Technical Change and the Distribution of Firm Growth*

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Abstract

We examine the effects of aggregate technology shocks on U.S. quoted companies' growth rates of real sales. A shock to firm technology does not simply translate into a mean-preserving spread in the density of firm growth, but it reflects a marked reallocation of probability mass over the growth domain. This fact has major implications for the amplification of shocks 'in the aggregate'. We also highlight some distinctive features of the transmission of consumption-specific and investment-specific technology improvements to the cross section of company growth rates.

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1 Introduction

Over the last two decades increasing interest has been devoted to understanding how microeconomic decisions affect the macroeconomy. Caballero (1992) has argued that probability theory imposes strong restrictions on the joint behavior of a large number of units that are less than fully synchronized. More recently, a number of authors have acknowledged the importance of tracking the business cycle behavior of firm-level dispersion over several economic dimensions, such as investment, sales growth, productivity and price-setting. Complementing the study of major macroeconomic aggregates with the analysis of the business cycle from the cross section has proven to be an important disciplining device for heterogeneous firm models (see, e.g., Bachmann and Bayer, 2011). This paper examines time-variation in the distribution of firm growth and its implications for the propagation of aggregate shocks to firm technology. We do so by estimating the quantiles of U.S. quoted companies' growth rates of real sales, conditioning them on both firm-specific characteristics and alternative measures of technical change. Unlike the approach followed so far – which focuses on a restricted subset of empirical moments – we characterize the cyclical behavior of the entire density of firm growth, as well as its response to shocks that are commonly regarded as important drivers of the business cycle. According to our evidence, shifts and contortions in the density of firm growth play a crucial role in shaping macroeconomic fluctuations, making a strong case for business cycle models that emphasize the role of microeconomic adjustment for aggregate dynamics (e.g., Caballero *et al.*, 1995, Caballero and Engel, 1999, Bachmann *et al.*, 2013).

Historically, a great deal of attention has been devoted to exploring the static properties of the distributions of firm size and growth. A number of theoretical and empirical contributions have focused on the assessment of the theoretical proposition known as Gibrat's law (Gibrat, 1931), which predicts randomness of firm growth rates.[1] Until recently, this literature has never taken a business cycle standpoint, so as to examine the role of firm-level size and growth in the genesis and propagation of business fluctuations. In this respect, Gabaix (2011) represents a remarkable exception, as he shows how idiosyncratic shocks to a handful of large firms explain a non-trivial fraction of aggregate fluctuations if the distribution of firm size is fat-tailed and the central limit theorem breaks down. The present paper takes a different path, exploring the transmission of aggregate technology shocks to the entire cross-section of firms in the COMPUSTAT dataset. In doing so, we acknowledge the importance of allowing for asymmetric time-variation in the density of company growth rates, as it is warranted by recent findings of Holly, Petrella, and Santoro (2013). According to this study, systematic changes in the density display leading properties with respect to the business cycle, so that shifts in the probability mass may propagate and amplify macroeconomic fluctuations, as originally hinted by Caballero (1992) and Caballero and Engel (1992, 1993).

We appreciate marked heterogeneity in the response of firms to productivity shocks. Technical change induces a divergence between the median and the mean growth rate, with the latter displaying greater responsiveness. The resulting swing in the skewness reflects a substantial reallocation of probability mass on either side of the density, so that aggregate disturbances do not simply reflect into a mean-preserving spread. We also highlight sizeable differences in the cross-sectional responsiveness to changes in consumption goods technology and investment goods technology. From an aggregate viewpoint,

we report evidence in line with Basu, Fernald, Fisher, and Kimball (2013), who show that consumption-technology improvements tend to be expansionary, while investment-technology improvements are contractionary at short horizons. Conditional quantiles help interpreting these facts. While a technological advance in the production of consumption goods initially tends to favor a wider group of firms, positive shocks to investment goods technology only benefit a relatively small part of the population of firms – those which perform the best. Such differences point to the existence of implementation lags and fixed costs entailed in the adoption of a new technology for the production of investment goods.

A large body of theoretical and empirical literature is expanding around the analysis of the business cycle from the cross section.[2] In this respect, the importance of our results traces back to the core implications for designing and validating heterogeneous firm business cycle models. From an empirical viewpoint, we show that resorting to methods that go "beyond the mean" may unveil relevant information when firm-level heterogeneity is pervasive and matters for the transmission of aggregate disturbances. We demonstrate that neither location shifts nor scale shifts (i.e., changes in the degree of dispersion) of the density of firm growth play a key role in the amplification of technology shocks. By contrast, technology shocks may produce major aggregate effects to the extent they are capable of inverting the asymmetry of the density of firm growth. Formulating models that emphasize the role of heterogeneity in amplifying and propagating aggregate technology shocks should start from this fact.

The remainder of the paper is laid out as follows: Section 2 introduces the quantile regression framework and reports some preliminary evidence on the dynamics of the density of firm growth; Section 3 explores the transmission of alternative measures of

technical change on the cross-section of firm growth and the associated aggregate dynamics; Section 4 discusses the implications of our results for the theoretical literature on heterogeneous firm business cycle models; Section 5 concludes.

2 Quantile Regression Analysis

The ultimate scope of our study is to understand whether changes in the density may propagate and amplify technology shocks. To address this task, estimation methods that "go beyond the mean" have to be used. In fact, there is no reason to anticipate that the marginal effects of certain covariates on the shape of the density are invariant over the spectrum of growth. Conditional quantile regressions have become increasingly popular and may usefully serve our purpose (Koenker and Bassett, 1978 and Koenker, 2005). Quantile regressions are especially useful when dealing with non-identically distributed data. In these situations, one should expect to observe significant discrepancies in the estimated 'slopes' at different quantiles with respect a given set of covariates (Machado and Mata, 2000). Such discrepancies may reflect not just into shifts of the density location, but also into scale shifts (i.e., changes in the degree of dispersion) and/or asymmetry reversals (i.e., changes in the sign of the skewness).

