Lecture Note 4

Modern formulations of Adam Smith's price and growth theory

Smith uses a historical approach, beginning with a primitive society (with no scarcity of land) where labour is the single productive factor. In such a society, goods are exchanged against each other on proportions which correspond to the labour which has used on achieving the goods. Measuring value in terms of labour content may seem to give problems: If the value of labour equals the value of the bundle of goods needed to feed the labourer, and this value is again measured in terms of labour, then where should we start?

Here it is useful to be explicit on commodities and labour. The following modern version of Adam Smith is due to Samuelson (1977), here given in a simplified form following Negishi (1989).

Labour only. Suppose that there are fixed input coefficients a_{ij} specifying the use of commodity i per unit output of commodity j, for $i, j = 1, \dots, n$, and similarly fixed coefficients a_{Lj} for use of labour per unit output of j. Also, we let m_i be the consumption of commodity i per unit of labour. If $q = (q_1, \dots, q_n)$ is produced, then we have that

$$\sum_{j=1}^{n} a_{Lj} q_j = L \tag{1}$$

(use of labour in production of all goods equals available labour), and

$$q_i = \sum_{j=1}^{n} a_{ij} q_j + m_i L, \; i = 1, \dots, n,$$

production of good i equals its use as input or in the goods basket of labour. It is convenient to write the last system of equations in matrix form, so we let I be the unit matrix and $A = (a_{i,j})_{i=1}^{n}{}_{j=1}^{n}$ be the matrix of technical coefficients, then we have that

$$(I - A)q = mL$$

with m the column vector giving the composition of the goods basket to labour., which can be solved to give

$$q = (I - A)^{-1} mL$$

(where we have assumed that $I - A$ can be inverted, fortunately this is the case under standard assumptions on the technical coefficients of a linear production model, actually one even gets that all its elements are nonnegative). If the prices are $p_1, \dots, p_n$ and wage is w, then competition secures that price equals cost,

$$p_j = w a_{Lj} + \sum_{i=1}^{n} p_i a_{ij},$$

or, in matrix form

$$p(I - A) = w A_L \tag{2}$$

with A_L a row vector of labour coefficients, giving a solution

$$p = w A_L (I - A)^{-1}. \tag{3}$$

This is not mere empty formalism, it shows that prices can be defined using only the technical coefficients (no logical circles involved). It can also can be used for example to show that

$$pm = p(I - A) q \frac{1}{L} = w A_L q \frac{1}{L} = w,$$

where we first used (2) and then (1). Thus, wages are determined as the price of the subsistence bundle. Altogether we get a labour theory of prices.

Labour and Land. This is however only the first step in Smith's price theory, describing a prehistoric society. Technological progress, for example in the form of reducing some of the a_{ij}, for $i, j \in \{1, \dots, n\}$ means that the same labour can produce more, wages will exceed subsistence level, production increases, but eventually it will hit another constraint, namely that of available land. Let S be the maximal amount of land. Since the use of land matters, we must be explicit on this use, so there are also coefficients $a_S = (a_{S1}, \dots, a_{Sn})$ for its use in producing commodities $1, \dots, n$.

With two scarce factors of production, we have a problem of choosing the right way of combining them in production. Technically we assume that there are several techniques (specified by the coefficients in A, a_L and a_K) available. We assume that the society chooses *net production* v_1 of commodity 1 to be as large as possible with given lower bounds $\overline{v}_j$ for the net production of the other goods, so that v_1 is maximized by choosing suitable values of $q_1, \dots, q_n$ and technical coefficients (as far as there is a choice), satisfying the constraints

$$
\begin{aligned}
& v_1 - \left(q_1 - \sum_{j=1}^{n} a_{1j} q_j \right) = 0, \\
& q_i - \sum_{j=1}^{n} a_{ij} q_j \geq \overline{v}_i, i = 2, \dots, n, \\
& \sum_{j=1}^{n} a_{Lj} q_j \leq L, \ \sum_{j=1}^{n} a_{Kj} q_j \leq S.
\end{aligned}
\tag{4}
$$

