Optimal Monetary Policy with Durable Consumption Goods and Factor Demand Linkages*

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Abstract

This paper deals with the implications of factor demand linkages for monetary policy design. We develop a dynamic general equilibrium model with two sectors that produce durable and non-durable goods, respectively. Part of the output produced in each sector is used as an intermediate input of production in both sectors, according to an input-output matrix calibrated on the US economy. As shown in a number of recent contributions, this roundabout technology allows us to reconcile standard two-sector New Keynesian models with the empirical evidence showing co-movement between durable and non-durable spending in response to a monetary policy shock. A main result of our monetary policy analysis is that strategic complementarities induced by factor demand linkages amplify the loss of social welfare. As the degree of interconnection between sectors increases, the cost of misperceiving the correct production technology of each sector can rise substantially. In addition, the transmission of different sources of exogenous perturbation is altered, compared to what is commonly observed in standard two-sector models without factor demand linkages. In this respect, the role of the relative price of non-durable goods is crucial, as this does not only influence the user cost of durables through the conventional demand channel, but also affects in opposite directions the real marginal cost of production in either sector through the intermediate input channel.

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Introduction

This paper deals with the implications of factor demand linkages for monetary policy design. We build a dynamic stochastic general equilibrium (DSGE) model with two sectors that produce durable and non-durable goods, respectively. Goods produced in each sector serve either as a final consumption good, or as an intermediate production input in both sectors, according to an input-output matrix calibrated on the US economy. The model can reproduce positive co-movement between durable and non-durable spending in response to a monetary policy shock. As documented by various contributions (see, among others, Aoki, 2004; Erceg and Levin, 2006; Barsky et al., 2007), co-movement between non-durable and durable consumption is an inherent feature of the US economy that multi-sector DSGE models need to replicate and that raises a number of questions from the normative point of view.

Introducing factor demand linkages into otherwise standard general equilibrium models with durable and non-durable consumption goods is of key importance for two main reasons. First, it is well documented that standard sticky-price models incorporating sectoral heterogeneity in price stickiness - usually in the form of sticky non-durable goods prices and flexible durables prices - cannot generate co-movement between sectors (Barsky et al., 2007). Bouakez, Cardia, and Ruge-Murcia (2008) and Sudo (2008) show that this negative co-movement puzzle is not robust to a realistic feature of US data such as inter-sectoral linkages.[1] These turn out to be crucial to restore positive co-movement between non-durables and durables value added.[2] Moreover, they act as a potent amplifier of shocks to the system. However, these papers limit their focus to model economies without an explicit stabilization role for a benevolent central bank. Second, despite its empirical relevance, the role of the intermediate input channel in the transmission of exogenous and policy induced shocks is generally neglected, as well as the effective degree of price rigidity across inter-dependent sectors.

Horvath (1998, 2000) shows that input materials can reinforce the effect of sectoral shocks, generating aggregate fluctuations and co-movement between different sectors of the economy, due to the emergence of strategic complementarities acting through the intermediate input channel. More generally, inter-sectoral linkages are crucial to explain co-movement between macroeconomic aggregates.[3] Horvath (2000) shows how independent sectoral shocks exert a substantial impact on the volatility of aggregate output under a delayed application of the law of large numbers. The underlying mechanism is consistent with the propagation of sectoral shocks through the "sparse matrix"[4] form of the intermediate input-use and the capital-use tables in the US.[5] Carvalho (2009) generalizes the results put

[1] As detailed by Bouakez, Cardia, and Ruge-Murcia (2008), input-output interactions are empirically important. Dale Jorgenson's data on input expenditures by US industries shows that materials (including energy) account for roughly 50 percent of outlays, while labor and capital account for 34% and 16%, respectively. The Input-Output (I-O) accounts compiled by the Bureau of Labor Statistics (BLS) shows that 70 percent of the material-input expenditures by the durables sector goes into goods produced by the nondurables sector. The converse proportion is around 10 percent, which is much smaller but still not negligible. More generally, the US I-O matrix is far from being the perfectly diagonal matrix that is implicitly assumed in models without inter-sectoral linkages.

[2] An alternative line of enquiry is followed by Monacelli (2009), who stresses the role of financial market imperfections in generating co-movement. He envisages a heterogeneous agents setting according to which reletively more impatient agents' access to credit is constrained by the value of their collateral. The presence of this constraint de-links the connection between the user cost of durables and their relative price, thus allowing to resolve the co-movement puzzle.

[3] Huang and Liu (2001) show that a model with a vertical input-output structure can reproduce characteristic patterns of price movements at different stages of production and persistent responses of aggregate output following a monetary shock.

[4] Horvath (1998, 2000) explores the role of inter-sectoral linkages in economies where sector-specific shocks represent a source of aggregate fluctuations. He shows that certain classes of input-output matrices transmit a shock in one sector to the others. In particular, independent variations in the productivity of different sectors survive in the aggregate when few full rows and many sparse colums characterize the input-output matrix. The presence of input materials re-inforces the initial impulse, thus generating aggregate fluctuations and co-movement between different sectors of the economy.

[5] Kim and Kim (2006) show that a similar mechanism generates widespread co-movement of economic activity (e.g.

forward by Horvath (2000), showing that the strength of the propagation mechanism in a multi-sector economy can be related to the degree of fat-tailness of the distribution of input-supply linkages in the input-output matrix. Overall, these studies emphasize the need to reconsider the policy implications drawn from otherwise standard multi-sector models at the light of role played by inter-sectoral linkages in the transmission of macroeconomic fluctuations. We add to this literature, providing the first contribution exploring optimal monetary policy in a two-sector model with factor demand linkages.

We first show that the central bank cannot attain the Pareto optimal allocation consistent with the full stabilization of output and inflation, even after removing sources of distortion in the labour market (imperfect labour mobility)[6] and in the goods market (monopolistic competition). This result generalizes the one obtained by Huang and Liu (2005) in a two-sector model with vertical input structure, where the Pareto optimal allocation can be attained only if sectoral shocks have exactly the same magnitude or when no labour is required in the production of final goods. In our variant model economy the occurrence of sectoral shocks of the same magnitude does not ensure that these offset each other, given the presence of a roundabout technology. Unless no labour input is employed in the production process, the Pareto optimum is attainable only when the sectoral shock buffeting the non-durable goods sector equals the one buffeting the durable goods sector, scaled by a factor that depends on the relative labour income share in the two sectors. We conclude that bi-directional horizontal factor demand linkages impose a more restrictive set of conditions to the full stabilization of the model economy in the face of exogenous shocks, compared to models characterized by a vertical input structure.

We then explore optimal monetary policy under the assumption that the policy maker can credibly commit to an interest rate rule derived from the minimization of a social welfare function. Following Rotemberg and Woodford (1998), we obtain a quadratic approximation to the representative household's utility function. We show that the welfare criterion assumed by the central bank balances, along with sectoral inflation variability, fluctuations in aggregate consumption. This is a distinctive feature of our model, given that a non-trivial difference arises between consumption and production, due to factor demand linkages. The relative importance of sector-specific inflation variability is not only determined by the relative degree of sectoral price stickiness, but also by the steady state ratio of labour supplied in the non-durable goods sector on the total labour force (ϕ), which proxies the relative size of each sector. As ϕ tends to zero, meaning that in the steady state no labour is supplied to non-durable goods producing firms, the relative weight of the terms pertaining to this sector vanishes.

Importantly, deriving a second-order approximation for the components of welfare associated with the durables sector involves an additional term that reflects an attitude to smooth the accumulation of the stock of durable goods. This constitutes a primary objective for the monetary authority, compared to the relative weight of other terms involved in the period loss. This feature of the social welfare function is not stressed by Erceg and Levin (2006), although their loss function is isomorphic to the one obtained in this paper.

A main contribution of this paper is to show how the introduction of factor demand linkages into an otherwise standard two-sector model with durable and non-durable goods amplifies social welfare loss, compared to the benchmark economy without input materials. Crucially, as the actual importance of factor demand linkages increases, the cost of misperceiving the correct production technology of each sector can be substantial. Factor demand linkages alter the transmission of exogenous perturbations to the system. A distinctive feature of our model is that a technology shock to either sector also affects potential output in the other sector, even if preferences into different types of goods are separable,

in employment) across sectors. See also Hornstein and Praschnik (1997).

[6] Empirical evidence suggests that labor and capital are not perfectly mobile across sectors. Davis and Haltiwanger (2001) find limited labor mobility across sectors in response to monetary and oil shocks. Bouakez, Cardia, and Ruge-Murcia (2008) report evidence suggesting that perfect labor mobility across sectors, with its implication that sectoral nominal wages are the same, is an imperfect characterization of the data.

as assumed by Erceg and Levin (2006). In addition, the presence of input materials implies that the relative price of non-durable goods does not only effect the marginal rate of substitution between durable and non-durable consumption, but also exerts a positive (negative) effect on the real marginal cost in the durable (non-durable) goods sector. We explore this feature of the model under optimal monetary policy. A technology shock in the non-durable goods sector determines a rise in the relative price gap, thus inducing stronger inflationary pressures in the durable goods sector, compared what is commonly observed in the baseline scenario without input materials. The amplification of the initial impulse due to the presence of strategic complementarities between sector translates into strong aggregate price inflation, whose stabilization represents a prohibitive task for the central bank.

Another feature of the model with factor demand linkages deserves attention. In the face of a cost push shock to either sector, the intermediate input channel acts as an endogenous attenuator of the deflationary effect in the sector which is not hit by the shock. This works through the opposite impact that the relative price exerts on sectoral marginal costs. Imperfect labour mobility magnifies this effect, as it implies that input materials become a primary mechanism of endogenous adjustment in the system, provided that these can flow between the two sectors without encountering any friction.

We finally explore the welfare properties of the system under a wide set of alternative policy regimes, whereby the monetary authority selects an alternative policy objective, compared to the specification of the welfare criterion consistent with the model economy. Concurrently, we study the effect induced on strict and flexible inflation targeting regimes by different definitions of the price index, thus considering "core" and "aggregate" inflation.[7] Regardless of the presence of input materials, when sectoral cost push shocks buffet the model economy along with technology shocks, core inflation targeting is outperformed by every alternative regime we consider.[8] More importantly, a flexible inflation targeting regime, whereby the central bank balances fluctuations in aggregate (or core) inflation with those in the real value added, delivers a welfare loss close to the one attained under optimal monetary policy. This result emphasizes the distinction between consumption and production that naturally arises in frameworks with horizontally integrated sectors of production.

The remainder of the paper is laid out as follows: Section 1 introduces the theoretical setting; Section 2 reports the calibration of our model economy and shows how co-movement between durable and non-durable consumption emerges following a shock to an instrumental policy rule; Section 3 discusses the Pareto optimal outcome; Section 4 discusses the implementation of optimal monetary policy under commitment, and compares the resulting deadweight welfare loss with that attainable under a number of alternative policy regimes. Last section concludes.

1 The Model

We build a two-sector DSGE model. The model economy is populated by a large number of infinitely-lived households. Each of them is endowed with one unit of time and derives utility from consumption of durable goods, non-durable goods and leisure. The two sectors in the economy are characterized by factor demand linkages. Goods produced in each sector serve either as a final consumption good, or as an intermediate production input in both sectors. The size of the intermediate input flow from one sector to the other depends on the input-output structure, which is characterized by the off-diagonal elements of the input-output matrix.

[7] The two concepts are coincident when sectors have the same size and nominal rigidity.

[8] As it is well known, this result stands in contrast to the common view put forward, among others, by Aoki (2001) and Woodford (2003). In particular, Woodford (2003) shows that in a two-sector model with technology shocks as the sole source of exogenous perturbation, optimal commitment policy is nearly replicated by an inflation targeting regime, where the weights attached to sectoral inflations depend on the relative degree of nominal stickiness.

1.1 Producers

Consider an economy that consists of two distinct sectors producing durable (sector d) and non-durable goods (sector n). Each sector is composed by a continuum of firms producing differentiated products. Let Y_t^n (Y_t^d) denote the gross output of the non-durable (durable) goods sector:

$$Y_t^n = \left[\int_0^1 \left(Y_{jt}^n \right)^{\frac{\varepsilon_t^n - 1}{\varepsilon_t^n}} dj \right]^{\frac{\varepsilon_t^n}{\varepsilon_t^n - 1}} \qquad Y_t^d = \left[\int_0^1 \left(Y_{kt}^d \right)^{\frac{\varepsilon_t^d - 1}{\varepsilon_t^d}} dk \right]^{\frac{\varepsilon_t^d}{\varepsilon_t^d - 1}} \tag{1}$$

where ε_t^i ($i = \{n, d\}$) denotes the time-varying elasticity of substitution between differentiated goods in the production composite. The composite products are produced in aggregation sectors operating under perfect competition. It is possible to show that each aggregation sector expresses the following demand for goods produced by firm j in sector n and by firm k in sector d, respectively:

$$Y_{jt}^n = \left(\frac{P_{jt}^n}{P_t^n} \right)^{-\varepsilon_t^n} Y_t^n \qquad Y_{kt}^d = \left(\frac{P_{kt}^d}{P_t^d} \right)^{-\varepsilon_t^d} Y_t^d, \tag{2}$$

where P_t^n and P_t^d are the prices of the composite good in the non-durable and in the durable goods sector respectively. From (1) and (2) the relation between firm-specific and sectoral prices reads as:

$$P_t^n = \left[\int_0^1 \left(P_{jt}^n \right)^{1-\varepsilon_t^n} dj \right]^{\frac{1}{1-\varepsilon_t^n}} \qquad P_t^d = \left[\int_0^1 \left(P_{kt}^d \right)^{1-\varepsilon_t^d} dk \right]^{\frac{1}{1-\varepsilon_t^d}}. \tag{3}$$

In our model economy sectors are related by factor demand linkages. Therefore, the composite output of sector $i = \{n, d\}$ serves partly as a consumption good and partly as an intermediate input in either sector. The allocation of output produced in sector n requires that:

$$Y_t^n = C_t^n + M_t^{nn} + M_t^{nd}, \tag{4}$$

where C_t^n denotes the production of sector n consumed by the households, M_t^{nn} represents the intermediate inputs produced in sector n and consumed in the production process of the firms in the same sector, while M_t^{nd} is the aggregate quantity of intermediate goods produced in sector n used in the production process of the firms in sector d. Symmetrically, the allocation of output in sector d requires that:

$$Y_t^d = C_t^d + M_t^{dn} + M_t^{dd}. \tag{5}$$

The production technology of a generic firm f in sector i reads as:

$$Y_{ft}^i = Z_t^i \left[\frac{\left(M_{ft}^{ni} \right)^{\gamma_{ni}} \left(M_{ft}^{di} \right)^{\gamma_{di}}}{\gamma_{ni}^{\gamma_{ni}} \gamma_{di}^{\gamma_{di}}} \right]^{\alpha_i} \left(L_{ft}^i \right)^{1-\alpha_i}, \qquad i = \{n, d\} \tag{6}$$

where Z_t^i ($i = \{n, d\}$) is a sector specific productivity shock, L_{ft}^idenotes the number of hours worked in firm f in sector i, M_{ft}^{ji} ($j = \{n, d\}$) denotes material inputs produced in sector j and supplied to firm f in sector i. It is important to stress that the durable goods sector actually produces "services from durables" that only consumers can accumulate.[9] Material inputs are combined according to a CES

[9] In Bouakez, Cardia, and Ruge-Murcia (2008) and Sudo (2008) durables used as intermediate inputs are also modeled as fully depreciating goods on the production side. The BLS in the US publishes two different input-output tables: (i) the "Input-use Table", which considers goods that fully depreciate in the same period they are produced, and are usually referred to as "materials" in the traditional KLEM setting; (ii) the "Capital Flow Table" of the input-output accounts.

aggregator:

$$M_{ft}^{ji} = \left(\int_0^1 \left(M_{kf,t}^{ji} \right)^{\left(\varepsilon_t^j - 1\right)/\varepsilon_t^j} dk \right)^{\varepsilon_t^j/\left(\varepsilon_t^j - 1\right)}, \tag{7}$$

where $\left\{ M_{kf,t}^{ji} \right\}_{k \in [0,1]}$ is a sequence of intermediate inputs produced in sector j by firm k, that are employed in the production process of firm f in sector i.

The generic element γ_{ab} ($a = \{n, d\}$; $b = \{n, d\}$) of the (2×2) input-output matrix, Γ, corresponds to the steady state share of total intermediate goods used in the production of sector b supplied by sector a. The input-output matrix is normalized, so that the elements of each column sum up to one, i.e. $\sum_{i=\{n,d\}} \gamma_{ni} = 1$ (and analogously $\sum_{i=\{n,d\}} \gamma_{di} = 1$).

Firms in both sectors are monopolistic competitors in the consumption goods markets and are price takers in the input markets. They set prices given the demand functions reported in (2). Firms are also assumed to be able to adjust their price with probability $1 - \theta^i$ in each period. When they are able to do so, they set a price that maximizes expected profits according to the following problem:

$$\max_{P_{ft}^i} E_t \sum_{s=0}^{\infty} (\beta\theta^i)^s \Omega_{t+s} \left[P_{ft+s}^i \left(1 + \tau^i\right) - MC_{ft+s}^i \right] Y_{ft+s}^i, \qquad i = \{n, d\} \tag{8}$$

where Ω_{t+s} is the stochastic discount factor (consistent with the maximizing behavior of households, which is reported in the next subsection), τ^i denotes lump sum subsidies to producers in sector i, while MC_{fs}^i denotes the marginal cost of production for the f^{th} firm in sector i. The optimal pricing choice, given the sequence $\left\{ P_t^n, P_t^d, Y_t^n, Y_t^d \right\}$, reads as:

$$\overline{P}_{ft}^i = \frac{\varepsilon_t^i}{\left(\varepsilon_t^i - 1\right)\left(1 + \tau^i\right)} \frac{E_t \sum_{s=0}^{\infty} (\beta\theta^i)^s \Omega_{t+s} MC_{ft+s}^i Y_{ft+s}^i}{E_t \sum_{s=0}^{\infty} (\beta\theta^i)^s \Omega_{t+s} Y_{ft+s}^i}, \qquad i = \{n, d\}. \tag{9}$$

In every period each firm solves a cost minimization problem in order to meet demand at its stated price. The first order conditions from this problem result in the following relationships:

$$MC_{ft}^i Y_{ft}^i = \frac{W_t^i L_{ft}^i}{1 - \alpha_i} = \frac{P_t^n M_{ft}^{ni}}{\alpha_i \gamma_{ni}} = \frac{P_t^d M_{ft}^{di}}{\alpha_i \gamma_{di}}, \qquad i = \{n, d\}. \tag{10}$$

It is useful to express the sectoral real marginal cost as a function of the relative price, $Q_t = P_t^n / P_t^d$, and of the sectoral real wage:

$$\frac{MC_t^n}{P_t^n} = \frac{\overline{\phi}^n \left(Q_t^{-\gamma_{dn}} \right)^{\alpha_n} \left(RW_t^n \right)^{1-\alpha_n}}{Z_t^n}, \tag{11}$$

where $RW_t^n = W_t^n / P_t^n$ is the real wage in sector n, and $\overline{\phi}^n$ is a convolution of the production parameters. Analogously, for the durable goods sector:

$$\frac{MC_t^d}{P_t^d} = \frac{\overline{\phi}^d \left(Q_t^{\gamma_{nd}} \right)^{\alpha_d} \left(RW_t^d \right)^{1-\alpha_d}}{Z_t^d}. \tag{12}$$

where $RW_t^d = W_t^d / P_t^d$.

The relative price of durable goods affects the real marginal cost of each sector in opposite direc-

Our Γ is calibrated according to (i). It is important to stress that what we usually define as durables producing sectors (say e.g. automobiles) have non-zero entries in the "input-use" matrix.

tions.[10] Moreover, assuming time-varying elasticities of substitution results into differentiated cost-push shocks that allow us to account for sector-specific shift parameters in the supply schedules, whereas the relative price represents a common shifter.[11]

1.2 Consumers

Households derive income from working in the production sector, investing in bonds and from the stream of profits generated in the production sector. Their preferences are defined over a composite of non-durable goods (C_t^n), an "effective" stock of durable goods ($\mathfrak{D}_t$), and labour (L_t). They maximize the expected present discounted value of their utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{H_t^{1-\sigma}}{1-\sigma} - \varrho \frac{L_t^{1+v}}{1+v} \right], \tag{13}$$

where

$$H_t = (C_t^n)^{\mu^n} \mathfrak{D}_t^{\mu^d}$$

and β is the discount factor, μ^n and $\mu^d (= 1 - \mu^n)$ denote the expenditure shares on non-durable and durable goods, σ is the inverse of the intertemporal elasticity of substitution, v is the inverse of the Frisch elasticity of labour supply. Durable goods are accumulated according to the following law of motion:

$$D_t = C_t^d + (1-\delta) D_{t-1}, \tag{14}$$

where δ denotes the depreciation factor. The effective stock of durables scales the effect of a cost of adjustment:[12]

$$\mathfrak{D}_t = D_t - F(D_t, D_{t-1}). \tag{15}$$

Consequently, adjustment costs are implicitly included in the utility function. Specifically, we rely on a convex (quadratic) adjustment function, whose form is rather standard in the literature:[13]

$$F(D_t, D_{t-1}) = \frac{\Xi}{2} \frac{(D_t - D_{t-1})^2}{D}, \quad \Xi \geq 0, \tag{16}$$

where D denotes the steady state stock of durable consumption goods.

We assume that labour can be either supplied to sector n or d according to a CES aggregator:

$$L_t = \left[\phi^{-\frac{1}{\lambda}} (L_t^n)^{\frac{1+\lambda}{\lambda}} + (1-\phi)^{-\frac{1}{\lambda}} \left(L_t^d\right)^{\frac{1+\lambda}{\lambda}} \right]^{\frac{\lambda}{1+\lambda}}, \tag{17}$$

where λ indexes the elasticity of substitution in labour supply, and ϕ is the steady state ratio of labour supply in the non-durable goods sector over total labour supply ($\phi = L^n/L$). This functional form allows us to account for different degrees of labour mobility between sectors, depending on the value

[10] As it will emerge from our calibration, the absolute value of the marginal impact of changes in the relative price on the sectoral marginal cost is higher in the durable goods sector, as $\alpha_n = \alpha_d$ and $\gamma_{nd} > \gamma_{dn}$.

[11] However, notice that sectoral cost-push shocks exert an indirect effect on the relative price through their influence on sectoral rates of inflation.

[12] The inclusion of adjustment costs of the stock of durables is in line with the research on durable consumption over the business cycle. Adda and Cooper (2000) provide evidence on the discrete nature of durables purchases at the individual level. King and Thomas (2006) show how the partial adjustment mechanism helps at accounting for the aggregate effect of discrete and occasional microeconomic adjustment of a large number of heterogeneous consumers that face a fixed costs of consumption adjustment.

[13] See, e.g., Bernanke (1985).

assumed by λ. For $\lambda = 0$ labour is prevented from moving across sectors, whereas for $\lambda \to \infty$ we allow for perfect labour mobility. Unlike Horvath (2000), this specification of the CES aggregator allows us to neutralize the impact of labour market frictions in the steady state.