Let $\tau \in (0,1)$. The τth quantile of the distribution of a generic variable y, given a vector of covariates $\mathbf{x}$, is:

$$Q_\tau(y|\mathbf{x}) = \inf\{y|\, F(y|\,\mathbf{x}) \geq \tau\}, \tag{1}$$

where $F(y|\,\mathbf{x})$ denotes the conditional distribution function. A least squares estimator of the mean regression model would be concerned with the dependence of the conditional

mean of y on the covariates. The quantile regression estimator tackles this issue at each quantile. In other words, instead of assuming that covariates shift only the location or the scale of the density, quantile regression looks at the potential effects on the whole shape of the distribution. The statistical model we opt for specifies the τth conditional quantile of firms' growth rate as a linear function of the vector of covariates, $\mathbf{x}_{it}$, as well as time effects, $\gamma_{t,\tau}$:[3]

$$Q_\tau\left(g_{it}|\gamma_{t,\tau},\mathbf{x}_{it}\right)=\gamma_{t,\tau}+\mathbf{x}_{it}'\boldsymbol{\beta}_\tau,\ \tau\in(0,1)\,, \tag{2}$$

where g_{it} is the growth rate of the ith firm. As discussed by Koenker (2005), the marginal change in the jth element of $\mathbf{x}$ produces a marginal change in the τth quantile that does not represent the impact of the covariate of interest on the quantiles of the unconditional distribution of firm growth. Quantile estimation is influenced only by the local behavior of the conditional distribution of the response near a given quantile. Therefore, no parametric form of the error distribution is assumed. Estimates depend on the signs of the residuals: outliers in the values of the response variables influence the model's fit to the extent that they are above or below the fitted hyperplane.

2.1 Data and Preliminary Evidence

We employ annual accounting COMPUSTAT data over the 1950-2010 period. Nominal sales are deflated by the GDP deflator.[4] The resulting measure of real sales is taken as a proxy for firm size, which is denoted by s_{it}.[5] We then compute firm i's growth rate as $g_{it}\equiv(s_{it}-s_{it-1})\,/\,[(s_{it}+s_{it-1})/2]$.[6] This definition has become standard, as it shares some useful properties of log differences and has the advantage of accommodating entry and exit (Haltiwanger *et al.*, 2013).[7]

Holly, Petrella, and Santoro (2013) have extensively reported that the empirical distribution of growth rates in the US displays shifts and contortions that are correlated with the business cycle.[8] Table 1 summarizes their key findings, reporting different measures of co-movement between the sample moments[9] and the rate of growth of real GDP. All statistics show that standard deviation and skewness behave counter-cyclically, while kurtosis follows a marked pro-cyclical pattern. These features have been originally documented by Higson *et al.* (2002, 2004)[10] and may be usefully summarized in Figure 1, where we sketch the typical shape of the density during contractions and expansions in economic activity. An economic slowdown generally translates into a density that shifts to the left and a relative increase in the probability mass on the left-hand side of the mode (LHS henceforth). From a visual viewpoint the right-hand side of the resulting density (RHS henceforth) assumes a characteristic tent shape, which is typical of Laplace benchmark. By contrast, the LHS is more bell-shaped and resembles a Gaussian density. This picture reverses during expansions, though we appreciate lower dispersion about the modal rate of growth, as compared with contractions. Section 4 discusses the implications of these properties for the transmission of productivity shocks.

Insert Table 1 here

Insert Figure 1 here

Figure 2 shows the time path of the distribution quantiles. The visual inspection confirms that heterogeneity is pervasive at business cycle frequencies. This tendency is clearly at odds with the view that aggregate fluctuations must reflect a mean-preserving spread in the density function, which would instead imply all quantiles displaying the same cyclical behavior. The density has also become more sparse over time, as documented by Comin and Philippon (2006) and Comin and Mulani (2006), among others.

However, a key aspect may be appreciated: increasing dispersion emerges as a phenomenon that primarily hinges on the evolution of firms in the tails of the distribution, while the interquantile range displays very moderate trending behavior. This emphasizes the importance of employing quantile-based techniques to deal with the cross-sectional dynamics of firm growth, so as disentangle the heterogeneous behavior of different parts of the density.

Insert Figure 2 here

2.2 Time-variation and Firm Growth

The preliminary analysis imposes us to account for both the cyclicality of higher moments and trending dispersion of the distribution of firm growth. In light of this, we first specify a quantile regression framework that aims at providing deeper insights into the behavior of the cross section of firm growth at both business cycle and secular frequencies. To this end we include a business cycle indicator (Δy_t) and a time trend (t), along with firm-specific (lagged) size and age. This amounts to set $\gamma_{t,\tau} = \alpha_\tau \Delta y_t + \delta_\tau t$ in (2). The resulting framework generalizes the first order Galton–Markov model that has traditionally been used to explore the relationship between firm size and growth in the context of empirical tests of Gibrat's law.[11]

Insert Figure 3 here

Figure 3 graphs the estimates of α_τ and δ_τ.[12] The quantile treatment effect (QTE hereafter) associated with the time trend is symmetric, though it is not centered at zero. This pattern is typical of a location and scale shift of the distribution. According to this picture, not only dispersion increases over time, but also the median growth rate does,

though at a very small pace. This finding may be seen as providing indirect support to Davis and Kahn (2008) and Davis, Faberman, Haltiwanger, Jarmin, and Miranda (2010), according to whom upward trending dispersion in the distribution of public companies might be driven by a marked shift in the selection of publicly traded firms occurred in the early 1980s. In fact, the secular pattern of the median growth rate is compatible with including in the sample relatively small but rapidly-growing companies.

Some important aspects emerge from inspecting the QTE associated with GDP growth. Overall, the QTE displays a marked U-shaped pattern, confirming that skewness is negatively correlated with the cycle. Most importantly, lower quantiles are the ones that react the most to marginal changes in the GDP, as compared with the right tail of the density. This is consistent with the prediction that, during contractionary episodes, dispersion mostly increases on the LHS, as exemplified in Figure 1 and previously reported by Holly, Petrella, and Santoro (2013).

The key element we retrieve from this picture is that the business cycle primarily acts as a treatment capable of inverting the skewness of the density. In turn, countercyclical scale shifts are the manifestation of changes in the skewness being themselves asymmetric over contractions and expansions, with the poor end of the density displaying greater responsiveness to macroeconomic fluctuations. These aspects certainly deserve closer attention. In fact, the literature on heterogeneous firm models has fundamentally underestimated the role of higher moments in the transmission of aggregate disturbances, while focusing almost exclusively on the cyclical behavior of the cross-sectional dispersion over several dimensions of firm-level activity. To dig deeper on these aspects, the next section explores the reaction of different growth quantiles to aggregate shocks that have been classically considered as potential drivers of macroeconomic fluctuations.

