Chained Financial Frictions and Credit Cycles*

Federico Lubello† Ivan Petrella‡ Emiliano Santoro§

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Abstract

We focus on the connection between bank collateral and macroeconomic volatility. To this end, we develop a tractable credit economy à la Kiyotaki and Moore (1997) where bankers intermediate funds between savers and borrowers. Bankers' ability to borrow from the savers is bounded by the limited enforceability of deposit contracts. If bankers default, savers acquire the right to liquidate bankers' asset-holdings. However, due to the vertically integrated structure of our credit economy, savers anticipate that liquidating financial assets (i.e., bank loans) is subordinated to borrowers being solvent on their debt obligations. This friction limits the collateralization of bankers' financial assets beyond that of other assets that are not involved in more than one layer of financial contracting. In this context, increasing the pledgeability of financial assets reduces the spread between the loan and the deposit rate, attenuating capital misallocation as it emerges in this type of credit economies. We uncover a close connection between the collateralization of bank loans, macroeconomic amplification and the degree of procyclicality of bank leverage. A regulator may reduce macroeconomic volatility through the introduction of a countercyclical capital buffer, while a fixed capital adequacy requirement displays limited stabilization power.

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†Banque Centrale du Luxembourg. *Address*: 2, Boulevard Royal, L-2983 Luxembourg. *E-mail*: federico.lubello@bcl.lu.

‡Warwick Business School and CEPR. *Address:* Warwick Business School, The University of Warwick, Coventry, CV4 7AL, UK. *E-mail*: Ivan.Petrella@wbs.ac.uk.

§Department of Economics, University of Copenhagen. *Address*: Østerfarimagsgade 5, Building 26, 1353 Copenhagen, Denmark. *E-mail*: emiliano.santoro@econ.ku.dk.

1 Introduction

Lending relationships are typically plagued by information asymmetries that play a major role in shaping macroeconomic outcomes. In this respect, banks are of central importance, as they are involved in at least two layers of financial contracting through their intermediation activity between savers and borrowers. This paper focuses on the macroeconomic implications of banks' portfolio decisions over different assets in the presence of chained financial frictions, intended as the combination of limited enforceability of both deposit and loan contracts. To this end, we devise a tractable model that integrates collateralized borrowing as popularized by Kiyotaki and Moore (1997) (KM, hereafter) with the limited enforceability of the deposit contracts. As in recent contributions on banking and the macroeconomy (see, *inter alia*, Gertler and Kiyotaki, 2015), deposits are secured by a fraction of their assets that are pledged as collateral. They key departure from these studies consists of considering different degrees of liquidity – and, thus, pledgeability – for different types of bank assets.

Financial institutions typically resort to collateralized debt to raise funds, providing assets as a guarantee in case of default on their debt obligations. This is the case for non-traditional banking activities – with sale and repurchase agreements (repos) employed as a main source of funding – as well as for commercial banks – where securitized-banking often supplements more traditional intermediation activities. In fact, banks employ financial collateral both for currency management purposes and, more recently, as part of non-standard monetary policy frameworks.[1] A vast literature has focused on quantifying the dynamic multiplier emerging from the limited enforceability of debt contracts in economies à la KM.[2] While most of the contributions in this tradition have emphasized the role of borrowers' collateral for the amplification of macroeconomic shocks, the role of bank collateral has generally been overlooked.

In our model the ability of bankers to intermediate funds between savers and borrowers rests on the composition of different assets they are able to pledge as collateral. Along with extending bank loans, bankers may also invest in an infinite-lived asset, 'capital', whose main purpose is to serve as a buffer against which the intermediary is trusted to be able to meet its financial obligations (Bernanke and Gertler, 1985). Deposits are bounded from above by bankers' holdings of both types of collateral assets. However, due to the vertically integrated structure of our credit economy savers anticipate that, in case of bankers' default, liquidating their financial claims (i.e., bank loans) is subordinated to borrowers being solvent on their debt obligations. This friction affects savers' perceived liquidity of bankers' financial assets beyond that of capital,[3] inducing a transaction cost depositors have to bear in order to liquidate bank loans. If the latter are regarded as relatively illiquid, savers will be less prone to accept them as collateral.

[1] The set of assets that central banks accept from commercial banks generally includes government bonds and other debt instruments issued by public sectors and international/supranational institutions. In some cases, also securities issued by the private sector can be accepted, such as covered bank bonds, uncovered bank bonds, asset-backed securities or corporate bonds.

[2] See Kocherlakota (2000), Krishnamurthy (2003) and Cordoba and Ripoll (2004), *inter alia*.

[3] In the remainder we will refer to *financial assets* and *bank loans* interchangeably, while *capital goods* will also be referred to as *real assets*.

A key feature of the model is that combining limited enforceability of deposit and loan contracts reduces the interest rate on loans below the one that would prevail in a standard economy where only loans are secured by collateral assets. This allows borrowers to extend their capital holdings, contributing to increase total production in the steady state and alleviating capital misallocation as it emerges in economies à la KM, where borrowers hold too little capital in equilibrium, due to constrained borrowing. This property has a striking implication for equilibrium dynamics: as the propagation of technology shifts crucially rests on the distribution of real assets, envisaging financially constrained intermediaries into an otherwise standard KM economy produces a 'banking attenuator' that is neither linked to monopolistic competition in the intermediation activity and interest rate-setting rigidities (Gerali *et al.*, 2010), nor to the procyclicality of the external finance premium (Goodfriend and McCallum, 2007).

