Lecture Note 16:
Neoclassical growth models, the Cambridge controversy
National accounts

The neoclassical growth model. It was natural that the Harrod-Domar model, which introduced the topic of growth as a continuation of the Keynesian analysis of the short run, looking at the capacity effects of investment as well as its effect on aggregate demand here and now, should be extended by including capital explicitly into the analysis.

The extension achieved gave rise to the *Solow-Swan* model of economic growth, which was published independently by **Robert Solow** (1924 –) and **Trevor Swan** (1918 – 89) in 1956. The model is by now a standard ingredient in the textbook treatment of growth, and that it is called the Solow model is only due to the fact that Swan was sitting in out-of-the-way Australia.

The new ingredient in this model is the macro production function

$$Y = F(K, L)$$

which is homogeneous of degree one, so that one may define per a capita production function

$$y = f(k)$$

using the notation $y = \frac{Y}{L}$, $k = \frac{K}{L}$, and f is given by $f(k) = F(k, 1)$. The standard properties of F (considered as an ordinary production function with two inputs and one output) translates to properties of f, in particular $f'(k) = F_1'(k)$ is decreasing in k.

Letting y and k be functions of (continuous) time, and adding the savings condition

$$k'(t) = sy(t) - \delta k(t)$$

(investment equals savings in shortrun equilibrium, and investment is used to replace worn-out capital at rate δ and to increase the capital stock). On a steady state growth path, the capital-labour ratio will be constant, so that $k'(t) = 0$, and we get that

$$\frac{k(t)}{y(t)} = \frac{K(t)}{Y(t)} = \frac{s}{\delta}.$$

It is seen that if the capital-output ratio exceeds s/δ, then net investment per worker becomes negative, and if it falls short of s/δ, there will be positive net investment, so we get rid of knife-edge problems.

The model can be (and immediately was) extended to allow for per capita growth,

replacing inputs with "efficiency" versions to model technological progress, or by inserting human capital as an independent production factor. Using the production function one may draw on the neoclassical approach to factor pricing, determining relative shares of wage and income from capital.

The Cambridge controversy. Shortly after the appearance of the Solow-Swan model, it became the starting point of a protracted academic debate, initiated by a group of economists at the University of Cambridge, notably Joan Robinson (see previous comments), **Piero Sraffa** (1898 – 1983), **Luigi Pasinetti** (1930 –), and the American economists Robert Solow (1924 –) and **Paul Samuelson** (1915 – 2009) at MIT, situated in Cambridge, Massachusetts (just outside Boston).

The point of departure was the macro-production function, as can be seen from the following (often cited) citation from Robinson (1953 – 54):

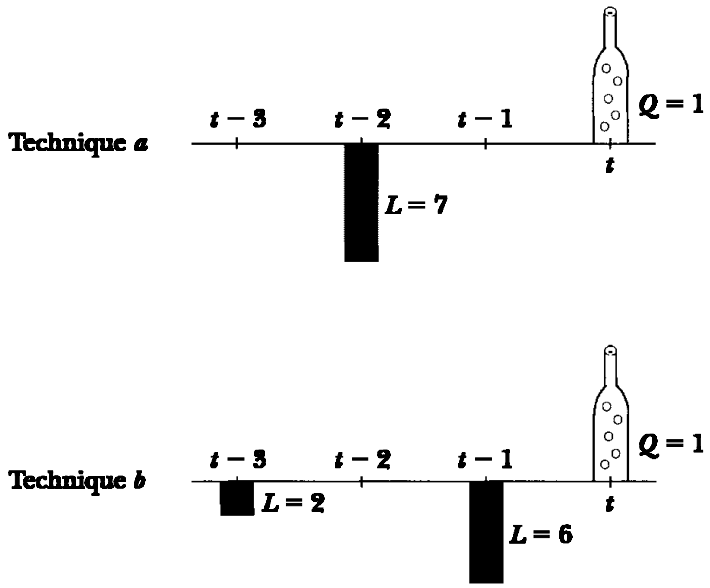
> . . . the production function has been a powerful instrument of miseducation. The student of economic theory is taught to write $Q = f(L, K)$ where L is a quantity of labor, K a quantity of capital and Q a rate of output of commodities. He is instructed to assume all workers alike, and to measure L in man-hours of labor; he is told something about the index-number problem in choosing a unit of output; and then he is hurried on to the next question, in the hope that he will forget to ask in what units K is measured. Before he ever does ask, he has become a professor, and so sloppy habits of thought are handed on from one generation to the next.

The basic problem is whether capital as such is a meaningful concept, and it has roots back in the history of economics. When utility was introduced into the price or value theory by the marginalists, their point of view was that value of consumption goods must be derived from consumers' utility of consumption, and after that value of production factors can be derived from the value of output. But if the output can be produced in several different ways, then this process may not be uniquely defined, so that remuneration of factors according to value of marginal product can have many solutions. In the neoclassical growth model, remuneration of capital is considered as an interest rate, and this takes us back to problems raised by Böhm-Bawerk and (in particular) Wicksell, the amount of capital used cannot be determined without knowing the interest rate, which in its turn should be determined as marginal product of capital.

The controversies became focussed on the problem of *reswitching*, a phenomenon noticed by Sraffa, who had studied Ricardo's work and from this established his own, neo-Ricardian economic theory (see the upcoming Lecture Note 17 for details). While the neo-Ricardians (Sraffa, Pasinetti and others) had little impact on other fields of economics, the reswitching debate continued for decades. Sraffa had shown in his model that the choice of technique in production may depend on factor prices in a non-monotonous way, so that a technique which is abandoned when the relative factor price increases may come back at even higher factor prices, and this is rather devastating for a useful theory of factor price remuneration. After some initial discus-

sions, the neoclassical side agreed that reswitching can indeed occur, and Samuelson gave a simple example, reproduced in Figure 1 (which is taken from Cohen and Harcourt (2003)).