3 The Transmission of Technology Shocks

The analysis so far has revealed varying degrees of co-movement between different parts of the distribution of firm growth and the business cycle. As it stands, this picture does not tell us much as to whether changes in the density may influence aggregate dynamics, or whether such movements are to be seen as simple cross-sectional projections of the business cycle. The next step in the analysis explicitly addresses these issues by exploring the transmission of structural shocks onto the cross-section of firm growth and, in turn, aggregate dynamics. To this end, we make use of local projections as indicated by Jorda (2005). This represents a very convenient methodology in our setting, as it does not require specifying a model and extrapolating responses from increasingly distant horizons. The response to a generic shock can directly be computed from predictive regressions. Specifically, we estimate the following quantile regressions, one at each horizon h:

$$Q_\tau\left(g_{it+h}|\mathbf{z}_{it}\right) = \mathbf{x}_{it}'\boldsymbol{\beta}_{h,\tau} + \phi_{h,\tau}v_t,\ \tau \in (0,1)\,,\quad h = 0,1,...,H, \tag{3}$$

where $Q_\tau\left(g_{it+h}|\mathbf{z}_{it}\right)$ denotes the time $t+h$ quantile estimate, conditional on $\mathbf{z}_{it} = [\mathbf{x}_{it}', v_t]$, with x_{it} denoting a generic set of covariates[13] and $v_t \sim iid\left(0, \sigma_v^2\right)$ representing the shock of interest. Following Jorda (2005), we can compute the impulse response function for the τth quantile as the difference between two conditional forecasts:

$$IRF_\tau\left(t, h, \sigma_\nu\right) = \mathbf{E}\left[Q_\tau\left(g_{it+h}|\mathbf{z}_{it}\right)|v_t = \sigma_\nu\right] - \mathbf{E}\left[Q_\tau\left(g_{it+h}|\mathbf{z}_{it}\right)|v_t = 0\right], \tag{4}$$

where we have implicitly assumed a one-standard deviation shock. Since the forecasts are directly computed from the predictive regression (3), at each h we can compute the impulse response function as $IRF_\tau\left(t, h, \sigma_\nu\right) = \phi_{h,\tau}\sigma_\nu$. This methodology allows us to

retrieve the response of the entire cross-section of firm growth. Thus, we can appreciate whether the shock is uniformely transmitted to the entire distribution – in which case we would assist to a simple location shift – or it affects the shape of the distribution. In the second case we could face two scenarios: the shock might just translate into a change in the scale of the density – in which case the conditional distribution would remain symmetric – or it might also affect its skewness. As we discuss in Section 3.1, the second scenario may have major implications for aggregate dynamics.

We consider alternative measures of technical change as computed by Fernald (2012).[14] Technology shocks are retrieved from adjusting the Total Factor Productivity (TFP henceforth) for factor utilization, as indicated by Basu, Fernald, and Kimball (2006). The series are then decomposed into utilization-adjusted TFP series for equipment investment, denoted by TFP_t^I, and consumption (intended as everything other than equipment investment and consumer durables), which is denoted by TFP_t^C. Figure 5 reports the QTE at each period after the shock has occurred. The upper panel graphs the responses to a TFP_t^C shock. Overall, we detect strong cross-sectional heterogeneity. A first striking finding is that, on impact (i.e., $h = 0$), the QTE is negative for the first few quantiles, while the others only display moderately positive responses. The reaction of lower quantiles gradually increases after the initial shock, implying that the response at the lower end of the distribution of firm growth takes some time to build up. As time goes by, lower quantiles are the ones benefiting the most from the technological advance. At $h = 2$ the QTE implies an asymmetry reversal: it is in this period that the shock exerts the strongest impact, with the tails denoting much stronger reactiveness, as compared with the central part of the density. Over the last two periods the lower end of the density is still the most reactive, but the shock gradually absorbs, and so dispersion

does, as signalled by the fact that the QTE mostly lies in the negative quadrant.

The responses to TFP_t^I are reported in the bottom panel of Figure 5. A crucial difference with the case of consumption-technology improvements is that "good performers" are the ones that initially benefit the most from the technological impulse. This reflects the potential existence of implementation lags and fixed costs entailed by the adoption of the new technology. As in the case of a TFP_t^C shock, the overall density reaches its peak response when asymmetry reverts, at $h = 2$. At this point the upper end of the density is still the most responsive. Otherwise, from $h = 3$ onwards the maximum response is attained at the poor end. This signals that relatively worse performers take much longer to pick up an investment-specific technological advance, while they are more responsive to shocks that do not entail major adjustments in the rate of capital utilization.

Insert Figure 5 here

3.1 Implications for Aggregate Dynamics

Section 3 has shown that the business cycle acts as a treatment capable of inverting the asymmetry of the distribution of firm growth. Also technology shocks have the power of changing the sign of the skewness and, importantly, they do so when the overall cross-sectional response reaches its peak after the initial impulse has taken place. In light of this, it seems relevant to pose the following question: once we look at the aggregate, do asymmetry reversals play any role in amplifying technology shocks? To address this point, we first need to define an overall measure of the density response to technical change. In this respect, the 'average' impulse response function appears as the most appropriate statistics:

$$\overline{IRF}\left(t, h, \sigma_\nu\right) = \int IRF\left(t, h, \sigma_\nu\right) f\left(g_{t+h}\right) dg_{t+h}. \tag{5}$$

The latter can be conveniently computed as $N^{-1}\sum_{\tau=1}^{N}\phi_{h,\tau}\sigma_{\nu}$, where N denotes the number of bins between the 5th and the 95th quantile. In turn, if we denote with $\phi_{h,50}$ the treatment effect associated with the median quantile, we can decompose the average response into $N^{-1}\left(\sum_{\tau=1,\tau\neq 50}^{N}\phi_{h,\tau}+\phi_{h,50}\right)\sigma_{\nu}$. Thus, it is immediate to derive the following condition:

$$IRF_{50}\left(t,h,\sigma_{\nu}\right)\gtreqless\overline{IRF}\left(t,h,\sigma_{\nu}\right)\Leftrightarrow\phi_{h,50}\gtreqless\bar{\phi}_{h,-50}, \tag{6}$$

where $IRF_{50}\left(t,h,\sigma_{\nu}\right)$ is the impulse response function associated with the median, while $\bar{\phi}_{h,-50}\equiv\left(N-1\right)^{-1}\sum_{\tau=1,\tau\neq 50}^{N}\phi_{h,\tau}$. According to (6) the mean response is greater than the median one whenever $\phi_{h,50}$ is smaller than the average of all other treatments. This condition allows us to compare the overall response of the density with a correct measure of central tendency, while keeping track of the degree of skewness.