The main distinction between different types of bank collateral lies in the way they affect bankers' incentives to intermediate funds. Both assets have the potential to relax bankers' financial constraint. However, while increasing real assets exacerbates capital misallocation and reduces lending through a negative externality on borrowers' demand for credit, increasing financial assests compresses the spread between the loan and the deposit rate, thus attenuating capital misallocation. This feature of the model has key implications for equilibrium dynamics under different degrees of collateralization of bank loans. A relatively scarce liquidity of bankers' financial assets amplifies the response of gross output to productivity shocks. As in KM, a positive technology shift reallocates capital from the lenders to the borrowers. On one hand, this allows borrowers to expand their borrowing capacity. On the other hand, a decline in bankers' real assets is typically counteracted by an expansion in bank loans. As the latter are perceived to be increasingly illiquid, the compensation effect is gradually muted, so that bankers need to cut their capital investment further to meet borrowers' higher demand for credit. In turn, the response of total production – which increases in borrowers' real assets, *ceteris paribus* – is amplified, relative to situations in which deposit contracts involve relatively low transaction costs in case of bankers' default.

Importantly, the model produces a countercyclical 'flight to quality' in bankers' optimal asset allocation (see, e.g., Lang and Nakamura, 1995): during expansions (contractions), banks increase (decrease) their holdings of the inherently riskier assets – bank loans – while decreasing (increasing) their capital-holdings, which do not bear any risk of default. As a result, 'too much' borrowing capacity is allocated during boom states and 'too little' in bad states, inducing a procyclical bank leverage and generating excessive fluctuations in credit, output and asset prices. From a normative viewpoint, we study to which extent a hypothetical banking regulator may intervene to smooth the amplitude of these fluctuations by impairing the endogenous propagation mechanism of shocks that hinges on capital misallocation. A constant capital-to-asset ratio is shown to attenuate the transmission of technology shifts, although the gap between borrowers' and bankers' marginal product of capital cannot entirely be closed. By contrast, the regulator may successfully attenuate the economy's response to the productivity shock by devising a state-dependent capital buffer that induces a countercyclical bank leverage and stabilizes fluctuations in borrowers' collateral.

The present paper is strictly related to a growing literature that seeks to introduce financial intermediation into well established quantitative macroeconomic frameworks, so as to account for a number of distinctive features of the last financial crisis (see, *inter alia*, Gertler and Kiyotaki, 2010). To name a few, Gertler *et al.* (2012) allow intermediaries to issue outside equity, thus making risk exposure an endogenous choice of the banking sector, while Gertler and Kiyotaki (2015) devise a model of banking that allows for liquidity mismatch and bank runs. More recently, Hirakata *et al.* (2017) have introduced chained financial contracts into a dynamic general equilibrium model à la Bernanke *et al.* (1999).[4] The common trait of these contributions and many others in this tradition is to look at different sources of funding of financial intermediaries – thus emphasizing the composition of the right-hand side of banks' balance sheet – while typically considering only one type of asset – bank loans. We deviate from this approach and focus on the role of limited enforceability of deposit contracts in a setting where banks may invest in different assets,[5] whose distinctive trait is to bear different degrees of liquidity depending on whether they are involved in more than one layer of financial relationships.

The paper is also part of a rapidly developing banking literature on the role of macroprudential policy-making. Some recent examples include Van den Heuvel (2008), Admati *et. al* (2010), Hellwig (2010), Martinez-Miera and Suarez (2012), Angeloni and Faia (2013), Harris *et. al* (2015), Clerc *et. al* (2015) and Begenau (2015). The common trait of these contributions is to rely on medium to large scale dynamic general equilibrium models. While an obvious advantage of this modeling approach is to allow for a variety of shocks, transmission channels and alternative policy settings, our framework allows for a neater interpretation of the interplay between bankers' balance sheet and capital misallocation, providing a rationale for imposing countercyclical capital ratios. In this respect, our framework is more closely related to Gersbach and Rochet (2016), who show that complete markets do not sufficiently stabilize credit-driven fluctuations, thus providing a rationale for a countercyclical macroprudential policy intervention.

The rest of the paper is organized as follows: Section 2 presents the framework; Section 3 discusses the steady-state equilibrium; Section 4 focuses on equilibrium dynamics in the neighborhood of the steady state and the amplification of shocks to productivity in connection with the degree of financial collateralization; Section 5 examines the role of macroprudential policy-making in reducing capital misallocation to smooth macroeconomic fluctuations; Section 6 concludes.

[4] In contrast to the present framework – which is based on costly enforcement of both deposit and loan contracts – they consider costly state verification problems applying to both intermediaries and entrepreneurs.

[5] In this respect, our framework is closer to Chen (2001), who stresses the importance of of moral hazard behavior of both bankers and entrepeneurs in a quantitative model á la Holmstrom and Tirole (1997). However, in this framework there is no role for liquidity assessment of different bank assets.