The following sequence of (nominal) budget constraints applies:

$$\sum_{i=\{n,d\}} P_t^i C_t^i + B_t = R_{t-1} B_{t-1} + \sum_{i=\{n,d\}} W_t^i L_t^i + \sum_{i=\{n,d\}} \Psi_t^i - T_t, \tag{18}$$

where B_t denotes a one-period risk-free nominal bond remunerated at the gross risk-free rate R_t, and T_t denotes a lump-sum tax paid to the government. The term $\Psi_t^n + \Psi_t^d$ captures the nominal flow of dividends from both sectors of production.

Maximizing (13) subject to (18), (14), (15), and (16) leads to a set of first-order conditions, which can be re-arranged to obtain:

$$\mu^n H_t^{1-\sigma} \left(C_t^n\right)^{-1} = \beta R_t E_t \left[\frac{\mu^n H_{t+1}^{1-\sigma} \left(C_{t+1}^n\right)^{-1}}{\Pi_{t+1}^n} \right], \tag{19a}$$

$$\frac{\mu^n H_t^{1-\sigma} P_t^d}{C_t^n P_t^n} = E_t \left\{ \beta \left(1-\delta\right) \mu^n \frac{H_{t+1}^{1-\sigma} P_{t+1}^d}{C_{t+1}^n P_{t+1}^n} + \right. \tag{19b}$$

$$\left. + \frac{\mu^d H_t^{1-\sigma}}{\mathfrak{D}_t \left[1 - \frac{\Xi}{D}\left(D_t - D_{t-1}\right)\right]^{-1}} + \beta \frac{\Xi}{D} \frac{\mu^d H_{t+1}^{1-\sigma}}{\mathfrak{D}_{t+1} \left(D_{t+1} - D_t\right)^{-1}} \right\},$$

$$W_t^n \frac{\mu^n H_t^{1-\sigma} \left(C_t^n\right)^{-1}}{P_t^n} = \varrho \phi^{-\frac{1}{\lambda}} L_t^{v-\frac{1}{\lambda}} \left(L_t^n\right)^{\frac{1}{\lambda}}, \tag{19c}$$

$$W_t^d \frac{\mu^n H_t^{1-\sigma} \left(C_t^n\right)^{-1}}{P_t^n} = \varrho \left(1-\phi\right)^{-\frac{1}{\lambda}} L_t^{v-\frac{1}{\lambda}} \left(L_t^d\right)^{\frac{1}{\lambda}}. \tag{19d}$$

Thus, from (19c) and (19d):

$$\left(\frac{\phi}{1-\phi}\right)^{-\frac{1}{\lambda}} \left(\frac{L_t^n}{L_t^d}\right)^{\frac{1}{\lambda}} = \frac{W_t^n}{W_t^d}. \tag{20}$$

As $\lambda \to \infty$, and perfect labour mobility is attained, sectoral nominal wages are equalized.

1.3 The Government and the Monetary Authority

We assume that the government serves two purposes in the economy. First, it delegates monetary policy to an independent central bank. We assume that the short-term nominal interest rate is used as the instrument of monetary policy, and that the policy maker is able to pre-commit to a time-invariant rule. We consider alternative specifications of the monetary policy rule, including both rules that can be regarded as reasonable characterizations of the recent historical experience (Section 2), and rules derived from an explicit optimization problem from the perspective of a benevolent central banker (Section 4).

The second task consists of taxing households and providing subsidies to firms to eliminate distortions arising from monopolistic competition in the markets for consumption goods. We assume that this task is pursued via lump-sum taxes that maintain a balanced fiscal budget.

1.4 Market Clearing and Aggregation

Total production reads as:

$$Y_t = Y_t^n + Y_t^d. \tag{21}$$

The allocation of output produced by each sector requires that sectoral gross product is partly sold on the market for consumption goods, while a proportion is sold on the markets for input materials. Therefore, (4) and (5) must hold jointly.

It is important to recognize that C_t^i and Y_t^i are not equivalent in our setting, given the presence of intermediate inputs. In particular, C_t^i can be interpreted as value added in the i^{th} sector, while Y_t^i reflects sectoral gross output.[14] As sectoral gross output can be sold both on the intermediate goods market and on the final goods market, total production is typically greater than real value added. Thus, according to our model economy C_t^i, matches most closely the empirically relevant definition of real GDP.

2 Solution and Calibration

To solve the model, we log-linearize private sector's behavioral equations and resource constraints around the non-stochastic steady state and then take the deviation from their counterparts under flexible prices.[15] The difference between log-variables under sticky prices and their linearized steady state is denoted by the symbol "ˆ", while we use symbol "$*$" to denote percent deviations of variables in the efficient equilibrium (i.e. flexible prices and constant elasticities of substitution) from the corresponding steady state value.[16] Finally, we use symbol "˜" to denote the difference between linearized variables under sticky prices and their counterparts in the efficient equilibrium.[17]

The model is calibrated at a quarterly frequency. We assume that the discount factor $\beta = 0.993$. We set $\sigma = 1$, a value in line with Ngai and Pissarides (2007) which implies separability in the utility derived from different consumption goods. The expenditure share on non-durable goods is $\mu^n = 0.682$. The inverse of the Frisch elasticity of labour supply (v) is set to 3, while $\lambda = 1$, which reflects limited labour mobility.[18] The production parameters $\alpha_n = \alpha_d = 0.6$, while the entries of the two-sector input-use matrix are $\gamma_{nn} = 0.899$, $\gamma_{nd} = 0.688$, according to the calibration of the US economy used by Bouakez, Cardia, and Ruge-Murcia (2008).[19] This implies that the net flow of input materials from the non-durable goods sector to the durable goods sector is positive. The depreciation rate of the stock of durables is assumed at 2.5%.[20] In the baseline calibration we assume that the degree of nominal rigidity is the same across sectors, namely $\theta_n = \theta_d = 0.75$. We will also explore the implications of allowing for asymmetric degrees of price stickiness between the two sectors at different stages in the analysis. We assume that sectoral elasticities of substitution have a steady state value equal to 11. Finally, we set the cost of adjusting the stock of durable goods $\Xi = 600$, as in Erceg and Levin (2006).

As discussed above, the system features two sector-specific technology shocks, z_t^n and z_t^d. Exogenous

[14] See also Nakamura and Steinsson (2008) on this distinction in multi-sector models with input materials.

[15] Steady state conditions are reported in Appendix A.

[16] The economy under flexible prices is reported in Appendix B.

[17] The linearized system in extensive form is reported in Appendix C1. Appendix C2 reports the matrices of parameters for the canonical form of the model.

[18] This value is in line with the calibration proposed by Horvath (2000).

[19] These shares were computed using the table "The Use of Commodities by Industries" for 1992 produced by the BLS. Sudo (2008) shows that the composition of the matrix is rather stable over time.

[20] As in Erceg and Levin (2006), this choice reflects that the durables sector in our model includes both consumer durables and residential investment, which have quarterly depreciation rates of about 5% and 0.75%, respectively, and that the expenditure share of consumer durables in the composite is about two-thirds.

variables are assumed to follow a first-order VAR:

$$\mathbf{v}_t = \mathbf{P}\mathbf{v}_{t-1} + \mathbf{e}_t,$$

where $\mathbf{v}_t = \left[\begin{array}{cccc} z_t^n & z_t^d & \eta_t^n & \eta_t^d \end{array} \right]'$, $\mathbf{P}$ is a diagonal matrix whose eigenvalues lie within the unit circle, and $\mathbf{e}_t$ is a vector of *iid* innovations with diagonal contemporaneous covariance matrix.[21] The cost-push shocks, η_t^n and η_t^d, are reduced form expressions for the time-varying cost-shift parameters in the sectoral New Keynesian Phillips curves.

We choose the parameters describing the persistence and variance of the productivity growth stochastic process in both sectors so that $\rho^{z^n} = \rho^{z^d} = 0.95$ and $\sigma^{z^n} = \sigma^{z^d} = 0.02$ respectively. These values are in line with the standard business cycle literature.[22] As to the cost push shocks, we follow Jensen (2002), Walsh (2003) and Strum (2008), and assume that these are purely transitory, with $\sigma^{\eta^n} = \sigma^{\eta^d} = 0.02$.

2.1 Equilibrium Dynamics and Co-Movement in the Face of a Monetary Policy Shock

We briefly discuss the most significant dynamic features of the model before turning our attention to its equilibrium dynamics under optimal monetary policy. To close the model at this stage, it is necessary to specify how the monetary authority sets the nominal rate of interest. In the first instance we consider the following instrumental rule:[23]

$$\widehat{r}_t = \rho^r \widehat{r}_{t-1} + (1 - \rho^r)\, \chi_\pi \pi_t + \psi_t^r, \quad \chi_\pi > 0 \text{ and } 0 \leq \rho^r \leq 1 \tag{22}$$

where ψ_t^r is an $iid\,(0,1)$ monetary policy innovation. The interest rate smoothing parameter ρ^r is set to 0.7, while $\chi_\pi = 1.5$. We assume that the monetary authority responds to a convex combination of sector-specific rates of inflation. The corresponding price index is defined as:

$$\widetilde{p}_t = \phi \widetilde{p}_t^n + (1 - \phi)\widetilde{p}_t^d, \tag{23}$$

where the weights associated to sectoral inflations capture the relative size of each sector and depend on the steady state ratio of labour supplied in the non-durable goods sector on the total labour force (ϕ).

Figure 1 reports the effects of a one-standard-deviation shock to the monetary policy rule (22) at $t = 0$, for alternative values of the frequency of adjustment in durables prices (the degree of stickiness in the non-durable sector is kept constant at four quarters). Factor demand linkages induce co-movement in durable and non-durable goods spending.[24] Durable and non-durable consumption decrease following the monetary contraction and gradually return to their equilibrium level thereafter. As expected, the first-round impact of the shock and the degree of persistence in the durable goods sector increases in the degree of nominal rigidity. When input materials are excluded ($\alpha_n = \alpha_d = 0$) and prices in the durable goods sector are flexible, as reported in Figure 1, durable consumption increases, while non-durable consumption mirrors its path in the opposite direction. This negative co-movement is induced by the

[21] An alternative strategy could consist of assuming correlated innovations. This is not pursued, so as to avoid that co-movement is generated through an exogenously postulated mechanism.

[22] These values are in line with Strum (2008) and are consistent with empirical studies that show how technology shocks are generally small, but highly persistent (see, e.g., Cooley and Prescott, 1995; Gali, 1999; Huang and Liu, 2005).

[23] See di Pace (2008) for a sensitivity analysis to different types of instrumental rules.

[24] Not only value added, but also sectoral productions co-move. This feature of the model is in line with the evidence discussed by Cristiano and Fitzgerald (1998) and Rebelo (2005).

assumption of price flexibility in the durables sector, as opposed to the assumption of price stickiness in the non-durables sector, which implies that the relative price of durables decreases following the initial monetary contraction.[25] The economy reported in the right panel of Figure 1 behaves in line with the multi-sector model a' la Woodford (2003), where relative price only affects the marginal rate of substitution between different consumption goods. Notice that when input-output interactions are switched off, imperfect labour mobility is not enough to generate co-movement in the face of a monetary policy innovation. Thus, as discussed in Bouakez, Cardia, and Ruge-Murcia (2008), imperfect labour mobility and inter-sectoral linkages are mutually reinforcing elements in the transmission of monetary policy innovations.[26]

In accordance to the empirical evidence discussed in the introduction,[27] the sensitivity of durable spending to the monetary innovation is larger than the one of non-durable spending, despite the introduction of an adjustment cost of the effective stock of durables. This effect is more pronounced as the degree of nominal rigidity in the durable goods sector increases. This results from two inherent features of this class of models, as detailed by Erceg and Levin (2006): (i) firstly, durables demand is for a stock, so that changes in the stock amplificate changes in the flow demand for newly produced goods; (ii) secondly, sectoral price rigidities mitigate the role that changes in the relative price (of durables) play in insulating the durables sector from exogenous perturbations. In Section 4 we show that these factors have clear-cut implications for optimal monetary policy design.

3 The Pareto Optimum

Removing sources of distortion in the labour market (imperfect labour mobility) and in the goods market (monopolistic competition) represents a desirable situation for a benevolent central banker. At this stage of the analysis we are interested to understand whether, after removing these distortions, the monetary authority can attain a first best allocation where output gap and inflation in both sectors are jointly stabilized. As in Huang and Liu (2005), the answer to this question is negative for general parameter values and shock processes, but the structure of the model economy imposes a different explanation for this outcome. We consider a variant economy without cost-push shocks and perfect labour mobility. The following proposition formalizes our results.

Proposition 1 *In the model with sticky prices and perfect labour mobility across sectors there exists no monetary policy that can attain the Pareto optimal allocation, unless the shock buffeting the non-durable goods sector equals the one buffeting the durable goods sector scaled by the factor* $(1-\alpha_n)/(1-\alpha_d)$.
Proof. *Suppose there were a monetary policy under which the equilibrium allocation under sticky prices would be Pareto optimal. Then, in such an equilibrium, the gaps would be completely closed for every period. That is,* $\widetilde{rmc}_t^n = \widetilde{rmc}_t^d = 0$, $\forall t$. *It follows from the pricing conditions that* $\pi_t^n = \pi_t^d = 0$, $\forall t$. *Recall that the relative price evolves as:*

$$\widetilde{q}_t = \widetilde{q}_{t-1} + \pi_t^n - \pi_t^d - \Delta q_t^*.$$

[25] Monacelli (2009) explains that a baseline two-sector New Keynesian model with asymmetric price stickiness cannot replicate such effects. Whenever consumption contracts in one sector following a monetary tightening, it would expand in the other one. Under perfect financial markets the shadow value of durables (i.e. the discounted stream of marginal utilities of durables) is roughly constant. As a result, durable spending is highly sensitive to variations in the user cost. If durable goods prices are flexible and non-durables prices are sticky, a monetary contraction lowers the relative price of durables and subsequently the user cost, leading to opposite effects in the two sectors.

[26] Sudo (2008) shows that in a model with perfect labour mobility factor demand linkages are enough to generate co-movement.

[27] See Barsky, House, and Kimball (2007), Erceg and Levin (2006), and Monacelli (2009).

Since we also have that $\Delta\widetilde{q}_t = 0$, the equation above implies that $\pi_t^n - \pi_t^d = \Delta q_t^*$. It can be shown that (see Appendix D for the derivation of the relative price in the efficient equilibrium setting with perfect labour mobility):

$$q_t^* = \frac{(1-\alpha_d)\, z_t^n - (1-\alpha_n)\, z_t^d}{1+\varkappa},$$

where

$$\varkappa = \alpha_n \alpha_d (\gamma_{nn} + \gamma_{dd} - 1) - \alpha_n \gamma_{nn} - \alpha_d \gamma_{dd}.$$

Therefore, it cannot be that $\pi_t^n = \pi_t^d = 0$, unless $\Delta q_t^* = 0$, which translates into:

$$\frac{\Delta z_t^d}{\Delta z_t^n} = \frac{1-\alpha_d}{1-\alpha_n}.$$

■

Assuming that durable and non-durable goods are partly used as input materials in both sectors is crucial to these results. Allowing for imperfect labour mobility would only reduce the possibility for the monetary authority to neutralize the real and nominal effects of the technology shocks. Although leading to the same fundamental conclusion of Huang and Liu (2005) that the Pareto optimal allocation cannot be attained when labour is required in the production of both goods, a significant difference arises. Our model features the presence of a roundabout production structure ($0 < \alpha_d, \alpha_n < 1$), whereas Huang and Liu (2005) envisage vertically integrated sectors of production, with the intermediate goods sector employing a constant returns to scale technology. In their case the Pareto optimum can be achieved as long as the sectoral shocks are equal or when no labour is required in the production of final goods. In our case having the same shock in both sectors does not ensure that these wash out in the aggregate, given the presence of a roundabout technology. Unless no labour input is employed in the production process, the sole possibility to attain the Pareto optimum arises when then sectoral shock buffeting the non-durable goods sector equals the one buffeting the durable goods sector, scaled by a factor that depends on the relative labour income share in the two sectors. Therefore, bi-directional horizontal factor demand linkages impose more restrictive conditions to the full stabilization of the model economy in the face of exogenous shocks, compared to models characterized by a vertical input structure.

4 Optimal Monetary Policy

As shown in the previous section, the central bank cannot attain the Pareto optimal allocation even after different sources of distortion in the labour and in the goods market are removed. Therefore, we turn our attention to policy strategies capable to attain second best outcomes. We explore equilibrium dynamics under the assumption that the policy maker can commit to a rule derived from the minimization of his objective function. Optimal monetary policy consists of maximizing the conditional expectation of intertemporal household utility subject to private sector's behavioral equations and resource constraints, as discussed by Woodford (2003).[28] This policy regime constitutes a useful benchmark for assessing the performance of alternative discretionary policy outcomes.

[28] We pursue a "timeless perspective" approach, as in Woodford (1999). This involves ignoring the conditions that prevail at the regime's inception, thus imagining that the commitment to apply the rules deriving from the optimization problem had been made in the distant past. In this case, there is no dynamic inconsistency in terms of the central bank's own decision-making process.

The system is solved for the evolution of the endogenous variables by reyling on the common practice discussed, e.g., by Sims (2002).

To evaluate social welfare we take a second-order Taylor approximation to the representative household's lifetime utility.[29] As detailed in Appendix E1, our procedure follows the standard analysis of Rotemberg and Woodford (1998), adapted to account for the presence of factor demand linkages. The resulting intertemporal social loss function reads as:

$$\begin{aligned}\mathcal{SW}_0 \approx & -\frac{U_H(H)H}{2}\Theta E_0 \sum_{t=0}^{\infty}\beta^t \left\{ \frac{\sigma-1}{\Theta}\left(\mu^n \widetilde{c}_t^n + \mu^d \widetilde{d}_t\right)^2 + S\left(\widetilde{d}_t - \widetilde{d}_{t-1}\right)^2 \right. \\ & \left. + \varsigma \left[\varpi \left(\pi_t^n\right)^2 + (1-\varpi)\left(\pi_t^d\right)^2\right] + (1+\nu)\left[\omega \widetilde{c}_t^n + (1-\omega)\widetilde{c}_t^d\right]^2 \right\} \\ & + \text{t.i.p.} + O\left(\|\xi\|^3\right), \end{aligned} \tag{24}$$

where:

$$\begin{aligned} S &= \mu^d \Theta^{-1}\Xi + (1-\delta)(1-\omega)\delta^{-2}, \\ \Theta &= \frac{\mu^n[1-\beta(1-\delta)] + \mu^d\delta}{1-\beta(1-\delta)}, \\ \varpi &= \phi\varepsilon^n(\kappa^n\varsigma)^{-1}, \\ \omega &= \frac{\mu^n[1-\beta(1-\delta)]}{\mu^n[1-\beta(1-\delta)] + \mu^d\delta}, \\ \varsigma &= \phi\varepsilon^n(\kappa^n)^{-1} + (1-\phi)\varepsilon^d\left(\kappa^d\right)^{-1}, \\ \kappa^i &= \frac{\left(1-\beta\theta^i\right)\left(1-\theta^i\right)}{\theta^i}, \; i=\{n,d\} \end{aligned}$$

and t.i.p. collects the terms independent of policy stabilization, whereas $O\left(\|\xi\|^3\right)$ summarizes all terms of third order or higher.

Function (24) reveals that the welfare criterion assumed by the central bank balances, along with sectoral inflation variability, fluctuations in aggregate consumption (or, equivalently, value added). This is a distinctive feature of our model, which differs from traditional multi-sector models where output gap variability is generally accounted for in the loss function (see, e.g., Aoki, 2001 and Woodford, 2003). The introduction of input materials is crucial to this result, as it implies a non-trivial distinction between output and consumption.

Approximating the terms of social welfare associated with the durables sector involves an additional objective, which reflects a strong preference to smooth the accumulation of the stock of durable goods.[30] Quantitatively, this term constitutes a primary objective in this setting, compared to the weight of other components of the period loss. Notice that the second term in S is extremely high and increases in the degree of durability (i.e. decreases in δ). Even though (24) is isomorphic to the social welfare function derived by Erceg and Levin (2006), this feature is not stressed in their paper. The introduction of convex adjustment costs ($\Xi > 0$) reinforces the relative importance of reducing fluctuations in durables accumulation further. Assuming durables accumulation smoothing as a prominent objective helps at counteracting the amplification effect of changes in the stock demand of durables on the flow demand of newly produced durable goods.

The remaining weights associated to the time-varying terms in (24) can be interpreted as follows: (i) ς is a measure of the total degree of nominal stickiness in the economy, as it depends inversely from

[29] We assume that the shocks that hit the economy are not big enough to lead to paths of the endogenous variables distant from their steady state levels. This means that shocks do not drive the economy too far from its approximation point and, therefore, a linear quadratic approximation to the policy problem leads to reasonably accurate solutions.

[30] Details on the linear approximation of this term are available in the technical appendix.

κ^d and κ^n: the greater is price stickiness in a particular sector, the greater the relative importance of that sector's inflation rate in the loss function; (ii) ϖ accounts for the relative price stickiness in the non-durable goods sector; (iii) ω represents the relative weight of non-durable consumption over total consumption when durable goods are reported as a flow. This is an inverse function of Θ. In turn, Θ depends upon the degree of durability of goods produced in sector d. For $\delta = 0$ it reduces to μ^n, whereas for $\delta = 1$ it reduces to one. Therefore, as the degree of durability increases, the weight attached to non-durables consumption gap increases compared to the one attached to the durable component.

Denote with $\mathcal{W}_t^{n,d}\Big|_{\delta=1}$ the part of loss which depends on the consumption gap under the assumption that consumption goods produced in sector d perish at a 100% rate: in this case households derive utility from consuming only non-durable goods, as $\widetilde{c}_t^d = \widetilde{d}_t$. Conversely, $\mathcal{W}_t^{n,d}\Big|_{\delta=0}$ denotes the consumption-based component of deadweight loss under perfect durability. This implies that only non-durable consumption is accounted for in the quadratic term derived from labour disutility, as $\omega = 1$.

$$\widetilde{\mathcal{W}}_t^{n,d}\Big|_{\delta=1} = -\frac{U_H(H)\,H}{2}(1+v)\left(\mu^n\widetilde{c}_t^n + \mu^d\widetilde{c}_t^d\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right), \tag{25}$$

$$\widetilde{\mathcal{W}}_t^{n,d}\Big|_{\delta=0} = -\frac{U_H(H)\,H}{2}(1+v)\,\mu^n\left(\widetilde{c}_t^n\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right). \tag{26}$$

Notice also that the relative importance of sector-specific inflation variability depends on the steady state ratio of labour supplied in the non-durable goods sector on the total labour force (ϕ). As ϕ tends to zero, meaning that no labour is supplied to non-durable goods producing firms in the steady state, the relative weight of the terms pertaining to this sector vanishes. Figure 2 maps the evolution of ϕ for different values of α_n and α_d. The contour map clearly shows that ϕ increases along the main diagonal of the subspace considered. The elasticity of ϕ to changes in the production function parameters is generally greater for α_d, as compared to the marginal impact of changes in α_n. When no input materials are employed in both sectors ($\alpha_n = \alpha_d = 0$) the loss function reduces to the one obtained in traditional two-sector models with no factor demand linkages between sectors (e.g. Woodford, 2003).