Both $\bar{\phi}_{h,-50}$ and $\phi_{h,50}$ are reported in each panel of Figure 5: importantly, in the first few periods after the shock has occurred the inequality $\bar{\phi}_{h,-50}>\phi_{h,50}$ consistently holds true. In particular, the median response tends to lie well below $\bar{\phi}_{h,-50}$ when the QTE implies an asymmetry reversal, due to the tails of the density displaying much greater responsiveness. Figure 6 confirms that treatment effects that are capable of altering the asymmetry of the distribution imply a substantial amplification of the mean response, as compared with the median one. It must be stressed that scale shifts are not crucial to this result. In fact, the amplification of the mean response relative to the median one could also be appreciated with a symmetric (yet U-shaped) QTE. Note also that greater swings of the mean growth rate in the presence of asymmetry reversals are necessarily compatible with the mean lying at the left (right) of the median during contractions (expansions). In fact, the rule of thumb according to which positive (negative) skewness implies a mean lying at the right (left) of the median is often violated in the case under scrutiny. This

is due to the skewed part of the density being highly leptokurtic, as compared with its counterpart on the other side of the mode.[15]

Figure 6 also retains important information about differences in the effect of changes consumption goods technology and investment goods technology. As to TFP_t^C, both the mean and the median rate feature a positive reaction in the first three years after the shock occurs. By contrast, TFP_t^I induces a negative reaction a year after the shock hits. Both findings are in line with the evidence of Basu, Fernald, Fisher, and Kimball (2013), who show that investment-technology improvements are contractionary at short horizons, while consumption-technology improvements tend to be expansionary. It is also important to note that contractionary movements in the mean/median growth rate that follow investment-technology improvements are not due to changes in the asymmetry of the distribution. In fact, Figure 5 shows that at $h = 1$ the density only presents a location and scale shift: in fact, the mean and the median response tend to overlap in this period, implying that there are no changes in the degree of skewness. This amounts to say that the time $h = 1$ contraction in Figure 6 is predominantly driven by both an increase in the dispersion in the growth performance and a shift of the entire distribution towards the left over the growth spectrum. However, no major reallocation of the probability mass from one side of the distribution to the other takes place, as changes in investment goods technology initially tend to favor only a small part of the population of firms, those that perform the best.

Insert Figure 6 here

4 Discussion

Over the last two decades the business cycle literature has been seeking for alternative forms of non-linear micro adjustment that, combined with micro-level heterogeneity, may be relevant to aggregate outcomes. The basic premise of these contributions is that firm-level heterogeneity in terms of output, employment and investment implies a large, continuous pace of reallocation of real activity across production sites. In turn, such an adjustment process may involve substantial frictions, so that the ultimate impact of an aggregate shock depends on the location of individual firms with respect to their adjustment thresholds, which determines time-varying elasticities of macroeconomic aggregates to aggregate shocks (King and Thomas, 2006). Under such circumstances representative agent frameworks necessarily suffer from a "fallacy of composition", as they do not distinguish between statements that are valid at the individual level and those that only apply to the aggregate (Caballero, 1992). Heterogeneous firm models have emerged to address these issues. Nevertheless, a clear consensus on the relevance of microeconomic decisions for the aggregate economy is far from being reached. To give a quick account of how the debate has evolved around these issues, we find it indicative to focus on firm-level investment. After a first generation of partial equilibrium models that have supported the importance of lumpy investment for the macroeconomy (Caballero *et al.*, 1995, Caballero and Engel, 1999), Thomas (2002), Veracierto (2002) and Kahn and Thomas (2003, 2008) have shown that, in a general equilibrium setting, investment lumpiness is irrelevant to the cyclical properties of aggregate dynamics. More recently, this view has been questioned by Bachmann, Caballero, and Engel (2013) upon methodological grounds that mark the distinction between partial and general equilibrium components of the impact of aggregate shocks on aggregate endogenous variables (investment, in the specific case

under scrutiny).

Regardless of the specific structure of the model economy, our study makes a strong case for business cycle frameworks that emphasize the importance of microeconomic adjustment for aggregate dynamics. We go even further, indicating that non-convexities and lumpy adjustment at different margins of firm-level decisions should be tailored on some specific cross-sectional criteria. In fact, our evidence suggests that technical change should not simply induce a mean-preserving spread in the density function, nor do scale shifts play a major role in propagating and amplifying technology shocks. By contrast, mechanisms that are capable of inverting the skewness of the density are to be seen as promising avenues to impose sound empirical restrictions on heterogeneous firm models. So far plant-level dispersion over several domains of firm activity has represented a key disciplining device. However, replicating the cyclical behavior of firm growth volatility – mostly in the form of counter-cyclical scale shifts – does not ensure *per se* a powerful propagation and amplification of technology shocks. Furthermore, we should stress that asymmetry reversals are likely to enhance the capacity of heterogeneous firm models embedding non-convexities along different margins of plant-level activity to generate counter-cyclical volatility of gross production (see, e.g., Šustek, 2011). This should be seen as a promising avenue to reproduce non-trivial business cycle asymmetries (see, among others, Neftci, 1984; Hamilton, 1989; Sichel, 1993; Morley and Piger, 2012).

A final word is due on the interaction of aggregate shocks with the moments of the cross-sectional distribution. In this respect, Caballero and Engel (1993) have formulated increasing-hazard models where larger variance leads to larger responses of aggregate employment to aggregate shocks, due to direct interaction. The intuition behind this result is that more weight on the tails of the distribution reflects higher average hazard,

so that the fraction of firms that hire workers is proportionally larger (and so the one that fire workers) when the shock is large. There is a close connection between this property of partial adjustment frameworks and the behavior of conditional quantiles. Asymmetry reversals imply higher responsiveness of the tails, regardless of the size of the shock. Therefore, more weight on the tails of the density means greater reallocation of probability mass following an aggregate technology shock, due to a non-zero net flow of production units from one hand of the distribution to the other.

