside to feed labour), and similarly B is the case where all land is used for commodity 1 (for the moment, forget about all other points). The budget line (9) is the segment of AB which falls in the first quadrant.

So far we have basically reduced labour to land, so that what comes out is more



Cantillon than it is Ricardo. But we have not yet used the fundamentally new aspect introduced by Ricardo, namely that of diminishing returns of land: As more land is used, the productivity of the newly cultivated soil will be inferior to those already used. Since the price paid for using land is the same, it allows for a surplus to owners of productive soil, which is *rent*. For the poorest land there is no such gain, so the price of commodities raised on this land reflects only labour, and since competition assures that commodity prices are the same no matter where the commodity was produced, we finally get a full labour theory of value.

This is a remarkable way of reasoning, but it has its limitations. There is an implicit assumption about the way in which the soil becomes less productive, namely that production of all commodities should be affected in the same way. If this is not the case, then relative prices will depend on other things than just technology (namely, demand). This is where the remaining part of the figure comes in: Let *OAB* be all the output combinations possible on the first (and best) piece of land. Now we add a second, inferior, piece of land, which isolated would have given another triangle with top in O, but since we already have the first one, we can produce all combinations arising as a sum of a production in the two triangles. Geometrically this means that we move the last triangle to have its top in A and the slide it down along AB the whole way to B. The outcome is a transformation curve FCE which has a kink in C, so its slope is not constant, in other words, relative prices cannot be determined by embodied labour alone.

4. The rate of profit and the rent on superior soil

Ricardo's reasoning about the rate of profit, which plays a role in his theory about the eventual static state of the economy, uses a somewhat simpler model of the economy that the one above, in particular he assumes that

1. Say's law works, so that there is no lack of demand or overproduction,
2. No fixed capital, only circulating capital in the form of wages advanced to workers,
3. Wages are at subsistence level,
4. Workers consume only agricultural products ("corn")

In the agricultural sector producing corn, the working capital has the size wL_c where w is the wage rate, which can be measured directly in corn, and L_0 is the number of workers employed in agriculture, giving a profit rate

$$\pi_0 = \frac{q_0 - wL_0}{wL_0} = \frac{1 - wa_0}{wa_0},$$

where $a_0 = \dfrac{L_0}{q_0}$ is the labour embodied in one unit of corn. Turning now to some other sector of the economy, such as the production of cloth, we similarly get a profit rate of the form

$$\pi_1 = \frac{p_1 q_1 - wL_1}{wL_1} = \frac{p_1 - wa_1}{wa_1},$$

with a_1 the labour coefficient of this sector. Now Ricardo applies a principle taken from Adam Smith, taking into account also the distinction between natural and market prices, namely that profit rates in different sectors must be equal due to the forces of the market. Thus, p_1 can be found from

$$\frac{p_1 - wa_1}{wa_1} = \frac{1 - wa_0}{wa_0},$$

which gives the well-known expression $p_1 = \frac{a_1}{a_0}$ for the price determined by labor coefficients.

The same approach could in the case of different qualities of soil, considering them as different sectors with $a_0' > a_0$ but here the price of the output, corn, is the same, so equality of profit rates can only by adding a new variable r with

$$\frac{1 - wa_0 - r}{wa_0} = \frac{1 - wa_0'}{wa_0'},$$

interpreted as the rent (here measured per unit of output) to be paid by the capitalist to the landowner.

It is seen that the size of the profit rate is determined by the less productive sector. Adding here the assumption, also a standard one with the classics, that investment, here in the form of extension of the working capital, is an increasing function of the profits earned, we get that economic growth becomes slower as a consequence of the lower productivity of the land, eventually sending the profits to zero.

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