How do factor demand linkages influence deadweight loss? The left panel of Figure 3 reports the social welfare loss defined over the subspace of the production parameters α_n and α_d when both technology and cost-push shocks buffet the model economy. The right panel of Figure 3 reports analogous evidence under the assumption that technology shocks are the only source of exogenous perturbation. The general pattern suggests that social welfare loss decreases monotonically in the share of intermediate goods used in the non-durable goods sector, whereas the share of input materials in the durable goods sector exerts a negligible impact.

Yet, our main concern is not only on how factor demand linkages affect welfare loss, but also on how central banks' misperception about their magnitude generates excess loss with respect to the correctly specified model economy. To answer this question we implement optimal commitment policy under the assumption that the central bank neglects the relevance of factor demand linkages in the production process ($\alpha_n = \alpha_d = 0$). Table 1 reports the (percentage) excess loss under misperception of the input-output structure with respect to the loss under the correctly specified production structure of the economy. As the actual weight of factor demand linkages increases in both sectors, excess loss can be substantial. Notice that the marginal impact - in terms of excess loss - of misperceiving α_n is greater than the one associated to α_d, given that non-durable goods producers are the major suppliers of intermediate inputs in our calibrated model economy.

4.1 Impulse-Response Analysis under Optimal Monetary Policy

Figure 4 reports equilibrium dynamics of the model economy following a one-standard-deviation technology shock in the non-durable goods sector, while the effects of an analogous shock in the durable goods sector are reported in Figure 5. The graphs in the left panel report equilibrium dynamics with factor demand linkages between sectors, whereas on the right side we consider the model with no input materials ($\alpha_n = \alpha_d = 0$). Inflation and interest rates are annualized. Symmetric nominal rigidity is assumed, as $\theta_n = \theta_d = 0.75$.[31]

Factor demand linkages amplify the effects of exogenous shocks to either sector, compared to the case without input materials. A technology shock in the non-durable goods sector makes production of these goods relatively cheaper, thus increasing their production and consumption. However, their price is prevented from reaching the level consistent with the situation of flexible prices. This determines a negative non-durable consumption gap. As to the response of the central bank, the real interest rate (on non-durables) initially rises in the model with no input materials, thus preventing output (and consumption) in the non-durable sector from rising as much as it would under flexible prices. Concurrently, real rates do not rise enough to prevent output gap in the durables sector from rising too much. As discussed by Erceg and Levin (2006), keeping output at potential in the non-durable goods sector requires a "sharp and persistent fall" in the real interest rate. By contrast, a sharp rise in the policy instrument is required to keep output at potential in the durable goods sector. This is exactly what happens in our variant economy with no input materials. Conversely, in the model with factor demand linkages the real interest rate initially decreases, gradually converging to its equilibrium level thereafter. Keeping output at potential in the non-durable goods sector prevails over the alternative objective. This result is intimately related to the existence of factor demand linkages, which amplify the effect of the initial impulse on the output/consumption gap of non-durables. Moreover, when input materials are employed in the production process, durable consumption under flexible prices increases, thus helping to reduce durables consumption gap. This endogenous mechanism is not at work in the model without input materials, where durables consumption under flexible prices is not affected by the shock, as a result of setting $\sigma = 1$, which implies separability of households' preferences in durable and non-durable consumption.

A distinctive feature of the model with factor demand linkages deserves attention. When a roundabout technology is envisaged, the relative price does not only affect the marginal rate of substitution between durable and non-durable consumption. As reported in equations (11) and (12), Q_t exerts a positive (negative) effect on the real marginal cost in the durable (non-durable) goods sector. A technology shock in the non-durable goods sector determines an increase in the relative price gap, which implies a substitution away from non-durable to durable consumption goods. In addition, the intermediate input channel gives rise to higher inflationary pressures in the durable goods sector through its influence on the real marginal cost. Conversely, the drop in the marginal cost of production of non-durables and the negative value added (output) gap translate into a persistent deflation in this sector. These factors, combined with the expansionary policy pursued by the central bank, imply that the inflationary pressures developed in the sector producing durable goods are in absolute value stronger than the deflationary effect experienced by non-durable goods prices. In turn, this determines rising inflationary pressures at the aggregate level.

Figure 6 reports equilibrium dynamics following a cost-push shock in the non-durable goods sector. A distinctive feature of the model with factor demand linkages is that a cost push shock to non-durable goods inflation drives the relative price gap up. In turn, this element partially counteracts the de-

[31] As in Strum (2008) we opt for this choice to prevent the central bank from focusing exclusively on the stickier sector in the formulation of its optimal policy, as predicted by Aoki (2001). Thus, for the time being we avoid to favour a stabilization objective over another. In the next subsections we draw implications from the model under asymmetric degrees of nominal rigidity across sectors.

flationary effect that is otherwise observed in the model without input-output interactions and that operates through the conventional demand channel. The overall contractionary effect is magnified by the presence of factor demand linkages. This leads the central bank to initially pursue a weakly contractionary policy, accompanied by a negative real rate of interest (on non-durables). Notice the marked discrepancy between the consumption and the production gap in each sector. Following the initial shock, a drop in the demand of both final and intermediate input goods takes place, which leads production to fall by more than consumption. By contrast, following a sectoral technology shock, sectoral production gaps (and value added gaps) co-move negatively under optimal monetary policy. Moreover, a technology shock generally induces consumption to fluctuate more than production. In fact, a drop in the consumption gap of the sector that experiences the positive technology shock translates into a drop in the demand gap of intermediate goods from the other sector. Thus, each sector experiences opposite demand effects on the markets for the consumption and the intermediate goods. Importantly, imperfect labour mobility exacerbates this effect, increasing the wedge between consumption and production. When aggregate demand increases exogenously, as labour cannot flow across sectors without frictions, firms need to increase intermediate inputs by more than they would under the assumption of perfect labour mobility to meet the increased demand. Consequently, fluctuations in production and consumption are wider in the presence of imperfect labour mobility.

Monetary policy is weakly contractionary in the face of a positive cost-push shock to durable goods inflation in the model with input materials (Figure 7), whereas in the variant economy without input materials we observe a substantial rise in the real interest rate (on non-durables). The optimal response in the baseline scenario is accompanied by a small first-round increase in the nominal rate of interest. From this period onwards the policy instrument falls below its steady state level, and gradually returns to equilibrium thereafter. Weakly contractionary monetary policy determines a positive non-durables consumption gap in the model with factor demand linkages. In addition, non-durables deflation is counteracted by the presence of factor demand linkages, compared to the alternative scenario without input materials. Thus, non-durables inflation is substantially stable, given that the intermediate input channel acts as an endogenous stabilizer. Concurrently, this policy strategy keeps durable goods consumption closer to potential, compared to the alternative scenario with $\alpha_n = \alpha_d = 0$. This allows the central bank to place higher attention on controlling fluctuations in the consumption of the stock of durables. If there were no incentive to smooth durable goods accumulation - meaning that $S = 0$ in (24) - the policy response would be quantitatively closer to the one observed in traditional one-sector models or in our variant multi-sector economy without factor demand linkages (Figure 8).

4.2 Optimal Monetary Policy versus Alternative Policy Regimes

We now compare the relative performance of optimal monetary policy under commitment with respect to a number of alternative policy regimes, using the second order welfare approximation (24) as a benchmark. We compute the unconditional expected welfare loss as the percentage of steady state aggregate consumption (and multiply the resulting term by 100).

The following alternative regimes are considered:[32]

$$\widetilde{\mathcal{W}}_t' = \left\{ \widetilde{\mathcal{W}}_t^{IT}, \widetilde{\mathcal{W}}_t^{GT}, \widetilde{\mathcal{W}}_t^{FIT}, \widetilde{\mathcal{W}}_t^{ITDS} \right\},$$

where $\widetilde{\mathcal{W}}_t^{IT}$ corresponds to the period loss under strict core or aggregate inflation targeting: in the first case the weights attached to sectoral inflations depend on the relative degree of price rigidity, as well as from the relative size of each sector and the degree of substitutability among differentiated goods;

[32] The analytic specification of each regime is reported in Appendix F.

in the second case the weights attached to sectoral inflations only depend on the relative size of each sector.[33] $\widetilde{\mathcal{W}}_t^{GT}$ denotes the period loss under consumption or output gap targeting.[34] $\widetilde{\mathcal{W}}_t^{FIT}$ denotes the loss function under flexible core or aggregate inflation targeting: in this case fluctuations in core or aggregate inflation are balanced together with a term that penalizes fluctuations in aggregate consumption (or production). Our focus on "flexible" regimes is motivated by the fact that an explicit trade-off is introduced when considering sectoral cost-push shocks. Finally, as the durable goods smoothing quadratic term in (24) constitutes a substantial fraction of the overall loss, it seems relevant to consider a hybrid inflation targeting regime, $\widetilde{\mathcal{W}}_t^{ITDS}$. This should allow us to understand whether a regime based on a strict inflation targeting induces too much volatility on the accumulation of the stock of durables.

As reported in Appendix F, either under a strict or a flexible inflation targeting regime,[35] the volatility of aggregate (or core) inflation is accounted for, and not the volatility of sectoral inflations separately. This choice is motivated by two main considerations. First, from a quantitative viewpoint considering the square term of either aggregate or core inflation involves an additional term, compared to the case where the central bank cares about fluctuations in sectoral inflations, which can be related to the covariance between sectoral inflations. This term is not otherwise accounted for in the second-order welfare criterion derived in the paper. Second, from a strategic viewpoint we are willing to understand whether the central bank can approximate the optimal outcome under commitment, without focusing on "distinct" objectives, such as the sectoral rates of inflation. In principle, this should enable the monetary authority to provide the public with a more intelligible target, thus increasing the degree of accountability on its policy mandate. Svensson (1997) stresses the importance of assuming intermediate targets which are highly correlated with the goal, easy to control, and transparent, so as to enhance communication to the public. In this sense measures of overall inflation are meant to serve at this scope better than sectoral inflations. Moreover, considering unique measures of aggregate or core inflation allows us to partly address the criticism advanced by Nelson (2002), who suggests that aggregate inflation targeting might not be an undesirable policy regime in this class of multi-sector models, as hinted by Erceg and Levin (2006).

Table 2 reports the welfare loss under the optimal rule and various alternative policy regimes. The overall loss is disaggregated into that attributable to each term weighted in (24). To compare equilibrium paths under alternative regimes, we evaluate the associated loss by taking as a benchmark our second-order approximation to the representative household's utility function. Both technology and cost-push shocks are assumed to buffet the model economy. We assume that sectors are symmetric in the degree of nominal rigidity ($\theta_n = 0.75$, $\theta_d = 0.75$), therefore $\pi_t^{core} = \pi_t^{agg}$. A flexible inflation targeting regime, whereby fluctuations in consumption or production are balanced along with aggregate inflation, performs nearly as well as optimal commitment policy. Importantly, the central bank attains a welfare loss closer to the one under optimal monetary policy when fluctuations in aggregate (or core) inflation are balanced with those in real value added, compared to the loss induced by controlling fluctuations in the gross product. The presence of factor demand linkages weakens the perfect correlation between consumption and production. Selecting consumption as a stabilization objective in the hybrid inflation targeting regime allows the policy maker to achieve a close-to-optimal performance. This is also confirmed from the the second-order approximation of consumers' utility, where consumption variability arises as a stabilization objective, and not production variability.

Table 3 reports the relative performance of alternative policy regimes under various sources of exogenous perturbation. We consider both our benchmark model economy ($\alpha_n = \alpha_d = 0.6$), and a model with no factor demand linkages ($\alpha_n = \alpha_d = 0$). As observed at different stages of the analysis,

[33] Notice that, when $\alpha_n = \alpha_d$, the weights are such that $\phi = \frac{L^n}{L} = \frac{Y^n}{Y^n + Y^d}$.

[34] Consumption gap targeting can be alternatively interpreted as value added targeting, given that the two concepts are coincident in our setting.

[35] See Svensson (1999) for a distinction between "strict" and "flexible" inflation targeting.

deadweight loss is generally higher in the first case. Concurrently, the presence of cost-push shocks induces higher variability. Again, the flexible inflation targeting regime outperforms other alternative regimes. As to strict inflation targeting, it is worth noticing that this regime displays a "competitive" performance only in the case with only technology shocks, as expected on a priori grounds. Moreover, notice that output gap targeting constantly outperforms consumption gap targeting. Given the non trivial distinction between production and consumption in the model with factor demand linkages, targeting consumption gap volatility enables the central bank to control only "part" of the variability in the marginal cost - and in turn its effect on inflation - compared to what obtained by targeting the production gap, which also accounts for the role of factor demand linkages. In turn, also sectoral inflation volatility benefits from this effect (see Table 2, columns 3 and 4).

Table 4 considers the case of asymmetric price stickiness, in the form of durables prices being more flexible than non-durables prices ($\theta_n = 0.75$, $\theta_d = 0.25$).[36] This allows us to introduce a distinction between core and aggregate inflation, as discussed earlier. We compare each regime under the assumption that either only sectoral technology shocks buffet the model economy or in the presence of sectoral cost-push shocks.[37] Considering core inflation targeting as an alternative to aggregate inflation targeting is somewhat related to a long-standing debate on the information that the central bank can access when formulating its strategy. Woodford (2003) shows that in a two-sector model with no input materials optimal commitment policy is nearly replicated by an inflation targeting regime, whereby the weights attached to sectoral inflations depend on the relative degree of nominal stickiness.[38] This result can only be replicated in the presence of technology shocks, no matter whether factor demand linkages are at work. When only cost-push shocks buffet the model economy, or when both sources of exogenous perturbation are at work, core inflation targeting is by far outperformed by every alternative regimes. This results from the pronounced effect that cost-push shocks exert on consumption, and is further amplified in the presence of factor demand linkages. Focusing on the "stickier" sector under core inflation targeting implies that the central bank almost stabilizes inflation in the non-durables sector, at the cost of inducing high volatility on the remaining components of welfare loss, especially on aggregate consumption. Our parameterization implies a marked degree of asymmetry in terms of sectoral nominal stickiness ($\varpi = 0.992$), which in turn leads the central bank to focus exclusively on non-durables inflation.

4.3 On the Role of Sectoral Asymmetries

The main goal of this section is to examine the role of asymmetry in the degree of competition and in the degree of price stickiness across sectors for the optimal weighting of sectoral inflations and the resulting welfare properties of the model economy. Our exercise is performed by varying the sectoral Calvo parameters and the elasticities of substitution between goods produced within the same sector, under the assumption that their aggregate counterparts are fixed at values consistent with the symmetric

[36] Bils and Klenow (2004) report a higher frequency of price adjustment for consumer durables than services. Erceg and Levin (2006) assume that non-durables and durables are equally sticky. By contrast, Barsky, House, and Kimball (2007) and Carlstrom and Fuerst (2006) develop models where durables prices are flexible and non-durables prices are sticky. Studies in the housing literature also assume that house prices are flexible (for example, Aoki et al., 2004; Iacoviello, 2005).

[37] Notice that introducing both sources of exogenous perturbation would deliver a scenario similar to the case with only cost-push shocks, as these typically dominate the effect induced by technology shocks on equilibrium dynamics.

[38] Aoki (2001) shows that the welfare-theoretic loss function consistent with a multi-sector economy with heterogeneous degrees of price stickiness attaches higher weight on variations sectoral prices characterized by higher stickiness. This provides a theoretical basis for seeking to stabilize an appropriately defined measure of "core" inflation rather than an equally weighted price index. Benigno (2004) relies on this reasoning to argue that a monetary union would maximize welfare by seeking to stabilize an index that does not weight the different countries' inflation rates strictly in proportion to the size of their economies.

case.[39]

Given that the monetary policy authority operates in a timeless perspective, the resulting rule is optimal no matter the nature of the (additive) exogenous disturbances that buffet the model economy, as advocated by Giannoni and Woodford (2003). Thus, it is possible to analyze the effect of sectoral technology and cost-push shocks separately. Our results are reported in Figure 9**.** Specifically, the y-axis in each graph measures the stickiness gap and the x-axis reports the competitive gap, as defined in Lombardo (2006).[40] The exercise is performed both under the assumption that no input materials are employed, as well as under our baseline calibration of the input-output matrix, i.e. $\alpha_n = \alpha_d = 0.6$.

On average, sectoral interaction determines an increase in the general level of deadweight loss for different degrees of asymmetry in either dimension (price stickiness and degree of competition). This result emerges both when only technology shocks are accounted for, as well as in the model with only cost-push shocks. In the model with cost push shocks and in the presence of factor demand linkages the loss is about 15% to 55% higher than that attainable without horizontal flows of input materials between the two sectors. When only technology shocks are considered, excess loss ranges from 25% to 50%.

Model with Technology Shocks - We first consider technology shocks as the sole source of exogenous perturbation. Before turning to the analysis of asymmetry in our baseline model economy, it is instructive to provide an overview of the effects determined by each distinctive feature of the model separately.[41] We start by assuming a two-sector economy with sectors of equal size, no durability in either of the goods produced, and no input-output linkages (i.e. $\alpha_n = \alpha_d = 0$, $\delta = 1$ and $\mu^n = \mu^d = \omega = \phi$). In this case, the resulting loss function is concave in both dimensions of asymmetry. A hump-shaped pattern emerges over the subspace considered. Thus, as asymmetry in either dimension increases, overall unconditional variance decreases. This is in line with the evidence produced by Benigno (2004), Lombardo (2006)[42] and Woodford (2003), and is intimately related to the existence of a trade-off in the stabilization of sectoral inflations. When sectors have same size and analogous production characteristics, asymmetry in the degree of nominal stickiness and/or in the competition gap mitigates this trade-off, as the monetary authority can focus on keeping inflation under control in the sector with higher weight in the index of core inflation. In such circumstances a lower total deadweight loss derives from the combined effect of: (i) higher effectiveness of the policy action against fluctuations in the sector with higher price rigidity and/or higher elasticity of substitution in demand, and (ii) lower price rigidity and/or lower elasticity of substitution in demand in the other sector, which respectively imply lower inflation persistence and lower cross-sectional dispersion in prices. When factor demand linkages are introduced and sectors are of equal size ($\alpha_n = \alpha_d = 0.6$, $\delta = 1$ and $\mu^n = \mu^d$) the loss is still symmetric over the spectrum of the "nominal rigidity gap", but decreases in the gap between the elasticity of substitution of the sector that represents the main supplier in the economy (n) and the one of the other sector (d). Due to the existence of factor demand linkages, inefficiencies deriving from nominal rigidities in either sector are partially passed through onto the other sector. This effect helps at

[39] We set $\overline{\theta} = 0.75$ and $\overline{\varepsilon} = 11$, as in the baseline calibration with symmetric sectoral nominal stickiness and elasticities of substitution in demand. Thus, we map the loss under optimal monetary policy for different values of the "nominal rigidity gap" $(\theta_n - \theta_d)$ and the "competition gap" $(\varepsilon_n - \varepsilon_d)$, under the assumption that $\phi\theta_n + (1-\phi)\,\theta_d = \overline{\theta}$ and $\phi\varepsilon_n + (1-\phi)\,\varepsilon_d = \overline{\varepsilon}$.

[40] Benigno (2004) performs a thorough analysis of the impact of asymmetry in the degree of nominal stickiness in a monetary union on the welfare loss. Lombardo (2006) shows how asymmetries in the degree of competition can exacerbate or mitigate the effects of asymmetric price rigidity.

[41] For reasons of space we only describe of the evolution of the deadweight loss under different parameterizations, rather than reporting the corresponding graphs. These are available, upon request, from the authors.

[42] Lombardo (2006) shows that when prices are set in a staggered fashion, the amount of 'output dispersion' generated by a given deviation of prices from their average depends positively on the elasticity of substitution between goods (i.e. the degree of competition in our model). Therefore, the country with the largest degree of competition is the one that generates the greatest cost of inflation.

counteracting the influence of the "nominal rigidity gap" on welfare. Moreover, if the competition gap is positive ($\varepsilon_n > \varepsilon_d$), then the weight attached to non-durable goods inflation rises linearly in ε_n. In this sense the monetary authority has to place higher attention on inflation fluctuations in this sector (with respect to the symmetric case). In turn, provided that the non-durable goods sector is a net supplier of intermediate goods in the model economy, stabilizing π_t^n helps at reducing fluctuations in the marginal cost gap of the durable goods sector, which is also affected by the relative price gap. When durability is accounted for ($\alpha_n = \alpha_d = 0.6$, $\delta = 0.025$ and $\mu^n = \mu^d$), an additional objective emerges in the central bank's welfare criterion, which reflects a rather strong preference for smoothing the accumulation of the stock of durables. The presence of this term amplifies the impact of nominal rigidities in the durable goods sector, thus leading the total deadweight loss to increase for $\theta_d > \theta_n$ and $\varepsilon_d > \varepsilon_n$.

Eventually, under the baseline calibration of our model ($\alpha_n = \alpha_d = 0.6$, $\delta = 0.025$ and $\mu^n = 0.682$), asymmetry in the degree of nominal stickiness exerts stronger marginal impact on the evolution of the welfare loss. In particular, for a given level of asymmetry in sectoral competition, optimal monetary policy involves placing greater weight on the "stickier" sector. The minimum loss is attained when the economy is characterized by the both the highest competitive gap and the highest asymmetry in the degree of nominal stickiness over the parameter subspace considered, and specifically when the non-durable goods sector is more competitive and "stickier". Notice that, given the relative size of the two sectors, in order to keep aggregate stickiness at a fixed level, a marginal increase in θ_n is accompanied by a more than proportional decrease in θ_d. Analogous observations apply to the sectoral degrees of competition. Higher θ_n and ε_n mean that the central bank penalizes inflationary pressures relatively more in the broader sector of our model economy.

Model with Cost-Push Shocks - In the remainder we consider the model under the baseline calibration reported in Section 2. The introduction of sectoral cost-push shocks induces a trade-off between output and inflation stabilization. Total welfare decreases in the degree of asymmetry in nominal stickiness and competition between sectors. This can be explained intuitively. As it can be observed in (24), the Calvo parameters indexing the degree of nominal rigidity in either sector enter non-linearly in the component of loss associated with fluctuations in core inflation. By contrast, the elasticity of substitution between goods produced in either sector only affects the relative weight of core inflation with respect to other objectives, namely ϖ. The trade-off between inflation and consumption is such that inflation volatility induced by the sector-specific cost-push shocks is not completely stabilized by the monetary authority. Consequently, the contour map tracks the pattern of ϖ in the subspace defined over the asymmetry gaps. In particular, ϖ evolves convexically with respect to the nominal rigidity gap, with a minimum occurring at zero, and increases (decreases) for $\theta_n > (<)\ \theta_d$.