5 Concluding Remarks

Recent years have borne witness to the development of various heterogeneous agent frameworks whose main goal is to understand whether the dynamics of major macroeconomic aggregates is non-trivially affected by the decisions of different microeconomic actors. At the firm-level, a number of researchers have regarded higher moments of company growth rates as important elements to discipline and validate business cycle models. This paper has combined quantile regression techniques with projection methods to show that shifts and contortions in the density of firm growth of real sales matter for the transmission of aggregate disturbances. Projection methods allow us to extrapolate the responses of each quantile of firm growth to different sources of technical change, so that we characterize the behavior of the entire density in the face of aggregate perturbations to firm technology. The analysis highlights a deep connection between systematic changes in the skewness and the amplification of aggregate disturbances, as well as distinctive traits in the transmission of shocks to consumption goods technology and investment goods technology. The formulation of heterogeneous firm models that aim at describing business cycle dynamics should account for these facts.

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Notes

[1]See Sutton (1997) for a comprehensive review of this literature.

[2]See Bachmann and Bayer (2011), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) and Bachmann, Caballero, and Engel (2013), among others.

[3]Ideally, one would prefer to implement quantile panel regressions, allowing for both firm-specific and time effects (see, e.g., Powell, 2010). However, this is computationally demanding, even in the presence of a limited number of covariates. Appendix B shows that our estimates are robust to the exclusion of firm-specific effects, comparing Powell's panel estimates with those from a quantile regression with pooled data. We conclude that alternative assumptions about the error structure are of second order importance in the present context.

[4]Distante, Petrella, and Santoro (2013) show that employing sectoral price deflators leads to virtually unchanged results.

[5]Various measures – including the value of assets of a firm, employment and sales – have been traditionally used to proxy firm size. Where data have been available for the various measures the results have generally been invariant to the measure of size (see Evans, 1987a and Hall, 1987).

[6]We remove firms growing (declining) beyond a 100% rate. Replicating the analysis with growth rates defined as log-differences or under alternative cut-off intervals does not qualitatively affect the analysis.

[7]Along with being symmetric around zero and bounded between -2 (exit) and 2 (entry), this growth rate represents a second order approximation of the log-difference for growth rates.

[8]The p-values of the Kolmogorov-Smirnov, Cramer-von Mises and Kuiper tests all

agree on indicating that the distribution is not time invariant.

[9]These are computed from the quantiles of the distribution of firm growth. Quantile-based statistics are typically seen as more robust than sample moments in the presence of outliers (see, e.g., Pearson, 1895). The estimation of conditional quantiles is detailed in the next section.

[10]DeVeirman and Levin (2011) report analogous evidence on the relationship between firm growth volatility and the business cycle.

[11]See Sutton (1997) for a comprehensive review of the literature.

[12]Distante, Petrella, and Santoro (2013) focus on the impact of size and age on firm growth. Firm size generally acts as a scale shifter of the distribution, while age emerges as a location and scale shifter. In the present context the cross-sectional impact of firm size and age is robust to the inclusion of time effects. Appendix B reports the QTE associated with both regressors.

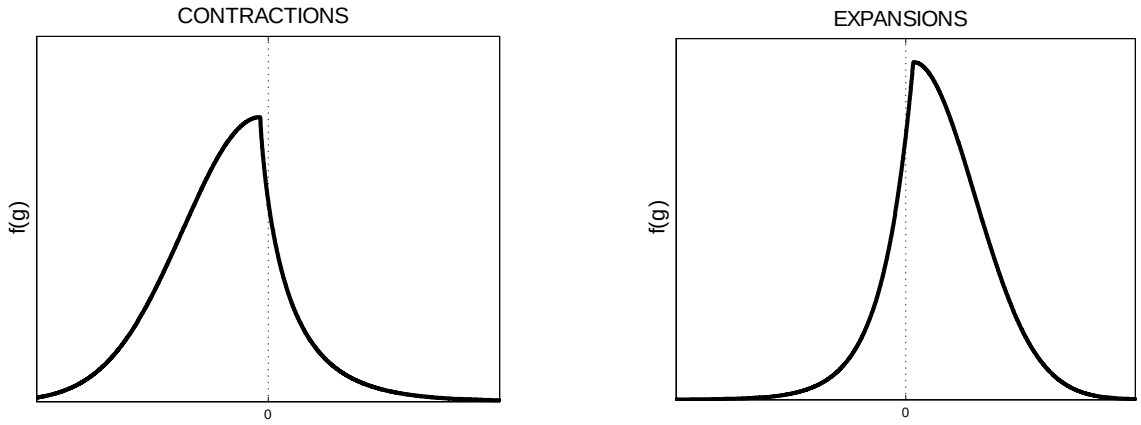
[13]This includes firm size, age and a time trend.

[14]The time series of these shocks span over the same time-window under investigation.

[15]von Hippel (2005) reviews cases in which skewness and unimodality need not imply a particular ordering of measures of location, presenting situations that are compatible with the evidence we report.