2 Model

The economy is populated by three types of infinitely-lived, unit-sized, agents: savers, bankers and borrowers.[6] There are two layers of financial relationships: savers make deposits to the bankers, who act as financial intermediaries and extend credit to the borrowers. Two goods are traded in this economy: a durable asset, 'capital', and a non-durable consumption good. Capital, which is held by both bankers and borrowers, does not depreciate and is fixed in total supply to one. All agents have linear preferences defined over non-durable consumption. The remainder of this section provides further details on the key characteristics of the actors populating the economy and their decision rules.

2.1 Savers

Savers are the most patient agents in the economy. In each period, they are endowed with an exogenous non-produced income. We assume that savers are neither capable of monitoring the activity of the borrowers, nor of enforcing direct financial contracts with them. As a result, savers make deposits at the financial intermediaries. The linearity of their preferences implies that savers are indifferent between consumption and deposits in equilibrium, so that gross interest rate on savings (deposits), R^S, equals their rate of time preference, $1/\beta^S$. Savers' budget constraint reads as:

$$c_t^S + b_t^S = u^S + R^S b_{t-1}^S, \tag{1}$$

where c_t^S denotes the consumption of non-durables, b_t^S is the amount of savings and u^S is an exogenous endowment.

2.2 Borrowers

Borrowers' ability to obtain external funding is bounded by the limited enforceability of debt contracts. In line with Jermann and Quadrini (2012) we assume that, should borrowers default, bankers acquire the right to liquidate the stock of capital, k_t^B. Based on the predicted outcomes of the renegotiation, borrowers are subject to an enforcement constraint. Neither bankers nor borrowers are able to observe the liquidation value before the actual default, though borrowers have all the bargaining power in the liquidation process. With probability $1 - \omega$ ($\omega \in [0, 1]$) bankers expect to recover no collateral asset after a default, while with probability ω bankers expect to be able to recover $E_t q_{t+1} k_t^B$, where q_t denotes the capital price at time t.

To derive the renegotiation outcome, we consider the following default scenarios:

1. *Bankers expect to recover* $E_t q_{t+1} k_t^B$. Since bankers can expropriate the whole stock of capital, borrowers have to make a payment that leaves bankers indifferent between liquidation and allowing borrowers to preserve the stock of collateral assets. This requires

[6] The model is a variation of the 'Credit Cycles' framework of KM.

borrowers to make a payment at least equal to $E_t q_{t+1} k_t^B$, so that the ex-post value of defaulting for the bankers is:

$$R^B b_t^B - E_t q_{t+1} k_t^B, \tag{2}$$

where R^B denotes the gross loan rate and b_t^B is the loan.

2. *Bankers expect to recover no collateral.* If the liquidation value is zero, liquidation is clearly not the best option for the borrowers. Therefore, borrowers have no incentive to pay the loan back. The ex-post default value in this case is:

$$R^B b_t^B. \tag{3}$$

Therefore, enforcement requires that the expected value of non defaulting is not smaller than the expected value of defaulting, that is:

$$0 \geq \omega \left[R^B b_t^B - E_t q_{t+1} k_t^B \right] + (1 - \omega) R^B b_t^B, \tag{4}$$

which reduces to

$$R^B b_t^B \leq \omega E_t q_{t+1} k_t^B. \tag{5}$$

According to (5), the maximum amount of credit borrowers may access is such that the sum of principal and interest, $R^B b_t^B$, equals a fraction of the value of borrowers' capital in period $t+1$. Borrowers also face a flow-of-funds constraint:

$$c_t^B + R^B b_{t-1}^B + q_t (k_t^B - k_{t-1}^B) = b_t^B + y_t^B, \tag{6}$$

where c_t^B and y_t^B denote borrowers' consumption and production of perishable goods, respectively. As in KM, borrowers are assumed to combine capital and labor – the latter being supplied inelastically – through a linear production technology, $y_t^B = \alpha_t k_{t-1}^B$, with α_t being a multiplicative productivity shifter: $\log \alpha_t = \rho \log \alpha_{t-1} + u_t$, where $\rho \in [0, 1)$ and u_t is an iid shock.

Borrowers maximize their utility under the collateral and the flow-of-funds constraints, taking R^B as given. The corresponding Lagrangian reads as:

$$\begin{aligned} \mathcal{L}_t^B \quad = \quad & E_0 \sum_{t=0}^{\infty} \left(\beta^B\right)^t \left\{ c_t^B - \vartheta_t^B \left[c_t^B + R^B b_{t-1}^B + q_t (k_t^B - k_{t-1}^B) - b_t^B - \alpha_t k_{t-1}^B \right] \right. \\ & \left. - \upsilon_t \left(b_t^B - \omega \frac{q_{t+1} k_t^B}{R^B} \right) \right\}, \end{aligned} \tag{7}$$

where β^B denotes borrowers' discount factor, while ϑ_t^B and υ_t are the multipliers associated with borrowers' budget and collateral constraint, respectively. The first-order conditions are:

$$\frac{\partial \mathcal{L}_t^B}{\partial b_t^B} = 0 \Rightarrow -\beta^B R^B E_t \vartheta_{t+1}^B + \vartheta_t^B - \upsilon_t = 0; \tag{8}$$

$$\frac{\partial \mathcal{L}_t^B}{\partial k_t^B} = 0 \Rightarrow -\vartheta_t^B q_t + \beta^B E_t \left[\vartheta_{t+1}^B q_{t+1}\right] + \beta^B E_t \left[\vartheta_{t+1}^B \alpha_{t+1}\right] + \omega \upsilon_t E_t \left[\frac{q_{t+1}}{R^B}\right] = 0. \tag{9}$$