Model with Technology and Cost-Push Shocks - The analysis of the loss function in the presence of both sources of exogenous sectoral perturbation suggests that the monetary authority faces an easier task in the stabilization of the variability due to the technology shocks, compared to its performance in the presence of cost-push shocks. Moreover, provided that cost-push shocks produce a non-trivial trade-off between inflation and output stabilization, they have a predominant impact on the pattern of the welfare loss. This is clearly displayed in the last panel of Figure 9, where aggregate loss is generally convex with respect to the nominal rigidity gap, whereas the effect of the competition gap is negative (positive) when the nominal rigidity gap is positive (negative).

5 Conclusions

We have integrated a roundabout production structure into a dynamic general equilibrium model where two sectors produce durable and non-durable goods. Part of the output produced in each sector is used as an intermediate input of production in both sectors, according to an input-output matrix calibrated on the US economy. This implies a non-trivial difference between consumption (or, equivalently, value added) and gross product, given the presence of input materials. The resulting input-output interactions have non-negligible implications for the formulation of policy recommendations aimed at reducing real and nominal fluctuations in the economy. The intermediate input channel generally amplifies the effects of exogenous perturbations to the system and alters the transmission mechanism, compared to standard multi-sector models. A key role is played by the relative price of non-durable goods, which acts not only as an allocative mechanism on the demand side, through its influence on the user cost of durable goods, but also on the supply side, through its competing effects on sectoral marginal costs of production.

The social welfare function derived through a second-order approximation of households' utility reveals that the policy maker is faced with the task of stabilizing fluctuations in sectoral inflations and aggregate value added. In addition, a term reflecting a strong preference for smoothing the accumulation of the stock of durable goods arises in the welfare criterion. This results as a primary objective of the central banker, no matter whether convex costs are assumed for adjusting the effective stock of durable goods. Factor demand linkages amplify overall deadweight loss. In particular, as the actual importance of factor demand linkages increases in both sectors, the cost of misperceiving the correct production structure of each sector can be substantial.

A distinctive feature of the model with input materials under optimal commitment policy is that, in the face of a cost push shock to either sector, the intermediate input channel acts as an endogenous stabilizer that attenuates the deflationary effect in the sector which is not hit by the shock. This mechanism works through the opposite impact that the relative price exerts on sectoral marginal costs.

Finally, we explore the welfare properties of the system under a wide set of alternative policy regime, whereby the monetary authority selects a "restricted" policy objective, compared to the specification of the welfare criterion consistent with the model economy. Importantly, a flexible inflation targeting regime, whereby the central bank balances fluctuations in aggregate or core inflation with those in real value added delivers a welfare loss close to the one attained under optimal monetary policy. This emphasizes the distinction between consumption and production that naturally arises in frameworks with horizontally integrated sectors of production.

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TABLES

TABLE 1: FACTOR DEMAND LINKAGES MISPERCEPTION

TECH SHOCKS					COST PUSH SHOCKS					BOTH SHOCKS				
α_n/α_d	0	0.3	0.6	0.8	α_n/α_d	0	0.3	0.6	0.8	α_n/α_d	0	0.3	0.6	0.8
0	0	4.664	22.468	52.119	0	0	2.984	9.400	16.554	0	0	3.075	10.210	18.322
0.3	2.348	1.012	14.763	41.453	0.3	3.904	1.234	4.970	10.061	0.3	3.552	1.209	5.601	11.267
0.6	17.232	0.906	5.713	27.242	0.6	28.415	11.303	10.324	13.976	0.6	25.684	8.912	9.517	14.037
0.8	49.424	8.468	1.160	17.336	0.8	92.084	45.841	36.050	38.283	0.8	77.991	30.982	27.664	35.721

Note: We report the percentage excess loss under misperception of the input-output structure with respect to the loss under the correctly specified production structure of the economy, for different shocks.

TABLE 2: WELFARE UNDER ALTERNATIVE POLICIES

	Loss Components				
	$\Delta\widetilde{d}_t$	π_t^n	π_t^d	x_t	Total
Optimal Policy	0.0417	0.6477	0.2918	0.2274	1.2086
Inflation Targeting	0.5486	0.0785	0.3472	2.6662	3.6405
Inflation Targeting (with Durable Smoothing)	0.1937	0.2486	0.3043	1.0450	1.7916
Consumption Gap Targeting	0.0234	1.6396	0.4026	0.0000	2.0656
Production Gap Targeting	0.0133	1.5380	0.3721	0.0067	1.9300
Flexible Inflation Targeting (with Consumption)	0.0476	0.6360	0.2900	0.2377	1.2113
Flexible Inflation Targeting (with Output)	0.0121	1.1501	0.3234	0.0243	1.5099

Note: $\Delta\widetilde{d}_t$ refers to the durable smoothing objective, and x_t to aggregate consumption (value added). The welfare loss is computed as a percentage of steady state aggregate consumption (multiplied by 100).

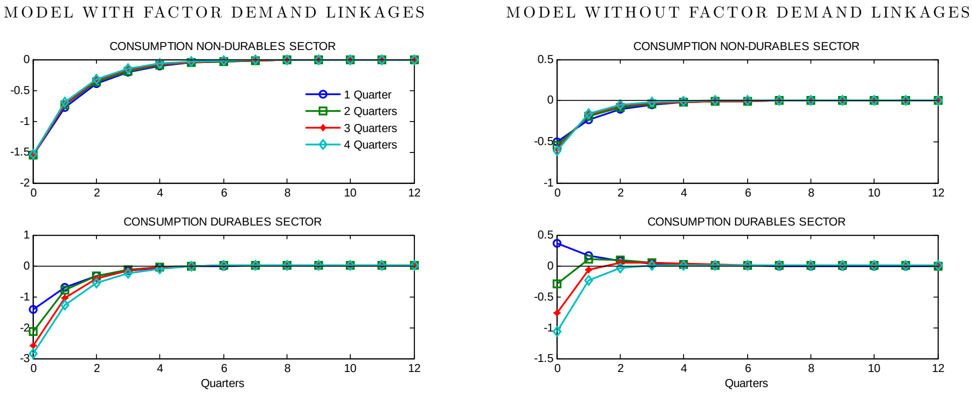
TABLE 3: WELFARE UNDER ALTERNATIVE POLICIES AND DIFFERENT PRODUCTION STRUCTURES				
MODEL WITH FACTOR DEMAND LINKAGES ($\alpha_n = \alpha_d = 0.6$)				
	Tech. Shocks	Cost Push Shocks	Dur. Sec. Shocks	Non-Dur. Sec. Shocks
Optimal Policy	0.1741	1.0645	0.2686	0.9658
Inflation Targeting	0.1768	3.5025	0.3891	3.3110
Inflation Targeting (with Dur. Smoot.)	0.1862	1.6418	0.2884	1.5459
Consumption Gap Targeting	0.2016	1.9235	0.2881	1.8400
Production Gap Targeting	0.1758	1.7903	0.2957	1.6749
Flex. Inflation Targeting (with Cons.)	0.1753	1.0654	0.2957	0.9665
Flex. Inflation Targeting (with Prod.)	0.1744	1.3664	0.2823	1.2581
MODEL WITHOUT FACTOR DEMAND LINKAGES ($\alpha_n = \alpha_d = 0$)				
	Tech. Shocks	Cost Push Shocks	Dur. Sec. Shocks	Non-Dur. Sec. Shocks
Optimal Policy	0.1156	0.6897	0.2854	0.5152
Inflation Targeting	0.1166	0.8662	0.3028	0.6778
Inflation Targeting (with Dur. Smoot.)	0.1157	0.7890	0.2911	0.6111
Output Gap Targeting	0.1217	1.6498	0.4196	1.3670
Flex. Inflation Targeting	0.1163	0.6907	0.2856	0.5168

Note: The first two columns report the loss attributable to technology shocks and cost-push shocks generated in both sectors, respectively. The last two columns report the loss due to both shocks in either sector. The welfare loss is computed as a percentage of steady state aggregate consumption (multiplied by 100).

TABLE 4: ASYMMETRY IN PRICE STICKINESS		
	Tech. Shocks	Cost Push Shocks
Optimal Policy	0.0388	0.8648
Core Inflation Targeting	0.0391	3.6864
Agg. Inflation Targeting	0.2000	1.3171
Core Inflation Targeting (with Dur. Smoot.)	0.0389	2.0198
Agg. Inflation Targeting (with Dur. Smoot.)	0.1822	1.1883
Consumption Gap Targeting	0.0408	1.6711
Production Gap Targeting	0.0396	1.6575
Flex. Core Inflation Targeting (with Cons.)	0.0402	0.8654
Flex. Agg. Inflation Targeting (with Cons.)	0.0936	0.8757
Flex. Core Inflation Targeting (with Prod.)	0.0396	1.1855
Flex. Agg. Inflation Targeting (with Prod.)	0.0463	1.1517

Note: We set the average duration of the non-durable goods' prices at 4 quarters, whereas we reduce the duration of durable goods prices to 1.3 quarters ($\theta^d = 0.25$). The welfare loss is computed as a percentage of steady state aggregate consumption (multiplied by 100).

FIGURE 1: IMPULSE RESPONSES TO A MONETARY POLICY TIGHTENING



Note: We employ the following instrumental rule $\widehat{r}_t = \rho^r \widehat{r}_{t-1} + (1-\rho^r)\,\chi_\pi \pi_t + \psi_t^r$, where ψ_t^r is an $iid\,(0,1)$ monetary policy innovation and the constant term involving the inflation target has been suppressed for simplicity. The interest rate smoothing parameter ρ^r is set to 0.7, while $\chi_\pi = 1.5$. We assume that the monetary authority responds to a convex combination of sector-specific rates of inflation.

FIGURE 2: LABOUR INCOME SHARE IN THE NON-DURABLE GOODS SECTOR (ϕ)

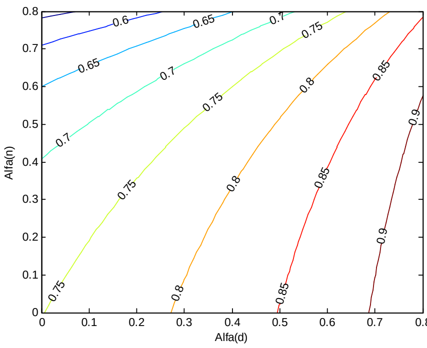
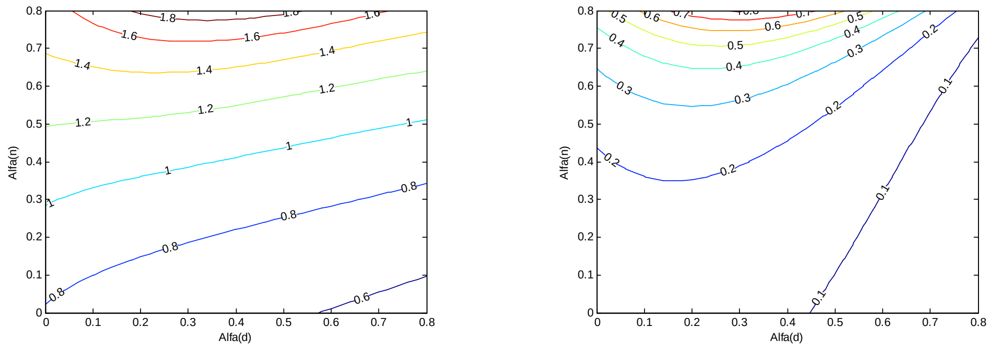
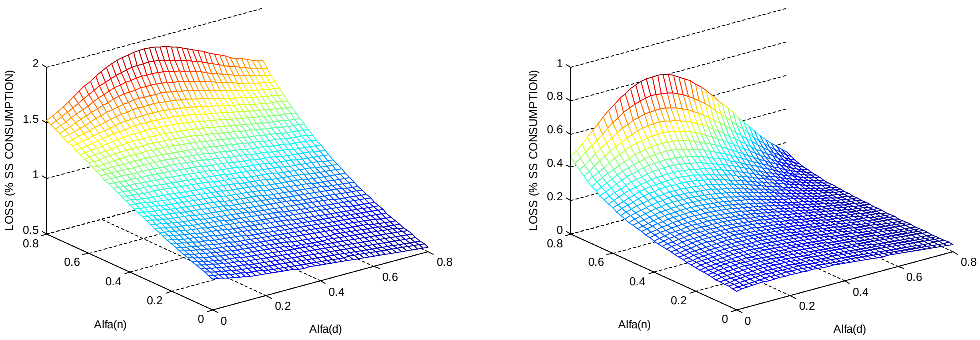


FIGURE 3: WELFARE UNDER OPTIMAL MONETARY POLICY FOR DIFFERENT α_n AND α_d

TECHNOLOGY AND COST-PUSH SHOCKS | ONLY TECHNOLOGY SHOCKS

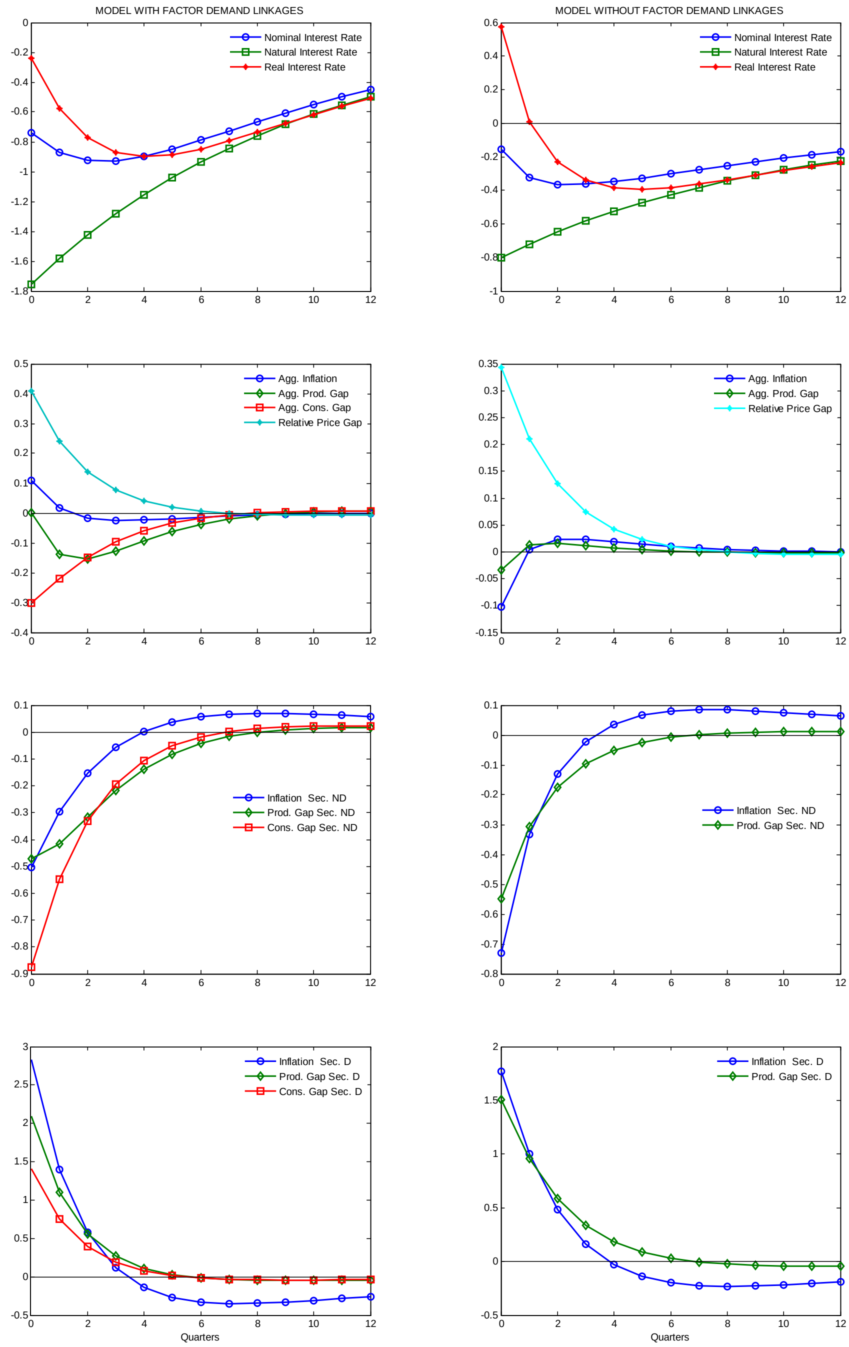


BOTH SHOCKS | ONLY TECHNOLOGY SHOCKS



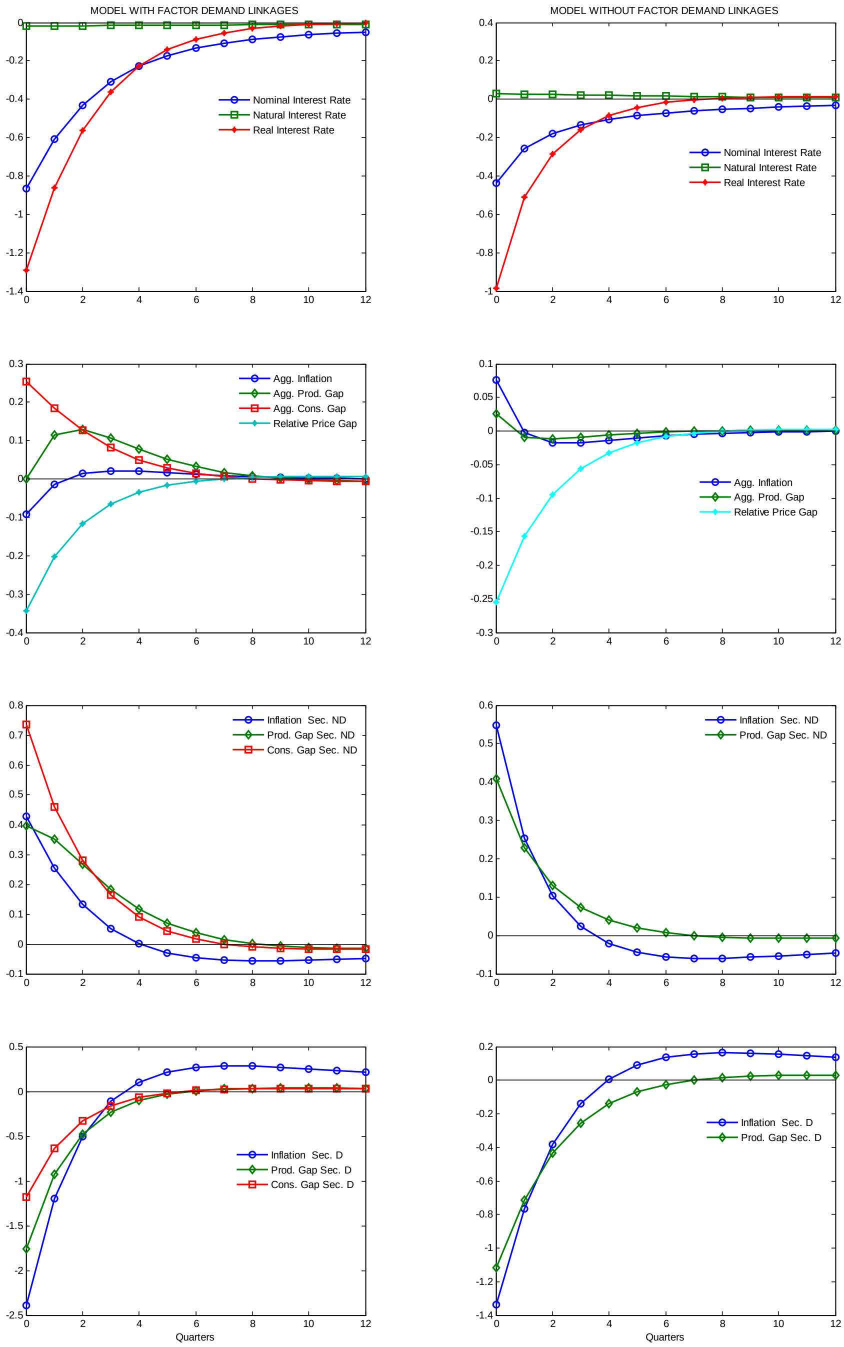
Note: The left panel of Figure 3 reports the welfare loss defined over the subspace of the production parameters in the two sectors when both technology and cost-push shocks buffet the model economy. The right panel reports analogous evidence under the assumption that technology shocks are the only source of exogenous perturbation.

FIGURE 4: IMPULSE RESPONSES TO A TECHNOLOGY SHOCK IN THE NON-DURABLE GOODS SECTOR



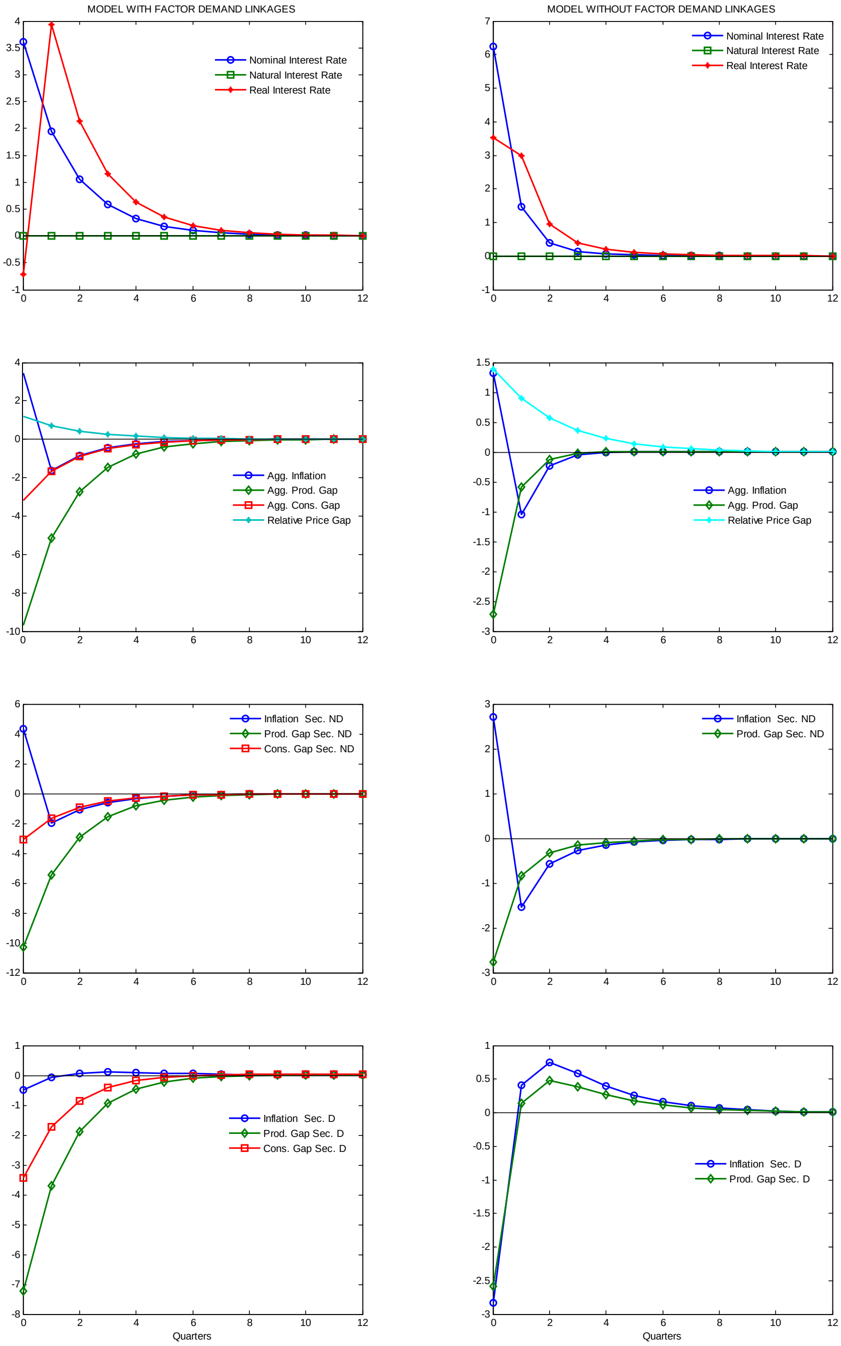
Note: The relative price gap is re-scaled by a factor of four to account for the marked effect brought by factor demand linkages in the amplification of the sectoral shocks.