Assuming that all the a_{ij} have been chosen optimally, we are let with an LP problem, which in tabular form can be written as

	v_1	q_1	q_2	$\cdots$	q_n	
	1	0	0		0	
p_1	1	-1	a_{12}	$\cdots$	a_{n1}	0
p_2	0	a_{21}	-1	$\cdots$	a_{2n}	$-\overline{v}_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
p_n	0	a_{n1}	a_{n2}		-1	$-\overline{v}_n$
w	0	a_{L1}	a_{L2}	$\cdots$	a_{Ln}	L
r	0	a_{S1}	a_{S2}	$\cdots$	a_{Sn}	S

where the dual variables, corresponding to the constraints (in which we have changed sign in the first n inequalities to have $\leq$ instead of $\geq$), are $p_1, \dots, p_n, w$ for commodities and labour and r for the land constraint. These dual variables must satisfy

$$p_j = \sum_{i=1}^{n} a_{ij} p_j + a_{Lj} w + a_{Sj} r, \quad j = 1, \dots, n$$

with $p_1 = 1$ (this can be seen either by looking at the dual of (4) or – simpler – by differentiating the Lagrangian of the maximization problem w.r.t. q_j). From the main theorem of LP (or alternatively – the complementary slackness condition of Kuhn-Tucker) we get that

$$p_1 v_1 = v_1 = -\sum_{j=2}^{n} p_2 \overline{v}_j + wL + rS,$$

or, assuming that all constraints are satisfied with equality,

$$\sum_{i=1}^{n} p_i v_i = wL + rS. \tag{5}$$

Summing up, in this more developed economy prices can be derived from labour *and* land (no further reduction possible), and (5) gives Smith's resolution of national income (value added in production) into wage and rent components.

Labour, Land and Capital. We now add a last component, namely capital, which however should be considered in the version which it took at the time of Adam Smith: Assume that the commodity bundle consumed by each worker must be handed over beforehand production as a *wage fund*. Thus, we have that capitalists initially dispose of a wage fund W to be paid to the workers, and then is production is carried out. Leaving everything else as above, we get a similar maximization problem where the solution again gives rise to dual variables (scarcity prices) $p_1, \dots, p_n$, w and r, only in our new version of the problem w is not the wage actually paid to the workers – the latter is found by as W/L, the value of the wage fund divided by the number of workers – but labour's contribution to the production realized.

Writing (5) as

$$\sum_{i=1}^{n} p_i v_i = W + (wL - W) + rS,$$

and we have the split of national income in wages, profits, and rent.

It should be noticed that Adam Smith's value theory does not preclude that wages can exceed subsistence minimum, i.e. that $W > pmL$, at least for some period of time.

Investment and growth. After the price and value theory of Adam Smith, we now consider his theory of a growing economy. For this purpose we consider aggregate output as a single commodity (just as in contemporary macroeconomics), keeping labour as the second one. We then let a be the labour coefficient of the aggregate product, and we let b be the consumption needed to reproduce one unit of labour.

In the initial primitive state, the value of the product is then a, and the value of the labour becomes ba, all measured per unit of available labour, and in a static situation, this unit is exactly reproduced so that $ab = 1$. Prices of the product p_1 and of labour power p_2 then satisfies

$$p_1 = ap_2 = a, \; p_2 = bp_1 = ba = 1. \tag{6}$$

To prepare for more advanced states of the society, we may assume that there is a time span between input and output, so that aggregate product $X(t)$ and available labour $L(t)$ depend on time and now satisfy

$$X(t) = bL(t+1),\ L(t) = aX(t+1)$$

(the labour available at $t+1$) is dependent on the stock of product $X(t)$ set aside to feed labour, etc.), if X and L do not depend on t we get the previous situation. But for the economy to develop, the stock of goods forwarded to feed labour must exceed the static value, and this will happen only if profit can be earned. Adam Smith introduces a *natural price* of a commodity defined as the sum of wages and profit, both at the natural rate, defined as an average of wages and profits in different employments. The natural prices must then satisfy

$$p(t+1) = (1+r)aw(t)$$

with r and w natural rates of profit and wages, and

$$w(t) = (1+r')bp(t),$$

so that the natural rate of wages is higher than the subsistence level in a growing economy. Assume that profits are all invested. If the growth is balanced so that X and L grow at a common rate g, then $ab(1+g)^2 = 1$, and relative prices $\frac{p(t)}{w(t)}$ remain constant over time, then this gives a modified version of (6) with

$$p = (1+r)aw = (1+r)a,\ w = 1 = (1+r')(1+r)ab.$$

Here w is the natural rate of wages, and p is the *commandable* labour value of the product which is larger than the *embodied* labour value a. Similarly, commandable labour value of labour power is larger than the embodied value ab. It may be shown that $r = r'$, so that $r = r' = g$. Thus, a high rate of wages and a high rate of profit coexist in economies with a high rate of growth (Adam Smith points her to the North American colonies). This differs from the approach taken later by Marx, who used a labour value closer to Cantillon than to Smith.