Condition (8) implies that a marginal decrease in borrowing today expands next period's utility and relaxes the current period's borrowing constraint. As for (9), acquiring an additional unit of capital today allows to expand future consumption not only through the conventional capital gain and dividend channels, but also through the feedback effect of the expected collateral value on the price of capital. As we consider linear preferences (i.e., $\vartheta_t^B = \vartheta^B = 1$), (8) implies $\upsilon_t = \upsilon = 1 - \beta^B R^B$.[7] Thus, the collateral constraint is binding in the neighborhood of the steady state as long as $R^B < 1/\beta^B$, which is imposed throughout the rest of the analysis. Finally, (9) can be rewritten as

$$q_t = \frac{\beta^B R^B + \omega\left(1 - \beta^B R^B\right)}{R^B} E_t q_{t+1} + \beta^B E_t \alpha_{t+1}. \tag{10}$$

2.3 Bankers

Bankers' primary activity consists of intermediating funds between savers and borrowers. However, their ability to attract savers' financial resources is bounded by the limited enforceability of the deposit contracts, given that bankers may divert assets for personal use (see also Gertler and Kiyotaki, 2015). At this stage of the analysis we abstract from the implementation of regulatory bank capital ratios to discourage bankers' moral hazard behavior, while focusing on the characteristics of the deposit contract. We assume that, upon bankers' default, savers acquire the right to liquidate bankers' asset-holdings.[8] At the time of contracting the amount of deposits, though, the liquidation value of bankers' assets is uncertain. In this respect, the enforcement problem is isomorphic to that characterizing bankers' lending relationship with the borrowers, in line with the arguments of Jermann and Quadrini (2012). However, due to the vertically integrated structure of the credit economy, we envisage an additional friction that limits the pledgeability of bank loans beyond that of capital. While real assets remain in the availability of the bankers for the entire duration of the deposit contract – so that savers can frictionlessly liquidate them in case of bankers' default – the resources corresponding to bankers' financial claims are in the availability of the borrowers. Therefore, from the perspective of the savers the possibility to liquidate b_t^B in the event of a default of the banking sector goes beyond the capacity of the bankers to honor the deposit contract, while being subordinated to borrowers' solvency. In light of this, we assume that savers account for a transaction

[7] Steady-state variables are reported without the time subscript.

[8] There are two main considerations why this assumption is a reasonable one: first, savers have no direct use of the collateral assets; second, even if collateral assets represent an attractive investment opportunity, savers have no experience in hedging.

cost they would have to bear for seizing bank loans: $(1-\xi)\, b_t^B$, where $\xi \in [0,1]$ indexes savers' perceived liquidity of bankers' financial assets. In the extreme case savers regard bank loans as completely illiquid and do not accept them as collateral, ξ is set to zero (i.e., financial frictions are no longer chained), while $\xi = 1$ corresponds to a situation in which savers attach no risk to their ability of liquidating financial assets in case bankers' default.

To derive the renegotiation outcome, we assume that with probability $1-\chi$ savers expect to recover no collateral, while with probability χ the expected recovery value is $E_t q_{t+1} k_t^I + \xi b_t^B$, where k_t^I denotes bankers' holdings of capital and ξb_t^B represents the amount of bank loans held as collateral, net of transaction costs. This implies the following default scenarios:

1. *Savers expect to recover* $E_t q_{t+1} k_t^I + \xi b_t^B$. Since savers expect to expropriate the stock of real and financial assets after bearing a transaction cost $(1-\xi)\, b_t^B$, bankers have to make a payment that leaves savers indifferent between liquidation and allowing borrowers to preserve the stock of collateral assets. This requires bankers to make a payment at least equal to $E_t q_{t+1} k_t^I + \xi b_t^B$, so that the ex-post value of defaulting for the bankers is:

$$R^S b_t^S - E_t q_{t+1} k_t^I - \xi b_t^B. \tag{11}$$

2. *Savers expect to recover no collateral.* If the liquidation value is zero, liquidation is clearly not the best option for the savers. Therefore, bankers have no incentive to pay deposits back. The ex-post default value in this case is:

$$R^S b_t^S. \tag{12}$$

Enforcement requires that the expected value of not defaulting is not smaller than the expected value of defaulting, so that:

$$0 \geq \chi \left[R^S b_t^S - E_t q_{t+1} k_t^I - \xi b_t^B\right] + (1-\chi) R^S b_t^S, \tag{13}$$

which reduces to

$$R^S b_t^S \leq \chi \left(E_t q_{t+1} k_t^I + \xi b_t^B\right), \tag{14}$$

according to which deposits should be limited from above by a fraction of the discounted expected collateral value. Notably, bankers' collateral constraint embodies the notion that real and financial assets have different degrees of liquidity (see also Bernanke and Gertler, 1985).[9] In fact, (14) echoes the liquidity constraint envisaged by Benigno and Nisticò (2017), where safe and pseudo-safe assets co-exist and both contribute to set the maximum amount of resources