FIGURE 5: IMPULSE RESPONSES TO A TECHNOLOGY SHOCK IN THE DURABLE GOODS SECTOR



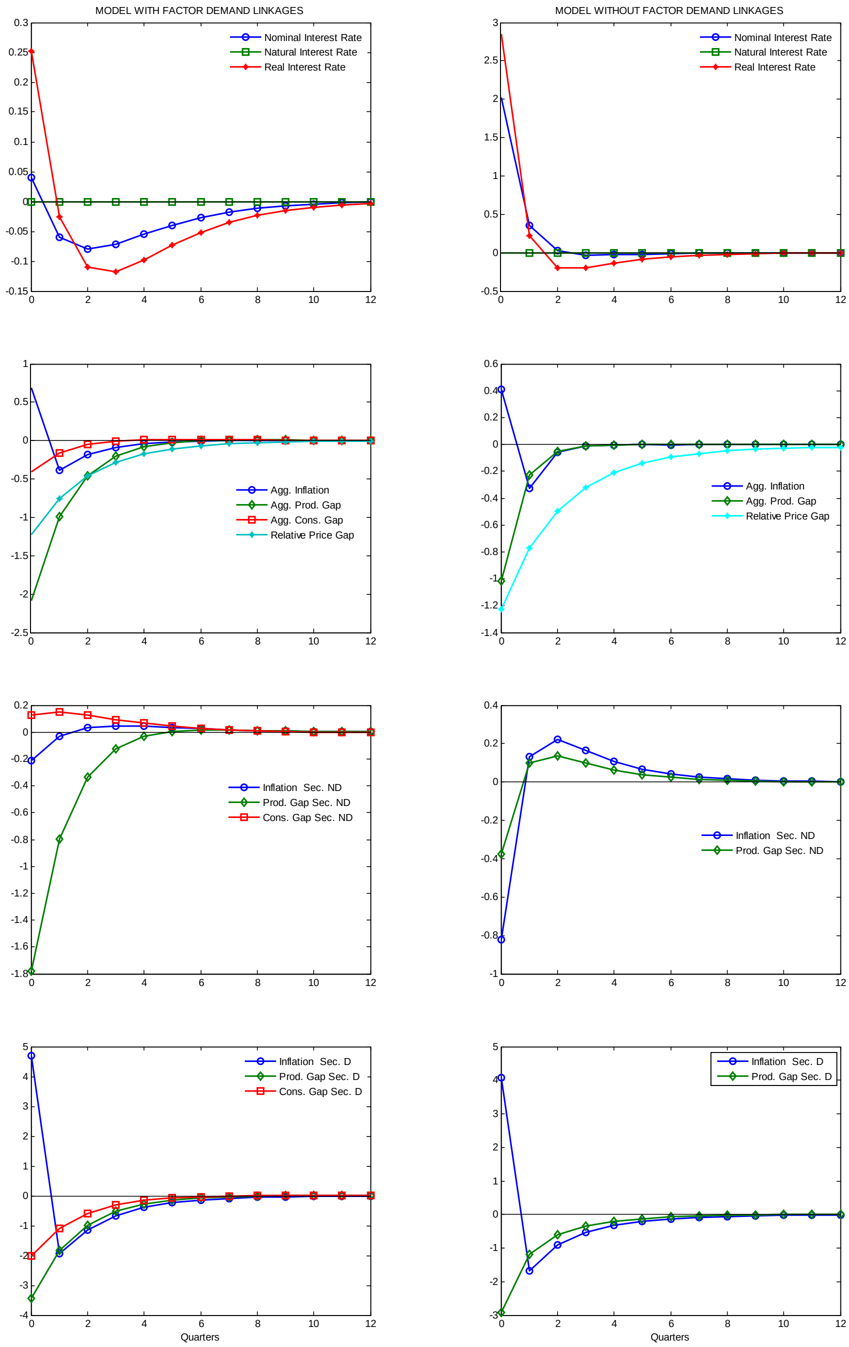
Note: The relative price gap is re-scaled by a factor of four to account for the marked effect brought by factor demand linkages in the amplification of the sectoral shocks.

FIGURE 6: IMPULSE RESPONSES TO A COST PUSH SHOCK IN THE NON-DURABLE GOODS SECTOR



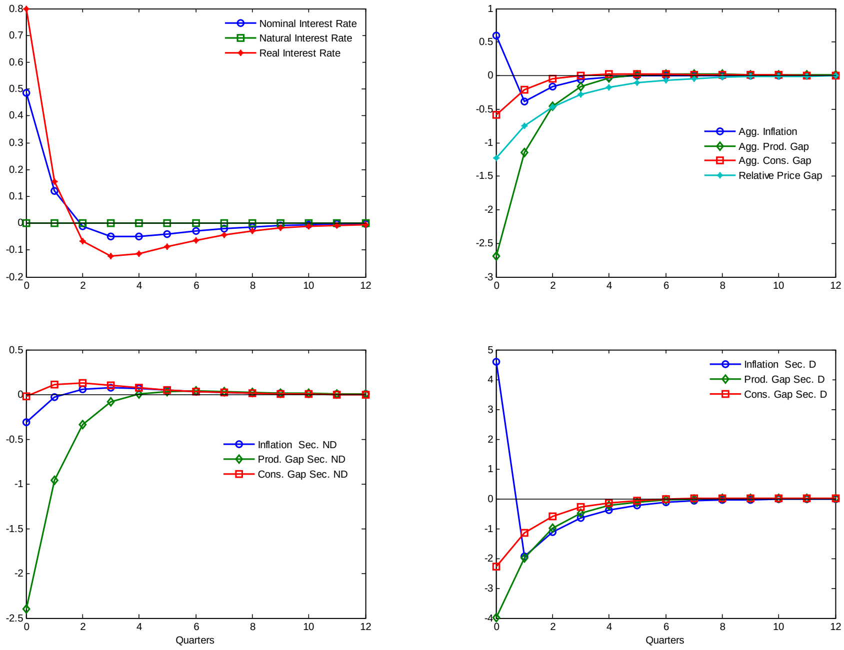
Note: The relative price gap is re-scaled by a factor of four to account for the marked effect brought by factor demand linkages in the amplification of the sectoral shocks.

FIGURE 7: IMPULSE RESPONSES TO A COST PUSH SHOCK IN THE DURABLE GOODS SECTOR



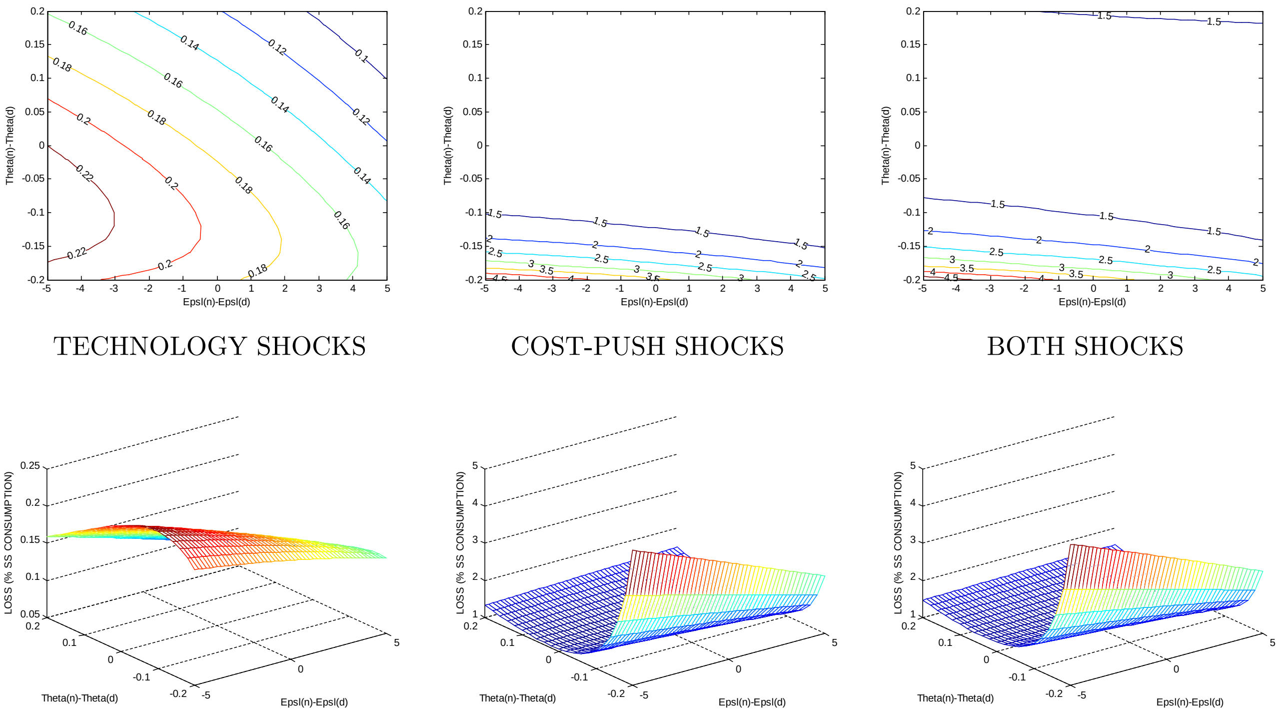
Note: The relative price gap is re-scaled by a factor of four to account for the marked effect brought by factor demand linkages in the amplification of the sectoral shocks.

FIGURE 8: IMPULSE RESPONSES TO A COST PUSH SHOCK IN THE DURABLE GOODS SECTOR
Optimal Commitment Policy with no Durables Accumulation Smoothing, $S = 0$.



Note: The relative price gap is re-scaled by a factor of four to account for the marked effect brought by factor demand linkages in the amplification of the sectoral shocks.

FIGURE 9: THE IMPACT OF ASYMMETRY ON WELFARE LOSS



Note: We set $\overline{\theta} = 0.75$ and $\overline{\varepsilon} = 11$, as in the baseline calibration with symmetric sectoral nominal stickiness and elasticities of substitution in demand. Thus, we map the loss under optimal monetary policy for different values of the "nominal rigidity gap" $(\theta_n - \theta_d)$ and the "competition gap" $(\varepsilon_n - \varepsilon_d)$, under the assumption that $\phi\theta_n + (1-\phi)\,\theta_d = \overline{\theta}$ and $\phi\varepsilon_n + (1-\phi)\,\varepsilon_d = \overline{\varepsilon}$.

APPENDIX A: Some Useful Steady State Relationships

As in the competitive equilibrium real wage in each sector equals the marginal product of labour. Thus, we can derive the following relationship between the production in non-durable and durable goods in the steady state:

$$\frac{Y^n}{Y^d} = \frac{(1-\alpha_d)\phi}{(1-\alpha_n)(1-\phi)} Q^{-1}.$$

Furthermore, the following relationship between durable and non-durable consumtpion can be derived from the Euler conditions:

$$\frac{C^n}{C^d} = (1-\beta(1-\delta)) \frac{\mu^n}{\mu^d} \frac{1}{\delta} Q^{-1}.$$

Moreover, the following shares of consumption and intermediate goods over total production are determined for the non-durable goods sector:

$$\begin{aligned}
\frac{C^n}{Y^n} &= \frac{(1-\alpha_n\gamma_{nn})\,\phi\,(1-\alpha_d) - (1-\alpha_n)(1-\phi)\,\alpha_d\gamma_{nd}}{\phi\,(1-\alpha_d)}, \\
\frac{M^{nn}}{Y^n} &= \alpha_n\gamma_{nn}, \\
\frac{M^{nd}}{Y^n} &= \frac{(1-\alpha_n)}{\phi} \frac{1-\phi}{1-\alpha_d} \alpha_d\gamma_{nd}.
\end{aligned}$$

Analogously, for the durable goods sector:

$$\begin{aligned}
\frac{C^d}{Y^d} &= \frac{(1-\alpha_d\gamma_{dd})(1-\phi)(1-\alpha_n) - (1-\alpha_d)\,\phi\alpha_n\gamma_{dn}}{(1-\phi)(1-\alpha_n)}, \\
\frac{M^{dn}}{Y^d} &= \frac{1-\alpha_d}{1-\phi} \frac{\phi}{1-\alpha_n} \alpha_n\gamma_{dn}, \\
\frac{M^{dd}}{Y^d} &= \alpha_d\gamma_{dd}.
\end{aligned}$$

These condtions turn out to be crucial in the second order approximation of consumers' utility, in order to eliminate linear terms. Moreover, they allow us to derive the steady state ratio of labour supply in the non-durable goods sector over the total labour supply (ϕ).

The Relative Price in the Steady State

We consider the steady state condition for the marginal cost in the non-durable goods sector:

$$\begin{aligned}
MC^n &= \Phi_n \left[(P^n)^{\gamma_{nn}} \left(P^d\right)^{\gamma_{dn}} \right]^{\alpha_n} (W^n)^{1-\alpha_n}, \\
\Phi_n &= \alpha_n^{\alpha_n} (1-\alpha_n)^{1-\alpha_n}.
\end{aligned}$$

As in the steady state production subsidies neutralize distortions due to imperfect competition:

$$\begin{aligned}
P^n &= MC^n \\
&= \Phi_n \left[(P^n)^{\gamma_{nn}} \left(P^d\right)^{\gamma_{dn}} \right]^{\alpha_n} (W^n)^{1-\alpha_n}.
\end{aligned}$$

After some trivial manipulations it can be shown that:

$$\Phi_n Q^{-\alpha_n \gamma_{dn}} \left(RW^n\right)^{1-\alpha_n} = 1.$$

Analogously, for the durables goods sector:

$$\Phi_d Q^{\alpha_d \gamma_{nd}} \left(RW^d\right)^{1-\alpha_d} = 1.$$

Using the fact that in steady state $W^n = W^d = W$:

$$\begin{aligned} \frac{RW^n}{RW^d} Q &= 1, \\ \frac{\left(\Phi_n^{-1} Q^{\alpha_n \gamma_{dn}}\right)^{\frac{1}{1-\alpha_n}}}{\left(\Phi_d^{-1} Q^{-\alpha_d \gamma_{nd}}\right)^{\frac{1}{1-\alpha_d}}} Q &= 1. \end{aligned}$$

Therefore:

$$\begin{aligned} Q &= \left(\Phi_n^{1-\alpha_d} \Phi_d^{-(1-\alpha_n)}\right)^{\frac{1}{\varphi}}, \\ \varphi &= (1-\alpha_n)(1-\alpha_d) + \alpha_n \gamma_{dn} (1-\alpha_d) + \alpha_d \gamma_{nd} (1-\alpha_n). \end{aligned}$$

Notice that, when $\alpha_n = \alpha_d = 1$:

$$Q = \Phi_n \Phi_d^{-1}$$

as in the case considered by Huang and Liu (2005) and Strum (2008).

APPENDIX B: Equilibrium Dynamics in the Efficient Equilibrium

In this appendix we outline the solution method of the linear model under the efficient equilibrium. This is obtained when both prices are flexible and elasticities of substitution are constant. Let us start from the pricing rule under flexible prices:

$$\begin{aligned} P_t^{n^*} &= \frac{\Theta^n}{1+\tau^n} MC_t^{n^*} \\ &= \frac{\Theta^n}{1+\tau^n} \frac{\overline{\phi}^n \left[\left(P_t^{n^*}\right)^{\gamma_{nn}} \left(P_t^{d^*}\right)^{\gamma_{dn}}\right]^{\alpha_n} \left(W_t^*\right)^{1-\alpha_n}}{Z_t^n} \end{aligned}$$

$$\begin{aligned} P_t^{d^*} &= \frac{\Theta^d}{1+\tau^d} MC_t^{d^*} \\ &= \frac{\Theta^d}{1+\tau^d} \frac{\overline{\phi}^d \left[\left(P_t^{n^*}\right)^{\gamma_{nd}} \left(P_t^{d^*}\right)^{\gamma_{dd}}\right]^{\alpha_d} \left(W_t^*\right)^{1-\alpha_d}}{Z_t^d} \end{aligned}$$

where Θ^n and Θ^d denote the mark-up terms. In log-linear form the conditions above reduce to:

$$(1-\alpha_n)\, rw_t^{n*} = z_t^n + \alpha_n \gamma_{dn} q_t^* \tag{27}$$

$$(1-\alpha_d)\, rw_t^{d*} = z_t^d - \alpha_d \gamma_{nd} q_t^* \tag{28}$$

We now recall some conditions under flexible prices from the linearized system:

$$\begin{aligned}
c_t^{d*} &= \frac{1}{\delta}d_t^* - \frac{1-\delta}{\delta}d_{t-1}^*, && (29)\\
rw_t^{n*} &= -\gamma c_t^{n*} - (1-\sigma)\,\mu^d d_t^* + \left[v\,(1-\phi) - \frac{1}{\lambda}\right] l_t^{d*} \\
&\quad + \left(\vartheta\phi + \frac{1}{\lambda}\right) l_t^{n*}, && (30)\\
l_t^{n*} &= \lambda\left(rw_t^{n*} - rw_t^{d*} + q_t^*\right) + l_t^{d*}, && (31)\\
y_t^{n*} &= \frac{C^n}{Y^n}c_t^{n*} + \frac{M^{nn}}{Y^n}m_t^{nn*} + \frac{M^{nd}}{Y^n}m_t^{nd*}, && (32)\\
y_t^{d*} &= \frac{C^d}{Y^d}x_t^* + \frac{M^{dn}}{Y^d}m_t^{dn*} + \frac{M^{dd}}{Y^d}m_t^{dd*}, && (33)\\
0 &= rw_t^{n*} + l_t^{n*} - y_t^{n*}, && (34)\\
0 &= rw_t^{d*} + l_t^{d*} - y_t^{d*}, && (35)\\
0 &= m_t^{nn*} - y_t^{n*}, && (36)\\
0 &= m_t^{nd*} + q_t^* - y_t^{d*}, && (37)\\
0 &= m_t^{dn*} - q_t^* - y_t^{n*}, && (38)\\
0 &= m_t^{dd*} - y_t^{d*}, && (39)
\end{aligned}$$

where $\vartheta = \left(v - \frac{1}{\lambda}\right)$, $\gamma = (1-\sigma)\,\mu^n - 1$ and $\phi = \frac{L^n}{L}$. We substitute (27) and (28) into (33) and (34) respectively:

$$\begin{aligned}
l_t^{n*} &= y_t^{n*} - \frac{1}{1-\alpha_n}z_t^n - \frac{\alpha_n\gamma_{dn}}{1-\alpha_n}q_t^*, && (40)\\
l_t^{d*} &= y_t^{d*} - \frac{1}{1-\alpha_d}z_t^d + \frac{\alpha_d\gamma_{nd}}{1-\alpha_d}q_t^*. && (41)
\end{aligned}$$

We can use conditions (31), (32), and (36)-(39), to get:

$$y_t^{n*} = \frac{C^n}{Y^n}c_t^{n*} + \frac{M^{nn}}{Y^n}y_t^{n*} + \frac{M^{nd}}{Y^n}\left(y_t^{d*} - q_t^*\right)$$

and

$$y_t^{d*} = \frac{C^d}{Y^d}c_t^{d*} + \frac{M^{dn}}{Y^d}\left(q_t^* + c_t^{g*}\right) + \frac{M^{dd}}{Y^d}y_t^{d*}.$$

We can find a VAR solution of this system, so that we can express y_t^{n*} and y_t^{d*}as a function of c_t^{n*}, c_t^{d*} and q_t^* :

$$\mathbf{A}\begin{bmatrix} y_t^{n*} \\ y_t^{d*} \end{bmatrix} = \mathbf{B}\begin{bmatrix} c_t^{n*} \\ c_t^{d*} \end{bmatrix} + \boldsymbol{\Upsilon} q_t^*,$$

where

$$
\begin{aligned}
\mathbf{A} &= \left[\begin{array}{cc} 1-\frac{M^{nn}}{Y^n} & -\frac{M^{nd}}{Y^n} \\ -\frac{M^{dn}}{Y^d} & 1-\frac{M^{dd}}{Y^d} \end{array}\right] = \left[\begin{array}{cc} \frac{C^n}{Y^n}+\frac{M^{nd}}{Y^n} & -\frac{M^{nd}}{Y^n} \\ -\frac{M^{dn}}{Y^d} & \frac{C^d}{Y^d}+\frac{M^{dn}}{Y^d} \end{array}\right], \\
\mathbf{B} &= \left[\begin{array}{cc} \frac{C^n}{Y^n} & 0 \\ 0 & \frac{C^d}{Y^d} \end{array}\right], \\
\mathbf{\Upsilon} &= \left[\begin{array}{c} -\frac{M^{nd}}{Y^n} \\ \frac{M^{dn}}{Y^d} \end{array}\right].
\end{aligned}
$$

Thus, we obtain:

$$
\left[\begin{array}{c} y_t^{n*} \\ y_t^{d*} \end{array}\right] = \mathbf{A}^{-1}\mathbf{B}\left[\begin{array}{c} c_t^{n*} \\ c_t^{d*} \end{array}\right] + \mathbf{A}^{-1}\mathbf{\Upsilon} q_t^*,
$$

or equivalently:

$$
\begin{aligned}
y_t^{n*} &= aq_t^* + bc_t^{n*} + ec_t^{d*}, \\
y_t^{d*} &= cq_t^* + dc_t^{d*} + fc_t^{n*}.
\end{aligned}
$$

These can be plugged into (40) and (41) to obtain:

$$
l_t^{n*} = bc_t^{n*} + ec_t^{d*} - \frac{1}{1-\alpha_n} z_t^n + \left(a - \frac{\alpha_n \gamma_{dn}}{1-\alpha_n}\right) q_t^* \tag{42}
$$

$$
l_t^{d*} = dc_t^{d*} + fc_t^{n*} - \frac{1}{1-\alpha_d} z_t^d + \left(\frac{\alpha_d \gamma_{nd}}{1-\alpha_d} + c\right) q_t^* \tag{43}
$$