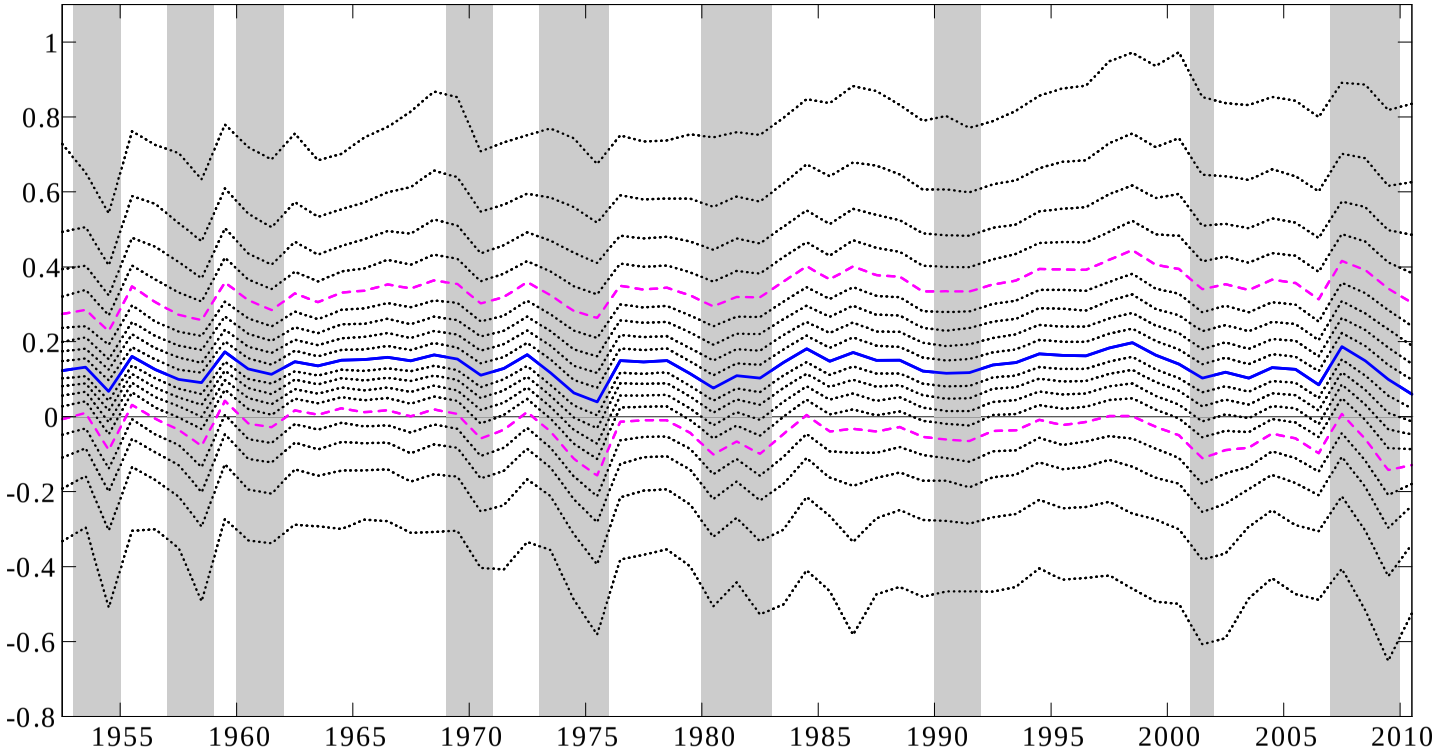
Figures and Tables

FIGURE 1. FIRM GROWTH DENSITIES



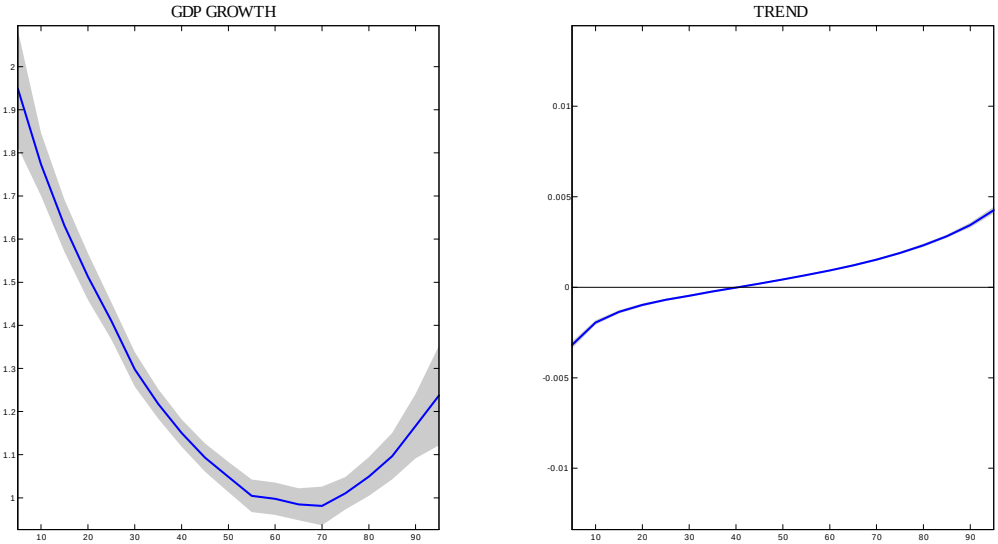
Notes: Figure 1 sketches the density of firm growth during contractions (LHS panel) and expansions (RHS panel). Contractions are generally characterized by positive skewness and higher dispersion about the modal rate of growth, while expansions tend to translate into positive skewness and lower cross-sectional dispersion.

FIGURE 2. QUANTILES OF FIRM GROWTH



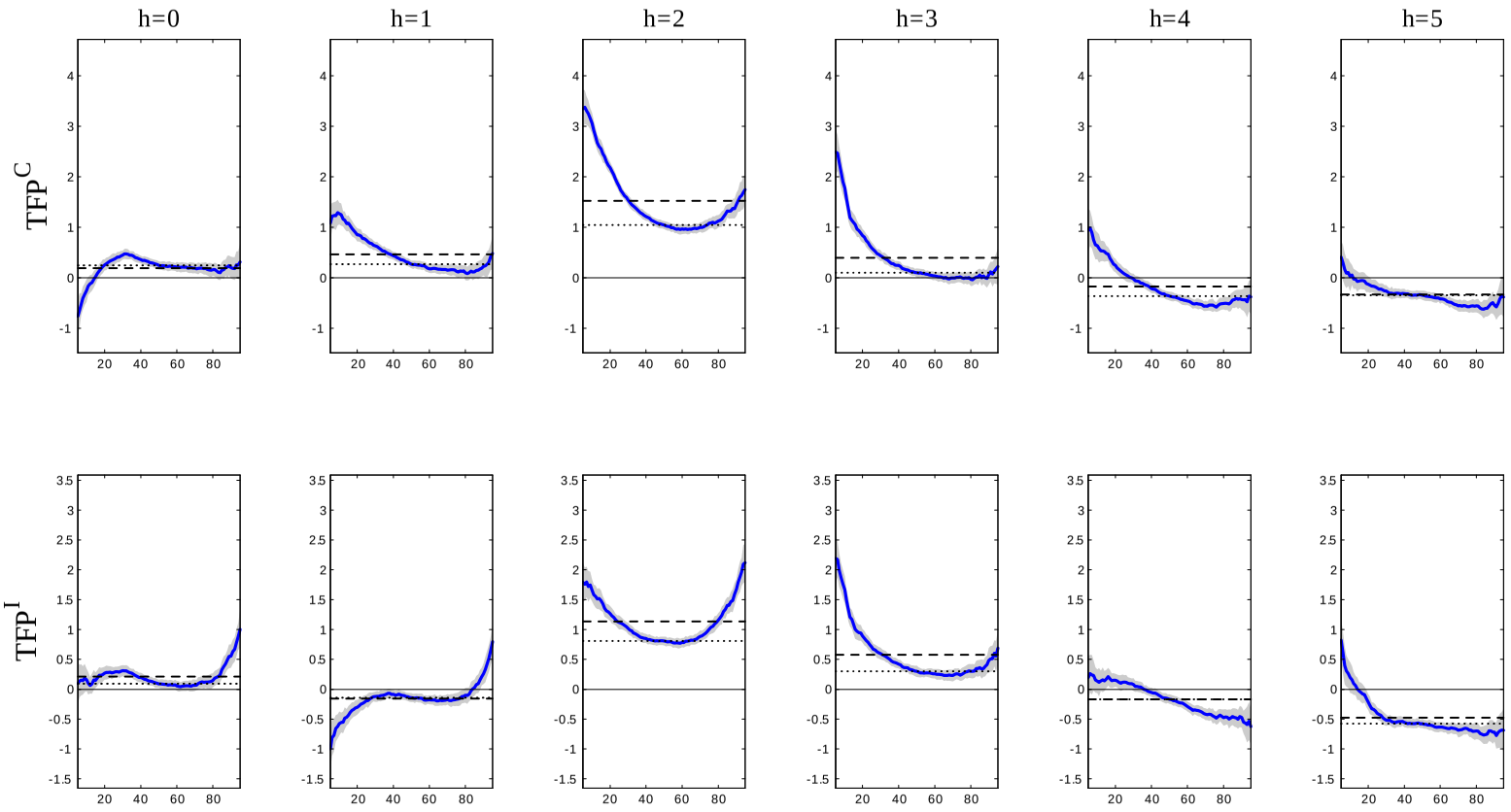
Notes: Figure 2 graphs the quantiles of firm-level growth of real sales over the 1950-2010 time window. The continuous line denotes the median, while the dashed lines denote the 25th and 75th quantiles. Recessionary episodes as identified by the NBER are denoted by the vertical bands.