Figure 1
Samuelson's (1966) Example of Wicksell Effects in a Simple Austrian Model



The cost equations are:
Technique a $7L(1 + r)^2$
Technique b $2L(1 + r)^3 + 6L(1 + r)$
When comparing costs, L cancels out for both techniques. Switchpoints occur when the costs are equal.

Technique *a* is less costly than *a* when $r < 50\%$ *and again* when $r > 100\%$, but for $50\% < r < 100\%$ technique b is the cheepest. It is also seen that *a* uses less labout than *b* so that it is more capital intensive.

The neoclassical response to what could be considered a fundamental blow to their theoretical construction (at least for the English Cambridgeans, it was seen as such) was that even if capital reswitching can occur, it does so only in specifically constructed cases, so that it is reasonably safe to go on as if it could never happen.

Paul Samuelson (1915 – 2009) contributed to many different parts of economic theory. He had a lasting influence on the way in which economics is taught at universities worldwide through his textbook *Economics,* the first edition of which appeared in 1948, after which it went through 19 reeditions (publishers insist on new editions to reduce the sale of used books, the last one in 2009. It set the style for introductory texts which has been followed in newer texts.

The book "Foundations of economic analysis" from 1947 was seen by many contemporaries as a path-breaking new approach to economics, emphasizing the role of constrained maximization as the basic model of behavior, but it has had limited impact on subsequent research. Also it marks the end of an era where new thoughts appeared in books. Most of the important contributions of Samuelsen (as of other researchers nowaday) appear in journal articles.

Samuelson initiated an alternative approach to consumer theory with what is called *revealed preferencs.* Instead of using the unobservable utilities, one might take the demand behavior as the primitive concept, and then check what can be derived from this given that it has some minimal consistency properties (formulated by Samuelsen as the *weak axiom:* if a bundles is revealed preferred to another bundle (it was bought in a price-income situation where the other bundle could have been bought), then this other bundles cannot be revealed preferred to the first one. It was later shown that with a somewhat stronger version (the strong axiom) demand behavior will be as if derived from a utility.

In the late fifties Samuelson introduced the *generation model* for considering problems of money and monetary policy. It has become a standard tool in modern equilibrium analysis. Also his contributions to international trade theory are worth mentioning, from the Stolper-Samuelson theorem (introduction of a tariff on a commodity will be useful for the production factor used intensively in its production) to the factor price equalization theorem and the reformulation of the Heckscher-Ohlin model of international trade (**Eli Filip Heckscher** (1879 – 1952) was economic historian and inspired by a study of mercantilism, **Bertil Gothard Ohlin** (1899 – 1979) had studied in Harvard and with Keynes).

National Accounts has a history which can be traced back to William Petty and the French mercantilists, but in the way in which we know it today, the System of National Accounts (abbreviated SNA) was initiated in the 1920s and 1930s with contributions of **Morris Albert Copeland** (1895 – 1989) in USA, and **Ragnar Frisch** (1895 – 1973) in Norway, both pointing to the use of double-entry accounting as the basis. This kind of work acquired official status after the 1929 crisis, and the conceptual background was reinforced by the Keynesian approach to macroeconomics, focussing on consumption, investment and saving as fundamental concepts. Work on establishing national income data was undertaken in many countries during the 1930s, in Sweden by the group around Lindahl, in the Netherlands by Jan Tinbergen.

The real breakthrough occurred in the years after 1945, as a result of work initiated during and after the war, the main driving force being **James Meade** (1907 – 1995) and **Richard Stone** (1913 – 1991) in England. An important stepping stone was establishing international standards for setting up national accounts, prepared by Richard Stone in 1945.

An alternative approach, parallel to the one based on accounting, had been de-

veloped in the USA by **Wassily Leontief** (1905 – 1999) in the form of **input-output analysis**. Originally designed as a simplified version of the Walrasian general equilibrium, adapted to the use of empirical data, it turned into a statistical method for measuring the inter-industrial flows in an economy. The development of input-output models was intensified in the US during the war but subsequently largely abandoned here, whereas it was taken up and continued in other countries.

The welfare theoretical meaning of national income has given rise to some debate. The national accounts display production and consumption in value terms, but clearly do not indicate utlity levels. If national income increases from one period to the next, this does not indicate that society's welfare has become larger in the sense of Pareto, indeed some individuals may have experienced a decrease in income and utility. The best one can hope for is a *potential* Pareto improvement in the sense that goods and services could have been redistributed in such a way that everybody had an increase in utility. This is the so-called *Kaldor criterion* for a welfare improvement ("winners can bribe losers"), after **Nicholas Kaldor** (1908 – 1986) (macroeconomist working on business cycles). But if the change in national income is not marginal, a problem of changing equilibrium prices arises. In the academic discussion, Kaldor's compensation criterion was met with the symmetric *Hicks criterion* (losers cannot bribe the winners to refrain from the change), and even a combination (the Scitovsky criterion proposed by **Tibor de Scitovsky** (1910 – 2002), will not guarantee that a welfare gain is attained.

We read: B&D Chapters 3 and 4, V&G Ch.28, pp.278–281.

References:

Robinson, J. (1953-54), The production function and the theory of capital, The Review of Economic Studies 21, 81 – 106.

Cohen, A.J. and G.C.Harcourt (2003), Whatever happened to the Cambridge capital theory controversies? Journal of Economic Perspectives 17, 199 – 214