Thus, we can substitute everything into (30) and (27):

$$
\frac{1+v\phi}{1-\alpha_n} z_t^n + \xi_1 z_t^d = \xi_2 c_t^{n*} - (1-\sigma)\,\mu^d d_t^* + \xi_3 c_t^{d*} + \xi_4 q_t^*, \tag{44}
$$

where:

$$
\begin{aligned}
\xi_1 &= \frac{v(1-\phi)\lambda - 1}{(1-\alpha_n)\,\lambda}, \\
\xi_2 &= \frac{\lambda\,(v\phi b - \gamma) + f\,[v(1-\phi)\lambda - 1]}{\lambda}, \\
\xi_3 &= \frac{\lambda v\phi e + d\,[v(1-\phi)\lambda - 1]}{\lambda}, \\
\xi_4 &= -\frac{\alpha_n \gamma_{dn}\,(1+v\phi)}{1-\alpha_n} + \left[\frac{v(1-\phi)\lambda - 1}{(1-\alpha_d)\,\lambda}\left(\frac{\alpha_d \gamma_{nd}}{1-\alpha_d} + c\right) + av\phi\right].
\end{aligned}
$$

In turn, we can plug (42), (43), (27) and (28) into (30):

$$
\xi_5 q_t^* = \frac{1}{1-\alpha_n} z_t^n - \frac{1}{1-\alpha_d} z_t^d - \frac{k}{(1+\lambda)}\left(c_t^{n*} - c_t^{d*}\right) \tag{45}
$$

where

$$
\xi_5 = \frac{a-c-\lambda}{1+\lambda} - \left[\frac{\alpha_n \gamma_{dn}}{(1-\alpha_n)} + \frac{\alpha_d \gamma_{nd}}{(1-\alpha_d)}\right].
$$

Conditions (44) and (45), together with the Euler conditions for the durable and the non-durable goods, and the law of accumulation for durable goods, allow us to determine a system of linear difference equations. Equlibrium dynamics under flexible prices can be reported in canonical form as:

$$E_t\left[\mathbf{F}_0\kappa_{t+1}^* + \mathbf{F}_1\kappa_t^* + \mathbf{F}_2\kappa_{t-1}^* + \mathbf{C}_1\varpi_t\right] = 0 \tag{46}$$

$$\begin{aligned}
\kappa_t^* &= \left[\begin{array}{ccccc} c_t^{n*} & c_t^{d*} & d_t^* & q_t^* & rr_t^* \end{array}\right]' \\
\varpi_t &= \left[\begin{array}{cc} z_t^n & z_t^d \end{array}\right]'
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{F}_0 &= \left[\begin{array}{c} \mathbf{0}_{2\times5} \\ -\gamma \quad 0 \quad -(1-\sigma)\mu^d \quad \mathbf{0}_{1\times2} \\ \beta(1-\delta)\left[(1-\sigma)\mu^n - 1\right] \quad 0 \quad F_0^{(4,3)} \quad 0 \quad -\beta(1-\delta) \\ \mathbf{0}_{1\times5} \end{array}\right], \\
F_0^{(4,3)} &= \beta\left\{(1-\delta)(1-\sigma)\mu^d + \Xi\left[1-\beta(1-\delta)\right]\right\}
\end{aligned}$$

$$\begin{aligned}
\mathbf{F}_1 &= \left[\begin{array}{c} F_1^{(1,1)} \quad F_1^{(1,2)} \quad (1-\sigma)\mu^d \quad F_1^{(1,4)} \quad 0 \\ \frac{k}{(1+\lambda)} \quad -\frac{k}{(1+\lambda)} \quad 0 \quad F_1^{(2,4)} \quad 0 \\ \gamma \quad 0 \quad (1-\sigma)\mu^d \quad 0 \quad -1 \\ \left[1-\beta(1-\delta)(1-\sigma)\mu^n\right] \quad 0 \quad F_1^{(4,3)} \quad 1 \quad 0 \\ 0 \quad 1 \quad -\frac{1}{\delta} \quad \mathbf{0}_{1\times2} \end{array}\right], \\
F_1^{(1,1)} &= -\left(\vartheta\phi + \frac{1}{\lambda}\right)b + \gamma - \vartheta(1-\phi)f, \\
F_1^{(1,2)} &= -\vartheta(1-\phi)d - \left(\vartheta\phi + \frac{1}{\lambda}\right)e, \\
F_1^{(1,4)} &= -\frac{\alpha_n\gamma_{dn}}{1-\alpha_n}\left[1 + \left(\vartheta\phi + \frac{1}{\lambda}\right)\right] + \vartheta(1-\phi)\left(\frac{\alpha_d\gamma_{nd}}{1-\alpha_d} + c\right) + a\left(\vartheta\phi + \frac{1}{\lambda}\right), \\
F_1^{(2,4)} &= \frac{a-c-\lambda}{1+\lambda} - \frac{\alpha_n\gamma_{dn}}{(1-\alpha_n)} - \frac{\alpha_d\gamma_{nd}}{(1-\alpha_d)}, \\
F_1^{(4,3)} &= -\left\{1 + \beta(1-\delta)\left[(1-\sigma)\mu^d - 1\right]\right\} - \Xi(1+\beta)\left[1-\beta(1-\delta)\right]
\end{aligned}$$

$$\begin{aligned}
\mathbf{F}_2 &= \left[\begin{array}{c} \mathbf{0}_{3\times5} \\ \mathbf{0}_{1\times2} \quad \Xi\left[1-\beta(1-\delta)\right] \quad \mathbf{0}_{1\times2} \\ \mathbf{0}_{1\times2} \quad \frac{1-\delta}{\delta} \quad \mathbf{0}_{1\times2} \end{array}\right], \\
\mathbf{C}_1 &= \left[\begin{array}{cc} \frac{1+\vartheta\phi+\frac{1}{\lambda}}{1-\alpha_n} & \frac{\vartheta(1-\phi)}{1-\alpha_d} \\ -\frac{1}{1-\alpha_n} & \frac{1}{1-\alpha_d} \\ \multicolumn{2}{c}{\mathbf{0}_{3\times2}} \end{array}\right].
\end{aligned}$$

Moreover, (46) can be re-written to include all the shocks $\mathbf{v}_t$ as

$$E_t\left[\mathbf{F}_0\kappa^*_{t+1}+\mathbf{F}_1\kappa^*_t+\mathbf{F}_2\kappa^*_{t-1}+\overline{\mathbf{C}}_1\mathbf{v}_t\right]=0 \tag{47}$$

with

$$\overline{\mathbf{C}}_1=\left[\begin{array}{cc}\mathbf{C}_1 & \mathbf{0}_{5\times 2}\end{array}\right].$$

APPENDIX C1: Log-linear Economy

Here we report the log-linear economy in extensive form:

$$\begin{aligned}
\widetilde{c}^n_t &= \frac{1}{\gamma}\left(\widehat{r}_t-E_t\pi^n_{t+1}-rr^*_t\right)+E_t\widetilde{c}^n_{t+1}+\frac{(1-\sigma)\,\mu^d}{\gamma}E_t\Delta\widetilde{d}_{t+1},\\
\widetilde{c}^n_t &= \frac{1}{\mu^n(1-\sigma)}\left\{\left[1-\mu^d(1-\sigma)\right]\widetilde{d}_t+\frac{1}{1-\beta(1-\delta)}\left[\left(\mu^n(1-\sigma)-1\right)\widetilde{c}_t+\mu^d(1-\sigma)\,\widetilde{d}_t-\widetilde{q}_t\right]+\right.\\
&\quad -\frac{(1-\delta)\,\beta}{[1-\beta(1-\delta)]}\left[\left(\mu^n(1-\sigma)-1\right)\widetilde{c}^n_{t+1}+\mu^d(1-\sigma)\,\widetilde{d}_{t+1}-\widetilde{q}_{t+1}\right]+\\
&\quad \left.+\Xi\left(\widetilde{d}_t-\widetilde{d}_{t-1}\right)-\beta\Xi\left(\widetilde{d}_{t+1}-\widetilde{d}_t\right)\right\}\\
\widetilde{c}^d_t &= \frac{1}{\delta}\widetilde{d}_t-\frac{1-\delta}{\delta}\widetilde{d}_{t-1},\\
\widetilde{rw}^n_t &= -\gamma\widetilde{c}^n_t-(1-\sigma)\,\mu^d\widetilde{d}_t+\vartheta(1-\phi)\,\widetilde{l}^d_t+\left(\vartheta\phi+\frac{1}{\lambda}\right)\widetilde{l}^n_t,\\
\widetilde{l}^n_t &= \lambda\left(\widetilde{rw}^n_t-\widetilde{rw}^d_t+\widetilde{q}_t\right)+\widetilde{l}^d_t,\\
\pi^n_t &= \beta E_t\pi^n_{t+1}+\frac{(1-\beta\theta^n)(1-\theta^n)}{\theta^n}\widetilde{rmc}^n_t+\eta^n_t,\\
\pi^d_t &= \beta E_t\pi^d_{t+1}+\frac{\left(1-\beta\theta^d\right)\left(1-\theta^d\right)}{\theta^d}\widetilde{rmc}^d_t+\eta^d_t,\\
\widetilde{y}^n_t &= \alpha_n\gamma_{nn}\widetilde{m}^{nn}_t+\alpha_n\gamma_{dn}\widetilde{m}^{dn}_t+(1-\alpha_n)\,\widetilde{l}^n_t,\\
\widetilde{y}^d_t &= \alpha_d\gamma_{nd}\widetilde{m}^{nd}_t+\alpha_d\gamma_{dd}\widetilde{m}^{dd}_t+(1-\alpha_d)\,\widetilde{l}^d_t,\\
\widetilde{y}^n_t &= \frac{C^n}{Y^n}\widetilde{c}^n_t+\frac{M^{nn}}{Y^n}\widetilde{m}^{nn}_t+\frac{M^{nd}}{Y^n}\widetilde{m}^{nd}_t,\\
\widetilde{y}^d_t &= \frac{C^d}{Y^d}\widetilde{c}^d_t+\frac{M^{dn}}{Y^d}\widetilde{m}^{dn}_t+\frac{M^{dd}}{Y^d}\widetilde{m}^{dd}_t,\\
\widetilde{rmc}^n_t &= \widetilde{rw}^n_t+\widetilde{l}^n_t-\widetilde{y}^n_t,\\
\widetilde{rmc}^d_t &= \widetilde{rw}^d_t+\widetilde{l}^d_t-\widetilde{y}^d_t,\\
\widetilde{rmc}^n_t &= \widetilde{m}^{nn}_t-\widetilde{y}^n_t,\\
\widetilde{rmc}^d_t &= \widetilde{m}^{nd}_t+\widetilde{q}_t-\widetilde{y}^d_t,\\
\widetilde{rmc}^n_t &= \widetilde{m}^{dn}_t-\widetilde{q}_t-\widetilde{y}^n_t,\\
\widetilde{rmc}^d_t &= \widetilde{m}^{dd}_t-\widetilde{y}^d_t,\\
\widetilde{q}_t &= \widetilde{q}_{t-1}+\pi^n_t-\pi^d_t-\Delta q^*_t.
\end{aligned}$$

where $\gamma=(1-\sigma)\,\mu^n-1$.

APPENDIX C2: The Model in Canonical Form

The log-linear system in canonical form reads as:

$$E_t\left[\mathbf{A}_0\widetilde{\boldsymbol{\xi}}_{t+1}+\mathbf{A}_1\widetilde{\boldsymbol{\xi}}_t+\mathbf{A}_2\widetilde{\boldsymbol{\xi}}_{t-1}+\mathbf{A}_3\widehat{r}_t+\mathbf{A}_4\boldsymbol{\kappa}_t^*+\mathbf{A}_5\boldsymbol{\kappa}_{t-1}^*+\mathbf{B}_1\mathbf{v}_t\right]=0, \tag{48}$$

$$\mathbf{v}_t=\mathbf{P}\mathbf{v}_{t-1}+\mathbf{e}_t, \tag{49}$$

where

$$\begin{aligned}
\widetilde{\boldsymbol{\xi}}_t &= \left[\begin{array}{cccccccccc} \widetilde{c}_t^n & \widetilde{y}_t^n & \widetilde{c}_t^d & \widetilde{y}_t^d & \widetilde{d}_t & \widetilde{rw}_t^n & \widetilde{rw}_t^d & \pi_t^n & \pi_t^d & \widetilde{q}_t \end{array}\right]', \\
\boldsymbol{\kappa}_t^* &= \left[\begin{array}{ccccc} c_t^{n*} & c_t^{d*} & d_t^* & q_t^* & rr_t^* \end{array}\right]', \\
\mathbf{v}_t &= \left[\begin{array}{cccc} z_t^n & z_t^d & \eta_t^n & \eta_t^d \end{array}\right]'.
\end{aligned}$$

The matrices of parameters are:

$$\begin{aligned}
\mathbf{A}_0 &= \left[\begin{array}{c}
-\gamma \quad 0_{1\times 3} \quad -\frac{\mu^d}{(1-\sigma)^{-1}} \quad 0_{1\times 2} \quad 1 \quad 0_{1\times 2} \\
(1-\delta)\beta\gamma \quad 0_{1\times 3} \quad A_0^{(2,5)} \quad 0_{1\times 4} \quad -\beta(1-\delta) \\
0_{3\times 10} \\
0_{1\times 7} \quad -\beta \quad 0_{1\times 2} \\
0_{1\times 8} \quad -\beta \quad 0 \\
0_{3\times 10}
\end{array}\right] \\
A_0^{(2,5)} &= \beta\left\{(1-\delta)(1-\sigma)\mu^d+\Xi\left[1-\beta(1-\delta)\right]\right\}
\end{aligned}$$

$$\mathbf{A}_1=\left[\begin{array}{c}
\gamma \quad 0_{1\times 3} \quad \frac{\mu^d}{(1-\sigma)^{-1}} \quad 0_{1\times 5} \\
\epsilon \quad 0_{1\times 3} \quad A_1^{(2,5)} \quad 0_{1\times 4} \quad 1 \\
0_{1\times 2} \quad 1 \quad 0 \quad -\frac{1}{\delta} \quad 0_{1\times 5} \\
\gamma \quad \frac{1-\vartheta\phi\lambda}{\lambda} \quad 0 \quad \frac{\vartheta}{(\phi-1)^{-1}} \quad \frac{\mu^d}{(1-\sigma)^{-1}} \quad \frac{\lambda+(1+\vartheta\phi\lambda)\alpha_n}{\lambda} \quad \frac{\vartheta\alpha_d}{(1-\phi)^{-1}} \quad 0_{1\times 2} \quad A_1^{(4,10)} \\
0 \quad -1 \quad 0 \quad 1 \quad 0 \quad (\alpha_n+\lambda) \quad -(\alpha_d+\lambda) \quad 0_{1\times 2} \quad (\alpha_n\gamma_{dn}+\alpha_d\gamma_{nd}+\lambda) \\
0_{1\times 5} \quad -\kappa^n(1-\alpha_n) \quad 0 \quad 1 \quad 0 \quad \kappa^n\alpha_n\gamma_{dn} \\
0_{1\times 6} \quad -\kappa^d(1-\alpha_d) \quad 0 \quad 1 \quad -\kappa^d\alpha_d\gamma_{nd} \\
-\frac{C^n}{Y^n} \quad \left(1-\frac{M^{nn}}{Y^n}\right) \quad 0 \quad -\frac{M^{nd}}{Y^n} \quad 0 \quad -\frac{(1-\alpha_n)M^{nn}}{Y^n} \quad -\frac{(1-\alpha_d)M^{nd}}{Y^n} \quad 0_{1\times 2} \quad A_1^{(8,10)} \\
0 \quad -\frac{M^{dn}}{Y^d} \quad -\frac{C^d}{Y^d} \quad \left(1-\frac{M^{dd}}{Y^d}\right) \quad 0 \quad -\frac{(1-\alpha_n)M^{dn}}{Y^d} \quad -\frac{(1-\alpha_d)M^{dd}}{Y^d} \quad 0_{1\times 2} \quad A_1^{(9,10)} \\
0_{1\times 7} \quad -1 \quad 1 \quad 1
\end{array}\right]$$

$$
\begin{aligned}
A_1^{(2,5)} &= -\left\{1+\beta(1-\delta)\left[(1-\sigma)\mu^d-1\right]\right\}-\Xi(1+\beta)\left[1-\beta(1-\delta)\right] \\
A_1^{(4,10)} &= -\vartheta(1-\phi)\alpha_d\gamma_{nd}+\left(\vartheta\phi+\frac{1}{\lambda}\right)\alpha_n\gamma_{dn} \\
A_1^{(8,10)} &= -\frac{M^{nd}}{Y^n}(\alpha_d\gamma_{nd}-1)+\frac{M^{nn}}{Y^n}\alpha_n\gamma_{dn} \\
A_1^{(9,10)} &= -\frac{M^{dn}}{Y^d}(1-\alpha_n\gamma_{dn})-\frac{M^{dd}}{Y^d}\alpha_d\gamma_{nd} \\
\kappa^n &= \frac{(1-\beta\theta^n)(1-\theta^n)}{\theta^n} \\
\kappa^d &= \frac{\left(1-\beta\theta^d\right)\left(1-\theta^d\right)}{\theta^d} \\
\epsilon &= \left[1-\beta(1-\delta)(1-\sigma)\mu^n\right]
\end{aligned}
$$

$$
\mathbf{A}_2=\begin{bmatrix} & 0_{2\times10} & \\ 0_{1\times4} & \frac{1-\delta}{\delta} & 0_{1\times5} \\ & 0_{6\times10} & \\ & 0_{1\times9} \quad -1 & \end{bmatrix}
$$

$$
\begin{aligned}
\mathbf{A}_3 &= \begin{bmatrix} -1 \\ 0_{9\times1} \end{bmatrix} \\
\mathbf{A}_4 &= \begin{bmatrix} 0_{1\times4} & 1 & \\ & 0_{8\times5} & \\ 0_{1\times3} & 1 & 0 \end{bmatrix} \\
\mathbf{A}_5 &= \begin{bmatrix} & 0_{9\times5} & \\ 0_{1\times3} & -1 & 0 \end{bmatrix}
\end{aligned}
$$

$$
\mathbf{B}_1=\begin{bmatrix} & 0_{5\times4} & \\ 0_{1\times2} & -1 & 0 \\ 0_{1\times3} & -1 & \\ & 0_{3\times4} & \end{bmatrix}
$$

Closing the Model with an Instrumental Policy Rule

Assume that the central bank settles the interest rate according to a simple rule of the form:

$$
r_t=\boldsymbol{\zeta}\boldsymbol{\xi}_t
$$

For instance, we can assume that:

$$
\begin{aligned}
r_t &= \chi_\pi\left[\omega\pi_t^n+(1-\omega)\pi_t^d\right], \\
\boldsymbol{\zeta} &= \begin{bmatrix} \mathbf{0}_{1\times7} & \chi_\pi\omega & \chi_\pi(1-\omega) & 0 \end{bmatrix}.
\end{aligned}
$$

Define:

$$
\overline{\boldsymbol{\xi}}_t=\begin{bmatrix} \boldsymbol{\xi}_t' & r_t \end{bmatrix}'.
$$

Thus (48) can be rewritten as:

$$
E_t\left[\overline{\mathbf{A}}_0\overline{\boldsymbol{\xi}}_{t+1}+\overline{\mathbf{A}}_1\overline{\boldsymbol{\xi}}_t+\overline{\mathbf{A}}_2\overline{\boldsymbol{\xi}}_{t-1}+\overline{\mathbf{A}}_4\boldsymbol{\kappa}_t^*+\overline{\mathbf{A}}_5\boldsymbol{\kappa}_{t-1}^*+\overline{\mathbf{B}}_1\mathbf{v}_t\right]=0
$$

$$
\begin{aligned}
\overline{\mathbf{A}}_0 &= \begin{bmatrix} \mathbf{A}_0 & \mathbf{0}_{10\times1} \\ \multicolumn{2}{c}{\mathbf{0}_{1\times11}} \end{bmatrix} \qquad \overline{\mathbf{A}}_1 = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0}_{10\times1} \\ -\boldsymbol{\zeta} & 1 \end{bmatrix} \\
\overline{\mathbf{A}}_2 &= \begin{bmatrix} \mathbf{A}_2 & \mathbf{0}_{10\times1} \\ \multicolumn{2}{c}{\mathbf{0}_{1\times11}} \end{bmatrix} \qquad \overline{\mathbf{A}}_4 = \begin{bmatrix} \mathbf{A}_4 \\ \mathbf{0}_{1\times5} \end{bmatrix} \\
\overline{\mathbf{A}}_5 &= \begin{bmatrix} \mathbf{A}_5 \\ \mathbf{0}_{1\times5} \end{bmatrix} \qquad \overline{\mathbf{B}}_1 = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{0}_{1\times4} \end{bmatrix}
\end{aligned}
$$

Following Sims (2002), we cast the model in the following form:

$$
\boldsymbol{\Gamma}_0 \mathbf{y}_t = \mathbf{c} + \boldsymbol{\Gamma}_1 \mathbf{y}_{t-1} + \boldsymbol{\Psi} \mathbf{z}_t + \boldsymbol{\Pi} \boldsymbol{\eta}_t \tag{50}
$$

where $\mathbf{c}$ is a vector of constant terms, $\mathbf{z}_t$ is an exogenously evolving, possibly serially correlated, random disturbance, and $\boldsymbol{\eta}_t$ is an expectational error, such that $E_t(\boldsymbol{\eta}_{t+1}) = 0$, $\forall\ t$. Moreover:

$$
\begin{aligned}
\mathbf{y}_t &= \begin{bmatrix} \mathbf{v}_t \\ \overline{\boldsymbol{\xi}}_t \\ E_t \overline{\boldsymbol{\xi}}_{t+1} \\ \kappa_t^* \\ E_t \kappa_{t+1}^* \end{bmatrix}, \qquad \boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\Sigma} \\ \mathbf{0}_{11\times4} \\ \mathbf{0}_{11\times4} \\ \mathbf{0}_{5\times4} \\ \mathbf{0}_{5\times4} \end{bmatrix}, \qquad \boldsymbol{\Pi} = \begin{bmatrix} \multicolumn{2}{c}{\mathbf{0}_{4\times16}} \\ \multicolumn{2}{c}{\mathbf{0}_{11\times16}} \\ \mathbf{I}_{11} & \mathbf{0}_{11\times5} \\ \multicolumn{2}{c}{\mathbf{0}_{5\times16}} \\ \mathbf{0}_{5\times11} & \mathbf{I}_5 \end{bmatrix}, \\
\boldsymbol{\Gamma}_0 &= \begin{bmatrix} \mathbf{I}_4 & \mathbf{0}_{4\times11} & \mathbf{0}_{4\times11} & \mathbf{0}_{4\times5} & \mathbf{0}_{4\times5} \\ \overline{\mathbf{B}}_1 & \overline{\mathbf{A}}_1 & \overline{\mathbf{A}}_0 & \overline{\mathbf{A}}_4 & \mathbf{0}_{11\times5} \\ \mathbf{0}_{11\times4} & \mathbf{I}_{11} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times5} & \mathbf{0}_{11\times5} \\ \overline{\mathbf{C}}_1 & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times11} & \mathbf{F}_1 & \mathbf{F}_0 \\ \mathbf{0}_{5\times4} & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times11} & \mathbf{I}_5 & \mathbf{0}_{5\times5} \end{bmatrix}, \\
\boldsymbol{\Gamma}_1 &= \begin{bmatrix} \mathbf{P} & \mathbf{0}_{4\times11} & \mathbf{0}_{4\times11} & \mathbf{0}_{4\times5} & \mathbf{0}_{4\times5} \\ \mathbf{0}_{11\times4} & -\overline{\mathbf{A}}_2 & \mathbf{0}_{11\times11} & -\overline{\mathbf{A}}_5 & \mathbf{0}_{11\times5} \\ \mathbf{0}_{11\times4} & \mathbf{0}_{11\times11} & \mathbf{I}_{11} & \mathbf{0}_{11\times5} & \mathbf{0}_{11\times5} \\ \mathbf{0}_{5\times4} & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times11} & -\mathbf{F}_2 & \mathbf{0}_{5\times5} \\ \mathbf{0}_{5\times4} & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times5} & \mathbf{I}_5 \end{bmatrix}, \\
\boldsymbol{\Sigma} &= diag \begin{bmatrix} \sigma^{z^n} & \sigma^{z^d} & \sigma^{\eta^n} & \sigma^{\eta^d} \end{bmatrix}.
\end{aligned}
$$

The $\boldsymbol{\eta}_t$ terms are not given exogenously. Instead, they are determined as part of the model solution. Models with more lags, or with lagged expectations, or with expectations of more distant future values, can be accommodated in this framework by expanding the $\mathbf{y}$ vector.