[9] As χ affects the collateralization of both real and financial assets, to disentangle between the two in the remainder we will refer to 'financial collateralization' as the pledgeability of bank loans that is exclusively captured by their liquidity, as indexed by ξ.

available for consumption. In their case, while the entire stock of safe assets (i.e., money) is available to finance private expenditure, only a fraction of pseudo-safe assets can be employed to cover consumption, hence displaying less than perfect liquidity.

A number of considerations are in order when examining the role of the capital goods held by the bankers. First, this asset mainly serves as a buffer against which the intermediary is trusted to be able to meet its financial obligations.[10] In addition, k_t^I is important in that it breaks the tight link between deposits and lending – which would be otherwise embodied by a binding deposit contract – thus allowing for the possibility that a countercyclical 'flight to quality' drives the supply of credit. In the present context, such an effect would translate into bankers' allocating relatively more resources to capital investment – which, unlike bank loans, does not bear any risk of default – during adverse peridos. In turn, this mechanism may open the route to the emergence of credit crunch episodes (Bernanke and Lown, 1991). Finally, as in Bernanke and Gertler (1985) we assume that capital is productive, being employed by bankers to invest in projects on their own behalf.[11] Specifically, bankers' production technology is assumed to feature the following properties:

$$y_t^I = \alpha_t G(k_{t-1}^I), \tag{15}$$

with $G^{'} > 0$, $G^{''} < 0$, $G^{'}\left(0\right) > \varrho > G^{'}\left(1\right)$,[12] and

$$\varrho \equiv \frac{R^B \beta^B \left[R^S \left(1-\beta^I\right) - \chi\left(1-\beta^I R^S\right)\right]}{R^S \beta^I \left[R^B \left(1-\beta^B\right) - \omega\left(1-\beta^B R^B\right)\right]}, \tag{16}$$

where β^I denotes bankers' discount factor and (16) is required to ensure an internal solution in which both bankers and borrowers demand capital.[13]

Bankers' flow-of-funds constraint reads as:

$$c_t^I + b_t^B + R^S b_{t-1}^S + q_t(k_t^I - k_{t-1}^I) = b_t^S + R^B b_{t-1}^B + y_t^I, \tag{17}$$

[10] Broadly speaking, k_t^I may be seen as corresponding to bankers' security holdings, which are typically more liquid, as compared with bank loans.

[11] One can envisage the banking sector as being a two-member household sharing externalities and pooling resources. One member of the household specializes in credit intermediation, whereas the other specializes in production. Therefore, the first household member has to determine the amount of resources to be lent to the borrower, as well as the amount to be extended to the other member of the household. When funds are kept within the household, agency costs are mitigated. Yet, this comes at the cost of bankers-producers accessing a decreasing returns to scale technology.

[12] Assuming a decreasing returns to scale technology available to the borrowers would not alter our key results. As we will see in the next section, it is the relatively higher impatience of borrowers, combined with their collateral constraint, that endows them with a suboptimal stock of capital. This point is also discussed in KM. Introducing a decreasing returns to scale technology would only hinder the analytical tractability of the model.

[13] The role of this property will be discussed further in Section 4.1.

where c_t^I denotes bankers' consumption. The Lagrangian for bankers' optimization reads as

$$\begin{aligned}\mathcal{L}_t^I &= E_0 \sum_{t=0}^{\infty} \left(\beta^I\right)^t \{c_t^I - \vartheta_t^I [c_t^I + R^S b_{t-1}^S + b_t^B + q_t (k_t^I - k_{t-1}^I) \\ &\quad -b_t^S - R^B b_{t-1}^B - \alpha_t G(k_{t-1}^I)] - \delta_t \left(b_t^S - \chi \frac{q_{t+1}}{R^S} k_t^I - \chi \xi \frac{b_t^B}{R^S} \right) \Big\},\end{aligned} \tag{18}$$

where ϑ_t^I and δ_t are the multipliers associated with bankers' budget constraint and enforcement constraint, respectively. The first-order conditions are:

$$\frac{\partial \mathcal{L}_t^I}{\partial b_t^S} = 0 \Rightarrow -R^S \beta^I E_t \vartheta_{t+1}^I + \vartheta_t^I - \delta_t = 0; \tag{19}$$

$$\frac{\partial \mathcal{L}_t^I}{\partial b_t^B} = 0 \Rightarrow R^B \beta^I E_t \vartheta_{t+1}^I - \vartheta_t^I + \frac{1}{R^S} \chi \xi \delta_t = 0; \tag{20}$$

$$\frac{\partial \mathcal{L}_t^I}{\partial k_t^I} = 0 \Rightarrow -\vartheta_t^I q_t + \beta^I E_t \left[\vartheta_{t+1}^I q_{t+1}\right] + \beta^I E_t \left[\vartheta_{t+1}^I \alpha_{t+1} G'(k_t^I)\right] + \delta_t \chi \frac{E_t \left[q_{t+1}\right]}{R^S} = 0. \tag{21}$$