FIGURE 3. QUANTILE TREATMENT EFFECTS (GDP Growth and Time Trend)



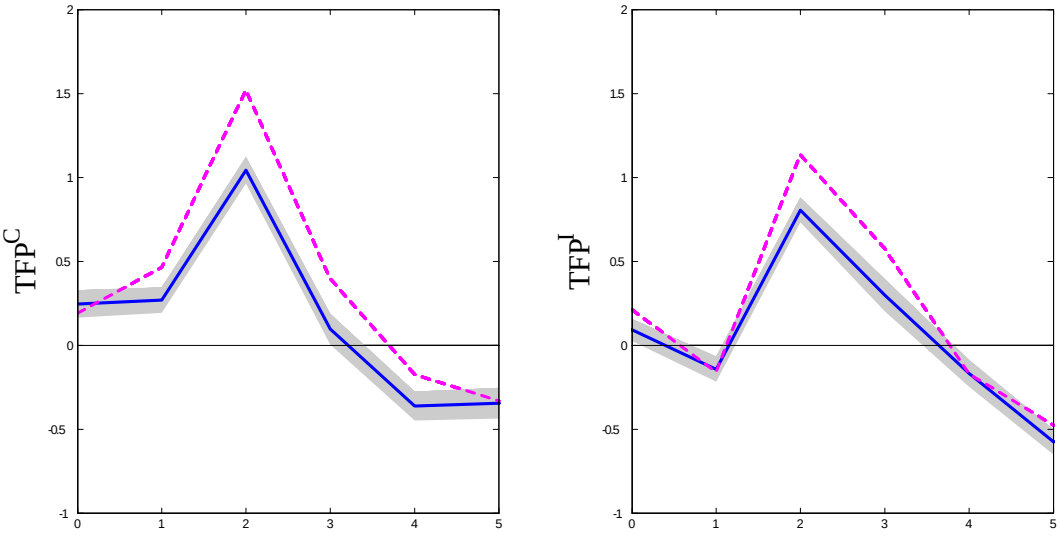
Notes: Figure 3 graphs the estimated QTE associated with real GDP growth (left panel) and a time trend (right panel), together with the 95% confidence interval.

FIGURE 4. QUANTILE TREATMENT EFFECTS (Impulse Response Functions)



Notes: Figure 4 graphs the estimated QTE associated with the responses to a TFP shock, together with the 95% confidence interval. The upper panel considers a TFP series for consumption, while the bottom one considers a TFP series for equipment investment. The dashed (dotted) line denotes the mean (median) response.

FIGURE 5. MEDIAN AND MEAN RESPONSE TO A TECHNOLOGY SHOCK



Notes: Figure 5 graphs the median QTE associated with a TFP shock (continuous line) and the mean response to the same disturbance (dashed line). The left hand panel graphs the responses to a TFP consumption shock, while the right hand panel considers a TFP series for equipment investment.

TABLE 1. SUMMARY STATISTICS

		Corr.	Dyn. Corr.	Conc.
Location	$Q_{0.50}$	0.6508	0.7023	0.5423
Scale	$(Q_{0.75}-Q_{0.25})/(Q_{0.75}+Q_{0.25})$	-0.6954	-0.7138	0.3898
Skewness	$(Q_{0.75}+Q_{0.25}-2Q_{0.50})/(Q_{0.25}-Q_{0.75})$	-0.7019	-0.6885	0.3220
Kurtosis	$(Q_{0.90}-Q_{0.10})/(Q_{0.75}-Q_{0.25})$	0.7824	0.8131	0.6271

Notes: *Corr.* is the correlation of the moment with the real GDP growth rate. *Dyn. Corr.* is a measure of dynamic correlation (Croux *et al.*, 2001), which accounts for correlation at a specific frequency band: in the present case we choose the business cycle frequency in the range $[\pi/4, 3\pi/4]$, which corresponds to a cycle of $6-32$ quarters. *Conc.* stands for the business cycle concordance indicator of Harding and Pagan (1999): this is bounded between 0 and 1 and indicates independence between two given series whenever it equals 0.5.