APPENDIX D: Relative Price in the Efficient Equilibrium with Perfect Labour Mobility

We now define the efficient equilibrium in the model with no frictions in both the goods and the labour market. On the labour market this condition, obtained for $\lambda \to \infty$, ensures that nominal salaries are equalized across sectors of the economy:

$$
W_t^{n^*} = W_t^{d^*} = W_t^*. \tag{51}
$$

Moreover, given the production subsidies that eliminate sectoral distortions due to monopolistic competition:

$$
P_t^{n^*} = MC_t^{n^*} \qquad\qquad P_t^{d^*} = MC_t^{d^*}. \tag{52}
$$

Conditions (51) and (52) imply that:

$$P_t^{n^*} = \left(\overline{\phi}^n\right)^{\frac{1}{1-\alpha_n\gamma_{nn}}} \left(P_t^{d^*}\right)^{\frac{\alpha_n\gamma_{dn}}{1-\alpha_n\gamma_{nn}}} \left(W_t^*\right)^{\frac{1-\alpha_n}{1-\alpha_n\gamma_{nn}}} \left(Z_t^n\right)^{-\frac{1}{1-\alpha_n\gamma_{nn}}}, \tag{53}$$

$$P_t^{d^*} = \left(\overline{\phi}^d\right)^{\frac{1}{1-\alpha_d\gamma_{dd}}} \left(P_t^{n^*}\right)^{\frac{\alpha_d\gamma_{nd}}{1-\alpha_d\gamma_{dd}}} \left(W_t^*\right)^{\frac{1-\alpha_d}{1-\alpha_d\gamma_{dd}}} \left(Z_t^d\right)^{-\frac{1}{1-\alpha_d\gamma_{dd}}}. \tag{54}$$

We then substitute (53) into (54) to eliminate W_t^*:

$$\left(P_t^{n^*}\right)^{\vartheta_n} = \Upsilon^{(1-\alpha_n\gamma_{nn})(1-\alpha_d)} \left(P_t^{d^*}\right)^{\vartheta_d} \left(Z_t^n\right)^{-(1-\alpha_d)} \left(Z_t^d\right)^{(1-\alpha_n)}$$

where

$$\Upsilon = \left(\overline{\phi}^n\right)^{\frac{1}{1-\alpha_n\gamma_{nn}}} \left(\overline{\phi}^d\right)^{-\frac{1}{1-\alpha_d}\frac{1-\alpha_n}{1-\alpha_n\gamma_{nn}}}$$

and

$$\vartheta_n = \vartheta_d = (1-\alpha_d)(1-\alpha_n\gamma_{nn}) + (\alpha_d\gamma_{nd})(1-\alpha_n).$$

Thus, after some trivial algebra we can show that the relative price reads as:

$$\begin{aligned} Q_t^* &= \frac{P_t^{n^*}}{P_t^{d^*}} = \Upsilon\left[\left(Z_t^n\right)^{-(1-\alpha_d)}\left(Z_t^d\right)^{1-\alpha_n}\right]^{\frac{1}{\varkappa+1}} \\ &= \Upsilon\left[\left(Z_t^n\right)^{-(1-\alpha_d)}\left(Z_t^d\right)^{1-\alpha_n}\right]^{\frac{1}{\varkappa+1}}. \end{aligned}$$

where

$$\varkappa = \alpha_n\alpha_d(\gamma_{nn}+\gamma_{dd}-1) - \alpha_n\gamma_{nn} - \alpha_d\gamma_{dd}.$$

APPENDIX E1: Second Order Approximation of the Utility Function

Following Rotemberg and Woodford (1998), we derive a well-defined welfare function from the utility function of the representative household:

$$\mathcal{W}_t = U\left(C_t^n, D_t\right) - V\left(L_t\right).$$

We start from a second order approximation of the utility from consumption of durable and non-durable goods:

$$\begin{aligned} U\left(C_t^n, D_t\right) \approx\; & U\left(C^n, D\right) + U_{C^n}\left(C^n, D\right)\left(C_t^n - C^n\right) + \frac{1}{2}U_{C^nC^n}\left(C^n, D\right)\left(C_t^n - C^n\right)^2 && (55) \\ & + U_D\left(C^n, D\right)\left(D_t - D\right) + \frac{1}{2}U_{DD}\left(C^n, D\right)\left(D_t - D\right)^2 + \frac{1}{2}\Xi U_D\left(C^n, D\right)\left(D_t - D_{t-1}\right)^2 \\ & + U_{C^nD}\left(C^n, D\right)\left(C_t^n - C^n\right)\left(D_t - D\right) + O\left(\|\xi\|^3\right), && (56) \end{aligned}$$

where $O\left(\|\xi\|^3\right)$ summarizes all terms of third order or higher. Notice that:

$$\begin{aligned}
U_D\left(C^n, D\right) &= \left(\mu^d C^n/\mu^n D\right) U_{C^n}\left(C^n, D\right), \\
U_{C^n C^n}\left(C^n, D\right) &= \left[\mu^n\left(1-\sigma\right)-1\right]\left(C^n\right)^{-1} U_{C^n}\left(C^n, D\right), \\
U_{DD}\left(C^n, D\right) &= \left[\mu^d\left(1-\sigma\right)-1\right]\left(\mu^d C^n/\mu^n D\right) U_{C^n}\left(C^n, D\right), \\
U_{C^n D}\left(C^n, D\right) &= \mu^d\left(1-\sigma\right) D^{-1} U_{C^n}\left(C^n, D\right).
\end{aligned}$$

As $\frac{C_t^n - C^n}{C^n} = \widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2$, where $\widehat{c}_t^n = \log\left(\frac{C_t^n}{C^n}\right)$ is the log-deviation from steady state under sticky prices, we obtain:

$$\begin{aligned}
U\left(C_t^n, D_t\right) \approx\;& U\left(C^n, D\right) + U_{C^n}\left(C^n, D\right) C^n\left[\widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2\right] + \\
& + \frac{1}{2}\left[\mu^n\left(1-\sigma\right)-1\right] U_{C^n}\left(C^n, D\right) C\left[\widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2\right]^2 + \\
& + U_D\left(C^n, D\right) D\left(\widehat{d}_t + \frac{1}{2}\widehat{d}_t^2\right) + \frac{1}{2}\left[\mu^d\left(1-\sigma\right)-1\right] U_D\left(C^n, D\right) D\left(\widehat{d}_t + \frac{1}{2}\widehat{d}_t^2\right)^2 + \\
& + \frac{1}{2}\Xi U_D\left(C^n, D\right) D\left(\widehat{d}_t - \widehat{d}_{t-1}\right)^2 + \\
& + \mu^d\left(1-\sigma\right) U_C\left(C^n, D\right) C^n\left[\widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2\right]\left(\widehat{d}_t + \frac{1}{2}\widehat{d}_t^2\right) + \text{t.i.p.} + O\left(\|\xi\|^3\right),
\end{aligned}$$

where t.i.p. collects terms independent of policy stabilization.

Next, we introduce a second order approximation to the transition law for the stock of durables. This will substitute out the linear term for durables in the expression above (see Erceg and Levin, 2006). The law of motion reads as:

$$D_t = \left(1-\delta\right) D_{t-1} + X_t.$$

For a general function $F\left(Y, X\right)$ the second orer Taylor approximation can be written as:

$$\begin{aligned}
F\left(Y, X\right) \approx\;& F_Y\left(Y, X\right) Y y + F_X\left(Y, X\right) X x + \frac{1}{2}\left(F_X\left(Y, X\right) X + F_{XX}\left(Y, X\right) X^2\right) x^2 \\
& + \frac{1}{2}\left(F_Y\left(Y, X\right) Y + F_{YY}\left(Y, X\right) Y^2\right) y^2 + F_{YX}\left(Y, X\right) Y X x y.
\end{aligned}$$

Now, we can rewrite the accumulation equation as:

$$F(D_{t-1}, C_t^d) = \log\left[\left(1-\delta\right) D_{t-1} + C_t^d\right].$$

Therefore:

$$\begin{aligned}
F_D &= \frac{(1-\delta)}{(1-\delta)D+C^d} = \frac{(1-\delta)}{(1-\delta)D+\delta D} = \frac{(1-\delta)}{D}, \\
F_{C^d} &= \frac{1}{(1-\delta)D+C^d} = \frac{1}{D}, \\
F_{DD} &= -\frac{(1-\delta)^2}{\left[(1-\delta)D+C^d\right]^2} = -\frac{(1-\delta)^2}{D^2}, \\
F_{C^dC^d} &= -\frac{1}{\left[(1-\delta)D+C^d\right]^2} = -\frac{1}{D^2}, \\
F_{DC^d} &= -\frac{1-\delta}{\left[(1-\delta)D+C^d\right]^2} = -\frac{1-\delta}{D^2}.
\end{aligned}$$

Considering that in the steady state $C^d = \delta D$:

$$\begin{aligned}
\widehat{d}_t &\approx \frac{(1-\delta)}{D} D\widehat{d}_{t-1} + \frac{1}{D}\delta D\widehat{c}_t^d + \\
&\quad + \frac{1}{2}\left[\frac{(1-\delta)}{D}D - \frac{(1-\delta)^2}{D^2}D^2\right]\widehat{d}_{t-1}^2 + \\
&\quad + \frac{1}{2}\left(\frac{1}{D}D - \frac{1}{D^2}D^2\right)\left(\widehat{c}_t^d\right)^2 - \frac{1-\delta}{D^2}\widehat{d}_{t-1}\widehat{x}_t \\
&\approx (1-\delta)\widehat{d}_{t-1} + \delta\widehat{c}_t^d + \frac{(1-\delta)\delta}{2}\widehat{d}_{t-1}^2 + \frac{(1-\delta)\delta}{2}\left(\widehat{c}_t^d\right)^2 - \frac{(1-\delta)\delta}{2}\widehat{c}_t^d\widehat{d}_{t-1} \\
&\approx (1-\delta)\widehat{d}_{t-1} + \delta\widehat{c}_t^d + \frac{(1-\delta)\delta}{2}\left(\widehat{d}_{t-1} - \widehat{c}_t^d\right)^2.
\end{aligned}$$

Thus:

$$\widehat{d}_t \approx (1-\delta)\widehat{d}_{t-1} + \delta\widehat{c}_t^d + \psi_t, \tag{57}$$

where:

$$\begin{aligned}
\widehat{\psi}_t &= \frac{(1-\delta)\delta}{2}\left(\widehat{c}_t^d - \widetilde{d}_{t-1}\right)^2 \\
&= \frac{(1-\delta)}{2\delta}\left(\widehat{d}_t - \widetilde{d}_{t-1}\right)^2.
\end{aligned}$$

Now, let us iterate backward (57), to obtain:

$$\textstyle\sum_{t=0}^{\infty}\beta^t\widehat{d}_t = \frac{1}{1-\beta(1-\delta)}d_0 + \sum_{t=0}^{\infty}\beta^t\left[\frac{\delta}{1-\beta(1-\delta)}\widehat{c}_t^d + \frac{1}{1-\beta(1-\delta)}\widehat{\psi}_t\right].$$

In turn, the term on the RHS will replace the one on the LHS into the intertemporal loss function.

The next step is to derive a second-order approximation for from labour disutility. Recall that:

$$\widehat{l}_t = \phi\widehat{l}_t^n + (1-\phi)\widehat{l}_t^d.$$

Therefore the second order approximation reads:

$$\begin{aligned} V\left(L_t\right) &\approx V_L\left(L\right)L\left[\phi\widehat{l_t^n}+\left(1-\phi\right)\widehat{l_t^d}+\frac{\phi\left(1+2\nu\phi\right)}{2}\left(\widehat{l_t^n}\right)^2+\frac{\left(1-\phi\right)\left[1+2\nu\left(1-\phi\right)\right]}{2}\left(\widehat{l_t^d}\right)^2\right]+ \\ &\quad +\text{t.i.p.}+O\left(\|\xi\|^3\right). \end{aligned}$$

After these preliminary steps, we need to find an expression for $\widehat{l_t^n}$ and $\widehat{l_t^d}$. Given the definition of the marginal cost, in equilibrium we get:

$$\begin{aligned} L_t^n &= \frac{\left(1-\alpha_n\right)MC_t^n}{W_t^n}\int_0^1 Y_{jt}^n dj=\frac{\left(1-\alpha_n\right)\overline{\phi}^n}{Z_t^n}\left(\frac{Q_t^{-\gamma_{dn}}}{RW_t^n}\right)^{\alpha_n}Y_t^n\int_0^1\left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n}dj, \\ L_t^d &= \frac{\left(1-\alpha_d\right)MC_t^d}{W_t^d}\int_0^1 Y_{kt}^d dk=\frac{\left(1-\alpha_d\right)\overline{\phi}^d}{Z_t^d}\left(\frac{Q_t^{\gamma_{nd}}}{RW_t^d}\right)^{\alpha_d}Y_t^d\int_0^1\left(\frac{P_{kt}^d}{P_t^d}\right)^{-\varepsilon_t^d}dk. \end{aligned}$$

Thus, we can report the linear approximation of the expressions above:

$$\begin{aligned} \widehat{l_t^n} &= -\alpha_n\gamma_{dn}\widehat{q}_t-\alpha_n\widehat{rw}_t^n-z_t^n+\widehat{y}_t^n+S_{nt}, \\ \widehat{l_t^d} &= \alpha_d\gamma_{nd}\widehat{q}_t-\alpha_d\widehat{rw}_t^d-z_t^d+\widehat{y}_t^d+S_{dt}, \end{aligned}$$

where:

$$S_{nt}=\log\left[\int_0^1\left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n}dj\right]\qquad S_{dt}=\log\left[\int_0^1\left(\frac{P_{kt}^d}{P_t^d}\right)^{-\varepsilon_t^d}dk\right]\tag{58}$$

If we set $\widehat{p}_{jt}^n$ to be the log-deviation of $\frac{P_{jt}^n}{P_t^n}$ from its steady state, which means that a second-order Taylor expansion of $\int_0^1\left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n}dj$ reads as:

$$\begin{aligned} \int_0^1\left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n}dj &\approx \int_0^1\left[1-\varepsilon^n\widehat{p}_{jt}^n-\varepsilon^n\widehat{p}_{jt}^n\widehat{\varepsilon}_t^n+\frac{1}{2}\left(\varepsilon^n\right)^2\left(\widehat{p}_{jt}^n\right)^2\right]dj+O\left(\|\xi\|^3\right) \\ &= 1-\varepsilon^n\mathbf{E}_i\widehat{p}_{jt}^n-\varepsilon^n\mathbf{E}_i\widehat{p}_{jt}^n\widehat{\varepsilon}_t^n+\frac{1}{2}\left(\varepsilon^n\right)^2\mathbf{E}_i\left(\widehat{p}_{jt}^n\right)^2+O\left(\|\xi\|^3\right), \end{aligned}$$

where $\mathbf{E}_i\widehat{p}_{jt}^n\equiv\int_0^1\widehat{p}_{jt}^n dj$ and $\mathbf{E}_i\left(\widehat{p}_{jt}^n\right)^2\equiv\int_0^1\left(\widehat{p}_{jt}^n\right)^2dj$. At this stage, we need an expression for $\mathbf{E}_i\widehat{p}_{jt}^n$. Let us start from

$$P_t^n=\left[\int_0^1\left(P_{jt}^n\right)^{1-\varepsilon_t^n}dj\right]^{\frac{1}{1-\varepsilon_t^n}},$$

which can be re-arranged as:

$$1\equiv\int_0^1\left(\frac{P_{jt}^n}{P_t^n}\right)^{1-\varepsilon_t^n}dj.$$

Following the procedure above, it can be shown that:

$$\left(\frac{P_{jt}^n}{P_t^n}\right)^{1-\varepsilon_t^n} \approx 1 + (1-\varepsilon^n)\,\widehat{p}_{jt}^n - \varepsilon^n \widehat{p}_{jt}^n \widehat{\varepsilon}_t^n + \frac{1}{2}(1-\varepsilon^n)^2 \left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right).$$

Substituting this into the preceeding equations yields:

$$0 = \int_0^1 \left[(1-\varepsilon^n)\,\widehat{p}_{jt}^n - \varepsilon^n \widehat{p}_{jt}^n \widehat{\varepsilon}_t^n + \frac{1}{2}(1-\varepsilon^n)^2 \left(\widehat{p}_{jt}^n\right)^2\right] dj + O\left(\|\xi\|^3\right),$$

which reduces to:

$$\mathbf{E}_i \widehat{p}_{jt}^n = \frac{\varepsilon^n - 1}{2}\mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right).$$

Thus:

$$\int_0^1 \left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n} dj = 1 + \frac{\varepsilon^n}{2}\mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right).$$

Now, notice that:

$$\mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 = \mathbf{E}_i \left[\left(p_{jt}^n\right)^2 - 2p_{jt}^n p_t^n + \left(p_t^n\right)^2\right] + O\left(\|\xi\|^3\right),$$

where lower case letters denote the log-value of the capital letters. Here we can use a first-order approximation of $p_t^n = \int_0^1 p_{jt}^n dj$, as this term is multiplied by other first-order terms each time it appears. With this, we have a second order approximation:

$$\mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 \equiv var_j p_{jt}^n.$$

Combining all of this the second-order approximation can be represented as:

$$S_{nt} = \frac{\varepsilon^n}{2} var_j p_{jt}^n + O\left(\|\xi\|^3\right).$$

Analogous steps in the sector producing durable goods lead us to:

$$S_{dt} = \frac{\varepsilon^d}{2} var_k p_{kt}^d + O\left(\|\xi\|^3\right).$$

Following Woodford (2003, Ch. 6, Proposition 6.3), we can obtain a correspondence between cross-sectional price dispersions in the two sectors and their inflation rates:

$$\begin{aligned} var_j p_{jt}^n &= \theta^n var_j p_{jt-1}^n + \frac{\theta^n}{1-\theta^n}\left(\pi_t^n\right)^2 + O\left(\|\xi\|^3\right), \\ var_k p_{kt}^d &= \theta^d var_k p_{kt-1}^d + \frac{\theta^d}{1-\theta^d}\left(\pi_t^d\right)^2 + O\left(\|\xi\|^3\right). \end{aligned}$$

Iterating these expressions forward leads to:

$$\sum_{t=0}^{\infty} \beta^t var_j p_{jt}^n = (\kappa^n)^{-1} \sum_{t=0}^{\infty} \beta^t (\pi_t^n)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right), \tag{59}$$

$$\sum_{t=0}^{\infty} \beta^t var_k p_{kt}^d = \left(\kappa^d\right)^{-1} \sum_{t=0}^{\infty} \beta^t \left(\pi_t^d\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right), \tag{60}$$

where

$$\begin{aligned} \kappa^n &= \frac{(1-\beta\theta^n)(1-\theta^n)}{\theta^n}, \\ \kappa^d &= \frac{\left(1-\beta\theta^d\right)\left(1-\theta^d\right)}{\theta^d}. \end{aligned}$$

After these preliminary steps, we can write $\mathcal{W}_t$ as:

$$\begin{aligned} \mathcal{W}_t \approx\ & U_{C^n}(C^n, D)\, C^n \left\{ \widehat{c}_t^n + \frac{1}{2}\left[\mu^n (1-\sigma)\right] (\widehat{c}_t^n)^2 + \left(\mu^d/\mu^n\right) \widehat{d}_t + \right. \\ & \left. + \frac{1}{2}\left[\mu^d (1-\sigma)\right] \left(\mu^d/\mu^n\right) \widehat{d}_t^2 + \mu^d (1-\sigma)\, \widehat{c}_t^n \widehat{d}_t + \frac{1}{2} \Xi \left(\mu^d/\mu^n\right) \left(\widehat{d}_t - \widehat{d}_{t-1}\right)^2 \right\} + \\ & - V_L(L)\, L \left\{ \phi \widehat{l}_t^n + (1-\phi)\, \widehat{l}_t^d + \right. \\ & \left. + \left(\frac{1+\nu}{2}\right) \left[\phi^2 \left(\widehat{l}_t^n\right)^2 + (1-\phi) \left(\widehat{l}_t^d\right)^2 + 2\phi(1-\phi)\, \widehat{l}_t^n \widehat{l}_t^d\right] \right\} + \\ & + \text{t.i.p.} + O\left(\|\xi\|^3\right). \end{aligned}$$

We now consider the linear terms in $\mathcal{W}_t$, which are collected under $\mathcal{LW}_t$:

$$\begin{aligned} \mathcal{LW}_t =\ & \frac{U_{C^n}(C^n, D)\, C^n}{\mu^n} \left\{ \mu^n \widehat{c}_t^n + \mu^d \widehat{d}_t \right\} + \\ & - \left\{ V_L(L)\, L\phi \left(-\alpha_n \gamma_{dn} \widehat{q}_t - \alpha_n \widehat{rw}_t^n + \widehat{y}_t^n\right) + \right. \\ & \left. + (1-\phi) \left(\alpha_d \gamma_{nd} \widehat{q}_t - \alpha_d \widehat{rw}_t^d + \widehat{y}_t^d\right) \right\} + \\ & + \text{t.i.p.} + O\left(\|\xi\|^2\right). \end{aligned}$$