As we assume linear preferences, $\vartheta_t^I = \vartheta^I = 1$. Therefore, conditions (19) and (20) imply that the financial constraint holds with equality in the neighborhood of the steady state (i.e., $\delta_t = \delta > 0$) as long as (i) $R^S \beta^I < 1$ and (ii) $R^B \beta^I < 1$. Specifically, condition (i) implies that bankers are relatively more impatient than savers,[14] while condition (ii) implies that, unless either χ or ξ equal zero, bankers charge a lending rate that is lower than their rate of time preference, as extending loans allows them to relax their financial constraint:

$$R^B = \frac{R^S - \chi \xi \left(1 - \beta^I R^S\right)}{\beta^I R^S}, \tag{22}$$

from which it is possible to write down the spread between the loan and the deposit rate:

$$R^B - R^S = \frac{\beta^S - \beta^I}{\beta^I \beta^S} - \chi \xi \frac{1 - \beta^I R^S}{\beta^I R^S}. \tag{23}$$

The first term on the right-hand side of the equality is the spread that would prevail if bankers could not borrow off their loans (i.e., if $\{\chi, \xi\} = 0$), while the second term captures how bankers' financial constraint affects their ability to intermediate funds. Increasing χ and/or ξ compresses the spread. As implied by (20), greater pledgeability of financial assets increases the collateral value that savers expect to recover in case of bankers' default. This relaxes the financial constraint, eases more deposits and translates into a higher credit supply, thus compressing the lending rate.

Although (20) and (21) imply that an increase in either real or financial assets relaxes borrowers' financial constraint, the distinction between the two types of collateral has crucial implications for bankers' incentives to intermediate funds between savers and borrowers. On

[14] In this respect, imposing $\beta^I R^S = 1$ reduces the model to the conventional KM economy.

one hand, while increasing k_t^I expands bankers' lending capacity, it also exerts a negative externality on borrowers' demand for credit by decreasing their collateral. On the other hand, increasing b_t^B attenuates the debt-enforcement problem between bankers and borrowers, as implied by the reduction of the spread between the loan and the deposit rate. As it will be discussed in Section 4.2, such a distinction has key implications for equilibrium dynamics.

Finally, from (21) we can retrieve the Euler equation governing bankers' investment in real assets:

$$q_t = \frac{R^S \beta^I + \chi \left(1 - \beta^I R^S\right)}{R^S} E_t q_{t+1} + \beta^I E_t \left[\alpha_{t+1} G'\left(k_t^I\right)\right]. \tag{24}$$

By relaxing (i) and allowing for $\beta^I R^S = 1$ (i.e., assuming that bankers are as impatient as savers), (24) reduces to lenders' euler equation in the conventional direct-credit economy à la KM. Under these circumstances, bankers are no longer financially constrained. As we shall see in the next section, this implies both a higher loan rate and a higher user cost of capital from the perspective of the bankers/lenders,[15] as compared with what observed when bankers face a binding collateral constraint. These properties will play a crucial role for both the long-run and the short-run behavior of the model economy.

2.4 Market Clearing

To close the model, we need to state the market-clearing conditions. We know that the total supply of capital equals one: $k_t^I + k_t^B = 1$. As for the consumption goods market, the aggregate resource constraint reads as:

$$y_t = y_t^I + y_t^B, \tag{25}$$

where y_t denotes the total demand of consumption goods.

The aggregate demand and supply for credit are given by the two enforcement constraints (holding with equality) faced by borrowers and bankers, respectively:

$$b_t^B = \omega \frac{E_t q_{t+1} k_t^B}{R^B}, \tag{26}$$

$$b_t^B = \frac{1}{\xi \chi} \left(R^S b_t^S - \chi E_t q_{t+1} k_t^I\right). \tag{27}$$

Finally, as savers are indifferent between any path of consumption and savings, the amount of deposits depends on bankers' capitalization. Thus, the markets for deposits and final goods are cleared according to the Walras' Law.

[15] In the original KM framework lenders are labelled as 'gatherers', while borrowers are named 'farmers'.

3 Steady State

Financial frictions characterizing both the savers-bankers relationship and the bankers-borrowers relationship deeply affect the welfare properties of the model. Examining their interaction in the long-run is key for understanding their propagation of technology shocks.

In the remainder we impose, without loss of generality, $G(k^I_{t-1}) = \left(k^I_{t-1}\right)^{\mu}$, with $\mu \in [0,1]$. Evaluating (10) in the non-stochastic steady state returns:

$$q = \frac{R^B \beta^B}{\left(1-\beta^B\right) R^B - \omega\left(1-\beta^B R^B\right)}. \tag{28}$$

From (24) we retrieve the marginal product of bankers' capital, as a function of its price:

$$G^{'}\left(k^I\right) = \frac{R^S\left(1-\beta^I\right) - \chi\left(1-\beta^I R^S\right)}{R^S \beta^I} q, \tag{29}$$

so that Equations (28), (29) and $k^I + k^B = 1$ pin down borrowers' and bankers' holdings of capital. In turn, these allow us to characterize the key inefficiency at work in the economy. Importantly, financial collateralization only affects the steady state of the economy through its impact on R^B, which in turn influences the capital price through borrowers' Euler, as implied by (28). Therefore, having savers investing in real assets in place of the bankers would alter neither the role of liquidity in bankers' financial collateral, nor the key aggregate properties of the model.