Appendix

Appendix A: Statistical Evidence

TABLE A1. COMPUSTAT DESCRIPTIVES

Variable	Mean	SD	Min	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	Max
Real sales s (mln \$)	1,298.66	5,814.29	0.001	31.75	141.61	602.64	267,265.90
Growth rate g	0.059	0.229	-1.00	-0.040	0.053	0.155	1.00
Age	15.70	11.66	2	7	12	21	61

Note: 216,282 observations for 10,478 firms over 60 years between 1951 and 2010. We kept only observations for which the growth rate of real sales was included in the interval $(-1, 1)$, dropping about 5,500 observations.

TABLE A2. SECTORAL REPRESENTATION

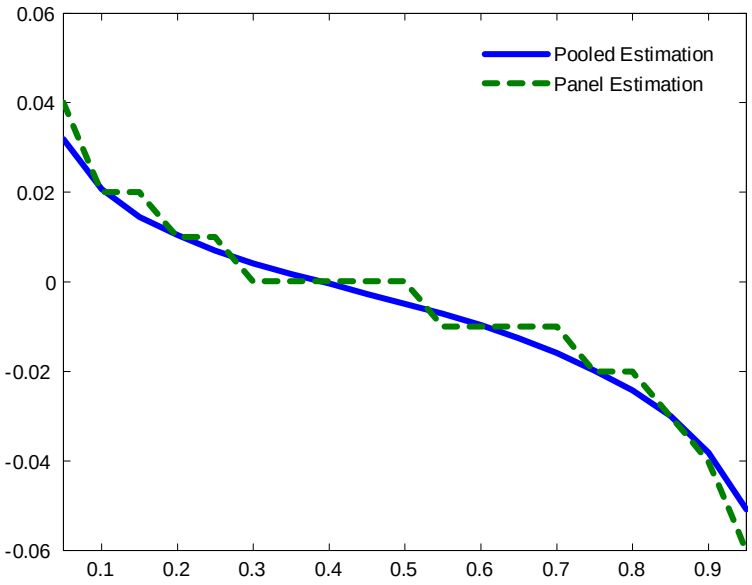
Sector	Frequency	Percent
Agriculture	718	0.33
Mining	8,294	3.83
Construction	2,303	1.07
Manufacturing (durables)	36,284	16.78
Manufacturing (nondurables)	62,370	28.84
Transportation	29,032	13.42
Trade	22,857	10.57
Financial and Other Services	54,424	25.16
TOTAL	216,282	100.00

TABLE A3. COMPUSTAT DESCRIPTIVES BY SECTOR

Agriculture					Mining				
Variable	Mean	SD	Min	Max	Variable	Mean	SD	Min	Max
Real Sales	342.54	787.21	1.67	7,313.79	Real Sales	698.61	2241.36	0.003	32,044.04
Growth	0.051	0.239	-0.882	0.991	Growth	0.063	0.310	-0.997	1.00
Age	14.15	0.4002	2	50	Age	14.64	10.55	2	61
Construction					Manufacturing (durables)				
Variable	Mean	SD	Min	Max	Variable	Mean	SD	Min	Max
Real Sales	733.25	1372.09	0.002	13,718.21	Real Sales	2,032.68	8,542.48	0.001	267,265.90
Growth	0.043	0.289	-0.984	0.994	Growth	0.053	0.208	-0.998	0.998
Age	15.54	11.09	2	61	Age	17.01	12.60	2	61
Manufacturing (nondurables)					Transportation				
Variable	Mean	SD	Min	Max	Variable	Mean	SD	Min	Max
Real Sales	1,050.65	5,445.49	0.001	149,939.4	Real Sales	1,555.49	4,432.01	0.003	78,068.14
Growth	0.053	0.242	-1.00	1.00	Growth	0.059	0.175	-0.997	0.998
Age	15.97	11.69	2	61	Age	19.40	14.07	2	61
Trade					Financial and Other Services				
Variable	Mean	SD	Min	Max	Variable	Mean	SD	Min	Max
Real Sales	1,648.62	7,104.13	0.003	253,339.4	Real Sales	937.54	4296.52	0.001	111,911.2
Growth	0.062	0.200	-0.995	0.996	Growth	0.068	0.248	-1.00	1.00
Age	14.71	10.87	2	61	Age	13.17	9.14	2	61

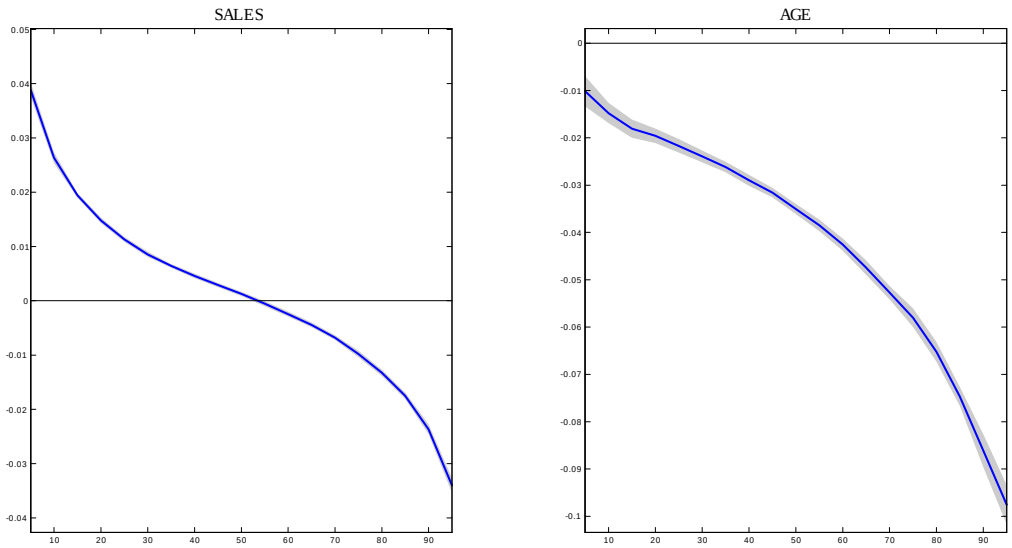
Appendix B: Pooling vs. Panel Estimates

FIGURE B1. QUANTILE TREATMENT EFFECTS (Size): PANEL VS. POOLED ESTIMATES



Notes: Figure B1 graphs the estimated QTE associated with lagged firm-level real sales.

FIGURE B2. QUANTILE TREATMENT EFFECTS (Size and Age)



Notes: Figure B2 graphs the estimated QTE associated with firm-specific lagged real sales (left panel) and age (right panel), together with the 95% confidence interval.