We subsitute for the real wage from marginal cost expressions to get:

$$\begin{aligned} \mathcal{LW}_t =\ & \frac{U_{C^n}(C^n, D)\, C^n}{\mu^n} \left\{ \mu^n \widehat{c}_t^n + \mu^d \widehat{d}_t \right\} + \\ & - V_L(L)\, L\phi \left(\frac{1}{1-\alpha_n} \widehat{y}_t^n - \frac{\alpha_n \gamma_{nn}}{1-\alpha_n} \widehat{m}_t^{nn} - \frac{\alpha_n \gamma_{dn}}{1-\alpha_n} \widehat{m}_t^{dn} \right) + \\ & - V_L(L)\, L (1-\phi) \left(\frac{1}{1-\alpha_d} \widehat{y}_t^d - \frac{\alpha_d \gamma_{nd}}{1-\alpha_d} \widehat{m}_t^{nd} - \frac{\alpha_d \gamma_{dd}}{1-\alpha_d} \widehat{m}_t^{dd} \right) + \\ & + \text{t.i.p.} + O\left(\|\xi\|^2\right). \end{aligned}$$

After substituting the second order approximation for the accumulation equation of durables we get:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\mathcal{L}\mathcal{W}_t &= U_{C^n}\left(C^n,D\right)C^n\sum_{t=0}^{\infty}\beta^t\left\{\widehat{c}_t^n+\frac{\delta}{1-\beta\left(1-\delta\right)}\frac{\mu^d}{\mu^n}\widehat{c}_t^d\right\}+ \\ &\quad -V_L\left(L\right)L\sum_{t=0}^{\infty}\beta^t\left\{\phi\left(\frac{1}{1-\alpha_n}\widehat{y}_t^n-\frac{\alpha_n\gamma_{nn}}{1-\alpha_n}\widehat{m}_t^{nn}-\frac{\alpha_n\gamma_{dn}}{1-\alpha_n}\widehat{m}_t^{dn}\right)+\right. \\ &\quad \left.+\left(1-\phi\right)\left(\frac{1}{1-\alpha_d}\widehat{y}_t^d-\frac{\alpha_d\gamma_{nd}}{1-\alpha_d}\widehat{m}_t^{nd}-\frac{\alpha_d\gamma_{dd}}{1-\alpha_d}\widehat{m}_t^{dd}\right)\right\}+ \\ &\quad +\text{t.i.p.}+O\left(\|\xi\|^2\right).\end{aligned} \tag{61}$$

Notice that the following linear approximations for the market clearing conditions hold:

$$\begin{aligned}\widehat{y}_t^n &= \frac{1-\alpha_n}{\phi}\mu^n\widehat{c}_t^n+\alpha_n\gamma_{nn}\widehat{m}_t^{nn}+\frac{\left(1-\alpha_n\right)\left(1-\phi\right)}{\phi\left(1-\alpha_d\right)}\alpha_d\gamma_{nd}\widehat{m}_t^{nd}, \\ \widehat{y}_t^d &= \frac{\delta\mu^d\left(1-\alpha_d\right)}{\left(1-\phi\right)\left[1-\beta\left(1-\delta\right)\right]}\widehat{c}_t^d+\frac{\left(1-\alpha_d\right)\phi}{\left(1-\phi\right)\left(1-\alpha_n\right)}\alpha_n\gamma_{dn}\widehat{m}_t^{dn}+\alpha_d\gamma_{dd}\widehat{m}_t^{dd}.\end{aligned}$$

It can be shown that, in the steady state, the following relationships hold:

$$V_{L^n}\left(L^n\right)L^n=\phi V_L\left(L\right)L \qquad V_{L^d}\left(L^d\right)L^d=\left(1-\phi\right)V_LL\left(L\right)$$

Moreover, the presence of production subsidies allows us to express the steady state marginal rate of substitution between labour supply and consumption of non-durable goods as:

$$\begin{aligned}\frac{-V_{L^n}\left(L^n\right)}{U_{C^n}\left(C^n\right)} &= \frac{Y^n\left(1-\alpha_n\right)}{L^n}, \\ \frac{-V_{L^d}\left(L^d\right)}{U_{C^n}\left(C^n\right)} &= \frac{Y^d\left(1-\alpha_d\right)}{L^dQ}.\end{aligned}$$

It is now convenient to express the marginal utility from non-durable consumption in terms of the marginal utility derived from total consumption:

$$U_{C^n}\left(C^n\right)=U_H\left(H\right)H\mu^n.$$

Therefore, we can re-write (61) as:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\mathcal{L}\mathcal{W}_t &= U_H\left(H\right)H\sum_{t=0}^{\infty}\beta^t\left\{\left(\mu^n\widehat{c}_t^n+\frac{\delta\mu^d}{1-\beta\left(1-\delta\right)}\widehat{c}_t^d\right)+\right. \\ &\quad -\mu^n\left(\frac{C^n}{Y^n}\right)^{-1}\left(1-\alpha_n\right)\left[-\alpha_n\gamma_{dn}\widehat{q}_t-\alpha_n\widehat{rw}_t^n-z_t^n+\widehat{y}_t^n\right]+ \\ &\quad \left.-\mu^n\left(\frac{C^n}{Y^d}\right)^{-1}\left(1-\alpha_d\right)Q^{-1}\left[\alpha_d\gamma_{nd}\widehat{q}_t-\alpha_d\widehat{rw}_t^d-z_t^d+\widehat{y}_t^d\right]\right\}+ \\ &\quad +\text{t.i.p.}+O\left(\|\xi\|^2\right).\end{aligned}$$

It is now possible to show, given the linearized market clearing conditions in the two sectors, that $\sum_{t=0}^{\infty}\beta^t\mathcal{L}\mathcal{W}_t=0$. Therefore, the linear term in $\mathcal{W}_t$ can be dropped. Thus we are left only with second order terms:

$$
\begin{aligned}
\sum_{t=0}^{\infty}\beta^{t}\mathcal{W}_{t} &\approx U_{H}(H)H\sum_{t=0}^{\infty}\beta^{t}\left\{\frac{1-\sigma}{2}\left(\mu^{n}\widehat{c}_{t}^{n}+\mu^{d}\widehat{d}_{t}\right)^{2}+\frac{1}{1-\beta(1-\delta)}\mu^{d}\widehat{\psi}_{t}+\frac{\mu^{d}}{2}\Xi\left(\widehat{d}_{t}-\widehat{d}_{t-1}\right)^{2}+\right.\\
&-\frac{\Theta}{2}\left[\phi\varepsilon^{n}(\kappa^{n})^{-1}(\pi_{t}^{n})^{2}+(1-\phi)\varepsilon^{d}\left(\kappa^{d}\right)^{-1}\left(\pi_{t}^{d}\right)^{2}\right]+\\
&\left.-\left(\frac{1+\nu}{2}\right)\Theta^{-1}\left[\mu^{n}\widehat{c}_{t}^{n}+\frac{\delta\mu^{d}}{1-\beta(1-\delta)}\widehat{c}_{t}^{d}\right]^{2}\right\}+\\
&+\text{t.i.p.}+O\left(\|\xi\|^{3}\right),
\end{aligned}
$$

where

$$
\Theta=\left(\frac{C^{n}}{Y^{n}}\right)^{-1}\frac{(1-\alpha_{n})\mu^{n}}{\phi}=\frac{\mu^{n}[1-\beta(1-\delta)]+\mu^{d}\delta}{1-\beta(1-\delta)}.
$$

We next consider the deviation of social welfare from its Pareto-optimal level:

$$
\begin{aligned}
\sum_{t=0}^{\infty}\beta^{t}\widetilde{\mathcal{W}}_{t} &= \sum_{t=0}^{\infty}\beta^{t}\left(\mathcal{W}_{t}-\mathcal{W}_{t}^{*}\right)\approx\\
&-\frac{U_{H}(H)H}{2}\Theta\sum_{t=0}^{\infty}\beta^{t}\left\{\frac{\sigma-1}{\Theta}\left(\mu^{n}\widetilde{c}_{t}^{n}+\mu^{d}\widetilde{d}_{t}\right)^{2}+\right.\\
&+\left[\mu^{d}\Theta^{-1}\Xi+(1-\delta)(1-\omega)\delta^{-2}\right]\left(\widetilde{d}_{t}-\widetilde{d}_{t-1}\right)^{2}+\\
&\left.+\varsigma\left[\varpi(\pi_{t}^{n})^{2}+(1-\varpi)\left(\pi_{t}^{d}\right)^{2}\right]+(1+\nu)\left[\omega\widetilde{c}_{t}^{n}+(1-\omega)\widetilde{c}_{t}^{d}\right]^{2}\right\}+\text{t.i.p.}+O\left(\|\xi\|^{3}\right),
\end{aligned}
$$

where the following notation has been introduced:

$$
\begin{aligned}
\omega &= \frac{\mu^{n}[1-\beta(1-\delta)]}{\mu^{n}[1-\beta(1-\delta)]+\mu^{d}\delta},\\
\varpi &= \frac{\phi\varepsilon^{n}(\kappa^{n})^{-1}}{\varsigma},\\
\varsigma &= \phi\frac{\varepsilon^{n}}{\kappa^{n}}+(1-\phi)\frac{\varepsilon^{d}}{\kappa^{d}}.
\end{aligned}
$$

APPENDIX E2: General Model Setup for Optimal Monetary Policy Analysis

The welfare function can be written in matrix form as:

$$
\mathcal{SW}_{0}=-\frac{U_{H}(H)H}{2}E_{0}\sum_{t=0}^{\infty}\beta^{t}\left\{\mathbf{X}_{t}'\mathbf{W}\mathbf{X}_{t}+\widehat{r}_{t}R\widehat{r}_{t}\right\}, \tag{62}
$$

where the state vector is augmented to include the past stock of durables:

$$
\begin{aligned}
\mathbf{X}_{t} &= \begin{bmatrix}\boldsymbol{\xi}_{t}' & \widetilde{d}_{t-1}\end{bmatrix}',\\
\widetilde{\boldsymbol{\xi}}_{t} &= \begin{bmatrix}\widetilde{c}_{t}^{n} & \widetilde{y}_{t}^{n} & \widetilde{c}_{t}^{d} & \widetilde{y}_{t}^{d} & \widetilde{d}_{t} & \widetilde{rw}_{t}^{n} & \widetilde{rw}_{t}^{d} & \pi_{t}^{n} & \pi_{t}^{d} & \widetilde{q}_{t}\end{bmatrix}'.
\end{aligned}
$$

Notice that $R=0$ in (62). Moreover, we define

$$\mathbf{W}=\mathbf{K}'\overline{\mathbf{Q}}\mathbf{K}$$

where $\overline{\mathbf{Q}}$ is a diagonal matrix and $\mathbf{K}$ reports the weighting coefficients included in the loss function:

$$\mathbf{K}=\begin{bmatrix} \mu^{n} & \mathbf{0}_{1\times3} & \mu^{d} & \mathbf{0}_{1\times6} \\ \mathbf{0}_{1\times2} & -1 & \mathbf{0}_{1\times7} & 1 \\ \mathbf{0}_{1\times7} & 1 & \mathbf{0}_{1\times2} & 0 \\ \mathbf{0}_{1\times8} & 1 & 0 & 0 \\ \mu^{n} & \mathbf{0}_{1\times3} & \frac{\delta}{1-\beta(1-\delta)}\mu^{d} & \mathbf{0}_{1\times6} \end{bmatrix},$$

$$\overline{\mathbf{Q}}=diag\left[\ (\sigma-1)\,,\ \ \left[\mu^{d}\Xi+\Theta\left(1-\delta\right)\left(1-\omega\right)\delta^{-2}\right],\ \ \Theta\varepsilon^{n}\phi\left(\kappa^{n}\right)^{-1},\ \ \Theta\left(1-\phi\right)\varepsilon^{d}\left(\kappa^{d}\right)^{-1},\ \ \Theta\left(1+\nu\right)\ \right].$$

To tackle the optimization problem we follow Dennis (2007) and consider the system under its structural (singular) form:

$$\begin{aligned} \mathcal{SW}_{0} &= E_{0}\sum_{t=0}^{\infty}\beta^{t}\left\{\mathbf{X}_{t}'\mathbf{W}\mathbf{X}_{t}+\right. \\ &\quad \left.+\,2\boldsymbol{\lambda}_{t}'\left[\mathbf{G}_{0}\mathbf{X}_{t+1}+\mathbf{G}_{1}\mathbf{X}_{t}+\mathbf{G}_{2}\mathbf{X}_{t-1}+\mathbf{G}_{3}r_{t}+\mathbf{G}_{4}\boldsymbol{\kappa}_{t}^{*}+\mathbf{G}_{5}\boldsymbol{\kappa}_{t-1}^{*}+\mathbf{N}_{1}\mathbf{v}_{t}\right]\right\}. \end{aligned}$$

For $t>t_{0}$ the following conditions hold:

$$\begin{aligned} \frac{\partial\mathcal{SW}_{0}}{\partial\boldsymbol{\lambda}_{t}} &= E_{t}\left[\mathbf{G}_{0}\mathbf{X}_{t+1}+\mathbf{G}_{1}\mathbf{X}_{t}+\mathbf{G}_{2}\mathbf{X}_{t-1}+\mathbf{G}_{3}r_{t}+\mathbf{G}_{4}\boldsymbol{\kappa}_{t}^{*}+\mathbf{G}_{5}\boldsymbol{\kappa}_{t-1}^{*}+\mathbf{N}_{1}\mathbf{v}_{t}\right]=\mathbf{0}, \\ \frac{\partial\mathcal{SW}_{0}}{\partial\mathbf{X}_{t}} &= \mathbf{W}\mathbf{X}_{t}+\mathbf{G}_{1}'\boldsymbol{\lambda}_{t}+\beta^{-1}\mathbf{G}_{0}'\boldsymbol{\lambda}_{t-1}+\beta\mathbf{G}_{2}'E_{t}\left(\boldsymbol{\lambda}_{t+1}\right)=\mathbf{0}, \\ \frac{\partial\mathcal{SW}_{0}}{\partial r_{t}} &= \mathbf{G}_{3}'\boldsymbol{\lambda}_{t}=0. \end{aligned}$$

Notice that the last condition suggests that the multiplier associated with the IS curve, λ_{t}^{IS}, is equal zero. The matrices of the system now read as:

$$\begin{aligned} \mathbf{G}_{0} &= \begin{bmatrix} \mathbf{A}_{0} & \mathbf{0}_{10\times1} \\ \mathbf{0}_{1\times10} & 0 \end{bmatrix}, & \mathbf{G}_{1} &= \begin{bmatrix} \mathbf{A}_{1} & \mathbf{0}_{10\times1} \\ \mathbf{0}_{1\times10} & 1 \end{bmatrix}, \\ \mathbf{G}_{2} &= \begin{bmatrix} \mathbf{A}_{2} & \mathbf{0}_{10\times1} \\ \mathbf{b} & 0 \end{bmatrix}, & \mathbf{b} &= \begin{bmatrix} \mathbf{0}_{1\times4} & -1 & \mathbf{0}_{1\times5} \end{bmatrix}, \\ \mathbf{G}_{3} &= \begin{bmatrix} \mathbf{A}_{3} \\ 0 \end{bmatrix}, & \mathbf{G}_{4} &= \begin{bmatrix} \mathbf{A}_{4} \\ \mathbf{0}_{1\times5} \end{bmatrix}, \\ \mathbf{G}_{5} &= \begin{bmatrix} \mathbf{A}_{5} \\ \mathbf{0}_{1\times5} \end{bmatrix}, & \mathbf{N}_{1} &= \begin{bmatrix} \mathbf{B}_{1} \\ \mathbf{0}_{1\times5} \end{bmatrix}. \end{aligned}$$

Following Sims, under the commitment optimal policy the economy evolves according to:

$$\boldsymbol{\Gamma}_{0}\mathbf{y}_{t}=\boldsymbol{\Gamma}_{1}\mathbf{y}_{t-1}+\mathbf{c}+\boldsymbol{\Psi}\mathbf{z}_{t}+\boldsymbol{\Pi}\boldsymbol{\eta}_{t}\qquad t=1,\ldots,T \tag{63}$$

where $\mathbf{c}$ is a vector of constants, $\mathbf{z}_{t}$ is an exogenously evolving, possibly serially correlated, random disturbance, and $\boldsymbol{\eta}_{t}$ is an expectational error, satisfying $E_{t}(\boldsymbol{\eta}_{t+1})=0$, all t. The $\boldsymbol{\eta}_{t}$ terms are not given exogenously, but instead are treated as determined as part of the model solution. Models with more lags, or with lagged expectations, or with expectations of more distant future values, can be accommodated in this framework by expanding the $\mathbf{y}$ vector. Specifically, the whole model can be cast in the form above by writing:

$$\mathbf{y}_t = \begin{bmatrix} \mathbf{v}_t \\ \mathbf{X}_t \\ r_t \\ E_t\mathbf{X}_{t+1} \\ \kappa_t^* \\ E_t\kappa_{t+1}^* \\ \boldsymbol{\lambda}_t \\ E_t\boldsymbol{\lambda}_{t+1} \end{bmatrix}, \qquad \mathbf{\Pi} = \begin{bmatrix} \mathbf{0}_{15\times27} \\ \mathbf{I}_{11} \quad \mathbf{0}_{11\times16} \\ \mathbf{0}_{5\times27} \\ \mathbf{0}_{5\times11} \quad \mathbf{I}_5 \quad \mathbf{0}_{5\times11} \\ \mathbf{0}_{11\times27} \\ \mathbf{0}_{11\times16} \quad \mathbf{I}_{11} \\ \mathbf{0}_{1\times27} \end{bmatrix}, \qquad \mathbf{\Psi} = \begin{bmatrix} \mathbf{\Sigma} \\ \mathbf{0}_{55\times4} \end{bmatrix},$$

$$\mathbf{\Gamma}_0 = \begin{bmatrix} \mathbf{I}_4 & \mathbf{0}_{4\times11} & \mathbf{0}_{4\times1} & \mathbf{0}_{4\times11} & \mathbf{0}_{4\times5} & \mathbf{0}_{4\times5} & \mathbf{0}_{4\times11} & \mathbf{0}_{4\times11} \\ \mathbf{N}_1 & \mathbf{G}_1 & \mathbf{G}_3 & \mathbf{G}_0 & \mathbf{G}_4 & \mathbf{0}_{11\times5} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times11} \\ \mathbf{0}_{11\times4} & \mathbf{I}_{11} & \mathbf{0}_{11\times1} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times5} & \mathbf{0}_{11\times5} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times11} \\ \overline{\mathbf{C}}_1 & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times1} & \mathbf{0}_{5\times11} & \mathbf{F}_1 & \mathbf{F}_0 & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times11} \\ \mathbf{0}_{5\times4} & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times1} & \mathbf{0}_{5\times11} & \mathbf{I}_5 & \mathbf{0}_{5\times5} & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times11} \\ \mathbf{0}_{11\times4} & \mathbf{W} & \mathbf{0}_{11\times1} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times5} & \mathbf{0}_{11\times5} & \mathbf{G}_1' & \beta\mathbf{G}_2' \\ \mathbf{0}_{11\times4} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times1} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times5} & \mathbf{0}_{11\times5} & \mathbf{I}_{11} & \mathbf{0}_{11\times11} \\ \mathbf{0}_{1\times4} & \mathbf{0}_{1\times11} & \mathbf{0}_{1\times1} & \mathbf{0}_{1\times11} & \mathbf{0}_{1\times5} & \mathbf{0}_{1\times5} & \mathbf{G}_3' & \mathbf{0}_{1\times11} \end{bmatrix},$$

$$\mathbf{\Gamma}_1 = \begin{bmatrix} \mathbf{P} & \mathbf{0}_{4\times11} & \mathbf{0}_{4\times1} & \mathbf{0}_{4\times11} & \mathbf{0}_{4\times5} & \mathbf{0}_{4\times5} & \mathbf{0}_{4\times11} & \mathbf{0}_{4\times11} \\ \mathbf{0}_{11\times4} & -\mathbf{G}_2 & \mathbf{0}_{11\times1} & \mathbf{0}_{11\times11} & -\mathbf{G}_5 & \mathbf{0}_{11\times5} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times11} \\ \mathbf{0}_{11\times4} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times1} & \mathbf{I}_{11} & \mathbf{0}_{11\times5} & \mathbf{0}_{11\times5} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times11} \\ \mathbf{0}_{5\times4} & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times1} & \mathbf{0}_{5\times11} & -\mathbf{F}_2 & \mathbf{0}_{5\times5} & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times11} \\ \mathbf{0}_{5\times4} & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times1} & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times5} & \mathbf{I}_5 & \mathbf{0}_{5\times11} & \mathbf{0}_{5\times11} \\ \mathbf{0}_{11\times4} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times1} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times5} & \mathbf{0}_{11\times5} & -\beta^{-1}\mathbf{G}_0' & \mathbf{0}_{11\times11} \\ \mathbf{0}_{11\times4} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times1} & \mathbf{0}_{11\times11} & \mathbf{0}_{11\times5} & \mathbf{0}_{11\times5} & \mathbf{0}_{11\times11} & \mathbf{I}_{11} \\ \mathbf{0}_{1\times4} & \mathbf{0}_{1\times11} & \mathbf{0}_{1\times1} & \mathbf{0}_{1\times11} & \mathbf{0}_{1\times5} & \mathbf{0}_{1\times5} & \mathbf{0}_{1\times11} & \mathbf{0}_{1\times11} \end{bmatrix},$$

where $\mathbf{\Sigma} = diag \begin{bmatrix} \sigma^{z^n} & \sigma^{z^d} & \sigma^{\eta^n} & \sigma^{\eta^d} \end{bmatrix}$.

APPENDIX F: Alternative Policy Regimes

The following alternative regimes are considered in Section 4.3:

$$
\begin{aligned}
\widetilde{\mathcal{W}}_t^{IT} &= \left(\pi_t^{IT}\right)^2, \\
\widetilde{\mathcal{W}}_t^{GT} &= \left(\widetilde{x}_t^{GT}\right)^2, \\
\widetilde{\mathcal{W}}_t^{FIT} &= \widetilde{\mathcal{W}}_t^{IT} + (1+v)\,\widetilde{\mathcal{W}}_t^{GT}, \\
\widetilde{\mathcal{W}}_t^{ITDS} &= \widetilde{\mathcal{W}}_t^{IT} + S\left(\Delta \widetilde{d}_t\right)^2,
\end{aligned}
$$

where

$$
\begin{aligned}
\pi_t^{IT} &= \left\{\pi_t^{core}, \pi_t^{agg}\right\} \\
\pi_t^{core} &= \varpi \pi_t^n + (1-\varpi)\,\pi_t^d, \\
\pi_t^{agg} &= \phi \pi_t^n + (1-\phi)\,\pi_t^d,
\end{aligned}
$$

and

$$
\begin{aligned}
\widetilde{x}_t^{GT} &= \left\{\widetilde{x}_t^c, \widetilde{x}_t^p\right\} \\
\widetilde{x}_t^c &= \omega \widetilde{c}_t^n + (1-\omega)\,\widetilde{c}_t^d, \\
\widetilde{x}_t^p &= \phi \widetilde{y}_t^n + (1-\phi)\,\widetilde{y}_t^d.
\end{aligned}
$$