[Insert Figure 1]

Figure 1 provides a sketch of the long-run equilibrium of the economy. On the horizontal axis, borrowers' demand for capital is measured from the left, while bankers' demand from the right. The sum of the two equals one. On the vertical axis we report the marginal product of capital of both borrowers and bankers. Borrowers' marginal product of capital is indicated by the line ACE^*, while bankers' marginal product is represented by the line DE^0E^*. The first-best allocation would be attained at E^0, where the product of capital owned by the bankers and the borrowers is the same, at the margin. In our economy, however, the steady-state equilibrium is at E^*, where the marginal product of capital of the borrowers ($mpk^B = 1$) exceeds that of the bankers ($mpk^I = \varrho$). That is, relative to the first best too little capital is used by the borrowers, due to their financial constraint. As discussed by KM, this type of capital misallocation implies a loss of output relative to the first-best, as indicated by the area CE^0E^*.[16] *Remark 1* elaborates on the relationship between borrowers' and bankers' marginal product of capital:

Remark 1 *As long as $\beta^B < \beta^I$, bankers' marginal product of capital is lower than that of the borrowers.*

[16] The area under the solid line, ACE^*D, is the steady-state output.

In fact, imposing $G^{'}(k^I) < 1$ returns the following inequality:

$$\beta^B - \beta^I < \frac{\beta^B \chi \left(1 - \beta^I R^S\right) \left(R^B + \omega\xi\right)}{R^S \left(R^B - \omega\right)}. \tag{30}$$

As we assume $\beta^B < \beta^I$, the left-hand side of the inequality is negative, while its right-hand side is positive, given that both $\beta^I R^S < 1$ and $R^B > \omega$ hold by assumption. Therefore, a defining feature of the equilibrium is that the marginal product of borrowers' capital is higher than that of the bankers, given that the former cannot borrow as much as they want. As a result, any shift in capital usage from the borrowers to the bankers will lead to a first-order decline in aggregate output, as it will become evident when exploring the linearized economy. With respect to this property, the present economy is isomorphic to that put forward by KM, as the suboptimality of the steady-state equilibrium allocation ultimately rests on borrowers' relatively higher impatience.

It is immediate to verify that, under $\beta^I R^S = 1$ (i.e., in a direct-credit economy à la KM, where savers and bankers have identical degrees of impatience), the productivity gap between bankers and borrowers would be even higher. This is due to bankers charging a higher loan rate and attaining a higher steady-state user cost of capital, which exacerbates the inefficiency in capital allocation. This is clearly indicated by the additional loss of output, relative to the first-best, as captured by the trapezoid $C_{KM}CE^*E^*_{KM}$ (where E^*_{KM} indicates the steady-state equilibrium in the KM setting). Therefore, a key result is that envisaging chained financial frictions induces bankers to hold less capital and increase their marginal product – thus setting the steady-state equilibrium on a more efficient allocation – as compared with the baseline KM economy.[17]

Given this key property of the setting under examination, it is important to understand how bankers' collateral impacts on capital misallocation. As we are primarily interested in the interplay between chained financial frictions and macroeconomic amplification, the remainder of the analysis will mostly focus on the impact of financial collateralization on both long-run and short-run outcomes. To formalize our next result, we define the productivity gap between borrowers and bankers as

$$mpk^B - mpk^I \equiv \Delta = 1 - \varrho. \tag{31}$$

The following summarizes the impact of financial collateralization on the productivity gap:

Proposition 1 *Increasing the collateralization of bank loans (ξ) reduces the gap between bankers' and borrowers' marginal product of capital (Δ).*

Proof. *See Appendix A.* ■

A higher degree of financial collateralization expands bankers' lending capacity and compresses the spread charged over the deposit rate. In turn, lower lending rates allow borrowers

[17] Importantly, raising the loan-to-value ratio implied by the loan contract up to its upper limit (i.e., $\omega = 1$) in the baseline KM economy would not allow to replicate the equilibrium outcome in our extended economy.

to increase their borrowing capacity through a higher collateral value, *ceteris paribus*. The combination of these effects is such that mpk^I increases in the degree of financial collateralization, reducing the productivity gap with respect to the borrowers.[18] This factor will play a key role in determining the size of the response of gross output to a technology shock, as we will see in Section 4.1.