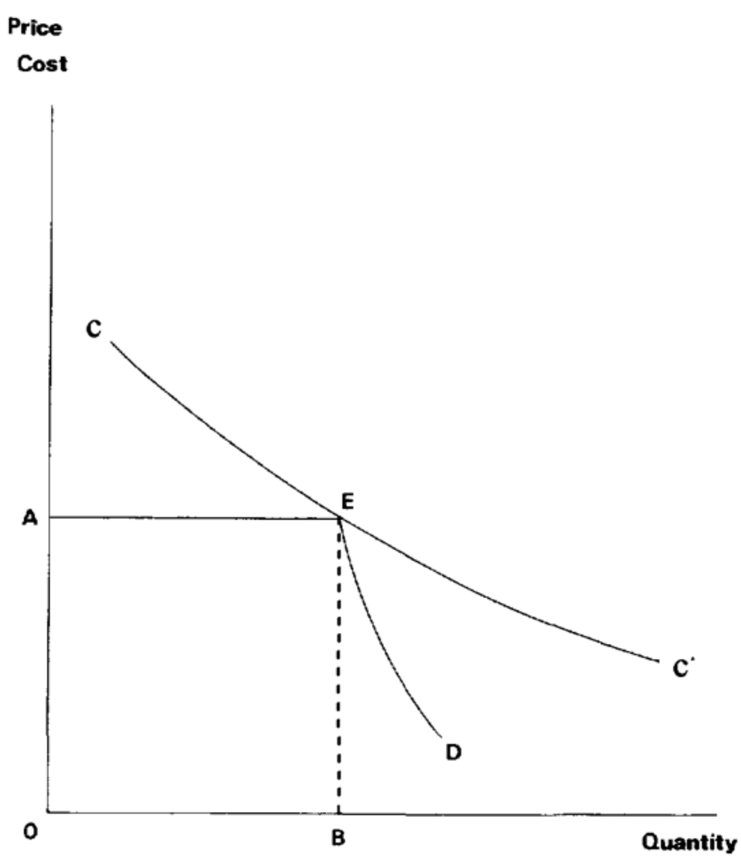
(Incidentally, the growth theory of Adam Smith fits into the linear multicommodity growth model known today as a von Neumann model.)

Division of labour. The famous story about the pin factory, where division of labour improved productivity enormously, is mentioned in most descriptions of Wealth of

Nations, and division of labour is indeed a fundamental concept which plays role for Smith's theory of competition, growth and trade.

In our contemporary understanding, division of labour is a way of achieving productivity gains when the number of workers gets larger. This will however give rise to another problem, since we have a case of *increasing returns to scale.* As is wellknown, the presence of increasing returns to scale destroys the possibility of allocating through a competitive market, since larger firms can produce cheaper than small firms, eventually giving rise to a monopoly. Even if Smith does mention monopolies (in another often-cited sentence) he does not consider them so widespread as to interfere with the general situation of competition, seemingly in contradiction with contemporary theory of competitive markets.

The explanation should be found in the way in which the firm and its competitive situation is understood by Smith. It should be remembered that a theory of demand is largely absent, so the decisions of the firm are based on considerations of cost and conjectures about the way in which the product can be sold, coming close to the contemporary notion of a *conjectural equilibrium* (Hahn, 1978). This can be illustrated as in the figure below, taken from Negishi, p.94.



In status quo, quantity is given at A and the firm breaks even with price equal to average cost at A. The firm does not expect the price to change if less than B is delivered to the market, and in the modern view of perfect competition, the firm would also expect the price to be unchanged if it delivers more than B, something which would make the market break. But in Smith's view, the firm will know that there are competitors out there, and it will expect them to act more or less in a similar way, so the price will drop if the quantity is increased.

Extending the market. With the views on competition and division of labour as above, it is natural that Smith should point to the size of the market as one of the important preconditions for economic growth. From the point of view of technology, a firm must have a certain size in order to exploit the division of labour, and to be able to profit from the increase in production, there must be sufficiently many buyers so that the production can be increased. As a consequence, countries with a large population have a larger growth potential than small countries.

One of the ways in which the market can be extended is by involving buyers of other countries. Therefore international trade is seen as generally beneficial for the exporter, but Smith does not consider the situation from the point of view of the other country.

References

Bréban, L. and A.Lapidus (2019), Adam Smith on lotteries: an interpretation and formal restatement, The European Journal of the History of Economic Thought 26, 157 – 197.

Hahn, F. (1978), On Non-Walrasian Equilibria, The Review of Economic Studies 45, 1 – 17.

Isard, W. (1956), Location and Space-economy; a General Theory Relating to Industrial Location, Market Areas, Land Use, Trade, and Urban Structure, Wiley.

Negishi, T, History of Economic Theory, North-Holland, 1989.

Samuelson, P.A. (1977), A modern theorist's vindication of Adam Smith, American Economic Review 67, 42 – 49.