4 Equilibrium Dynamics

To examine equilibrium dynamics, we log-linearize the Euler equations of both borrowers and bankers around the non-stochastic steady state.[19] As for the borrowers:

$$\hat{q}_t = \phi E_t \hat{q}_{t+1} + (1-\phi) E_t \hat{\alpha}_{t+1}, \tag{32}$$

where $\phi \equiv \frac{\beta^B R^B + \omega\left(1-\beta^B R^B\right)}{R^B}$. As for the bankers:

$$\hat{q}_t = \lambda E_t \hat{q}_{t+1} + (1-\lambda) E_t \hat{\alpha}_{t+1} + \frac{1-\lambda}{\eta}\hat{k}_t^B, \tag{33}$$

where $\lambda \equiv \frac{R^S \beta^I + \chi\left(1-\beta^I R^S\right)}{R^S}$ and η^{-1} is the elasticity of the bankers' marginal product of capital times the ratio of borrowers' to bankers' capital-holdings in the steady state (i.e., $\eta \equiv \frac{1-k^B}{k^B(1-\mu)}$).

Once we obtain the solutions for $\hat{q}_t$ and $\hat{k}_t^B$ as linear functions of the technology shifter, we can determine closed-form expressions for the equilibrium path of the other variables in the model. Thus, we first focus on (32), whose forward-iteration leads to:

$$\hat{q}_t = \gamma \hat{\alpha}_t, \tag{34}$$

where $\gamma \equiv \frac{1-\phi}{1-\phi\rho}\rho > 0$. With this expression for $\hat{q}_t$, we can resort to (33), obtaining

$$\hat{k}_t^B = v\hat{\alpha}_t, \tag{35}$$

where $v \equiv \frac{\eta}{1-\lambda}\frac{(\lambda-\phi)(1-\rho)\rho}{1-\phi\rho} > 0$.

4.1 Financial Collateral and Macroeconomic Amplification

We have now lined up the elements necessary to examine the economy's response to technology disturbances. Proposition 2 details the effect induced by a marginal change in the degree of financial collateralization on borrowers' capital-holdings and the capital price. Both variables

[18] As far as the pleadgeability of bankers' capital is concerned, this has mixed effects on mpk^I: on one hand, increasing χ affects the interest rate on loans in the same direction as ξ does, hence attenuating capital misallocation; on the other hand, it compresses the first factor on the right-hand side of (29), thus exerting a negative force on mpk^I.

[19] Variables in log-deviation from their steady-state level are denoted by a "^".

are crucial to determine the size of the dynamic multiplier at work in this type of credit economies, which will be examined in the remainder of this section.

Proposition 2 *Increasing the degree of collateralization of bank loans (ξ) attenuates the impact of the technology shock on both borrowers' holdings of capital and the capital price.*

Proof. *See Appendix A.* ■

According to *Proposition 2* the sensitivity of borrowers' capital-holdings to the technology shifter decreases in the degree of collateralization of bank loans. The intuition for this is twofold: on one hand, increasing ξ determines a more even distribution of capital goods, as reflected by the drop in η; on the other hand, being able to pledge a higher share of financial assets reinforces the sensitivity of the capital price to the capital gain component in borrowers' Euler equation, ϕ, through the drop in the loan rate, while reducing the sensitivity to the dividend component. The combination of these effects influences the overall degree of macroeconomic amplification of the system, as captured by the response of gross output to the technology shock. To dig deeper into this aspect, we linearize total production in the neighborhood of the steady state:

$$\hat{y}_t = \hat{\alpha}_t + \Delta \frac{y^B}{y} \hat{k}^B_{t-1}, \tag{36}$$

According to (36), the dynamics of gross output is shaped by $\hat{\alpha}_t$, as well as by borrowers' capital-holdings at time $t-1$: the second effect captures the endogenous propagation of productivity shifts on gross output. In fact, $\hat{y}_t$ depends on the past history of shocks not only through the direct impact of $\hat{\alpha}_t$, but also through the effect of $\hat{\alpha}_{t-1}$ on $\hat{k}^B_{t-1}$, as implied by (35). In light of this, we can rewrite (36) as

$$\hat{y}_t = \varpi \hat{\alpha}_{t-1} + u_t \tag{37}$$

where $\varpi \equiv \rho + \upsilon \Delta \frac{y^B}{y}$. This result is important in that it shows how eliminating the key source of steady-state inefficiency – i.e., attaining $\Delta = 0$ – implies that total output departures from its steady-state level would track the path of the technology shock, so that the model would feature no endogenous propagation of productivity shifts.[20] Moreover, we need to recall that envisaging limited enforceability of both deposit and loan contracts reduces capital misallocation as it emerges in the original KM economy, thus compressing Δ with respect to the case in which $R^S \beta^I = 1$. In this respect, the model produces a 'banking attenuator' that entirely rests on the functioning of financial frictions in banking activity, as compared with analogous effects stemming from monopolistic competition in the intermediation activity and staggered interest rate-setting schemes (Gerali *et al.*, 2010) or the procyclicality of the external finance premium (Goodfriend and McCallum, 2007).

[20] This property echoes the role of the steady-state inefficiency for short-run dynamics in the KM model. In their setting, closing the gap between the marginal products of capital of lenders and borrowers would imply no response at all to a productivity shift. In this respect, the key difference between the two frameworks lies in that we assume an autoregressive shock, while they consider an unexpected one-off shift in technology.

There are three different channels through which an increase in savers' perceived liquidity of bankers' financial assets affect the propagation of a technology shock. To see this, we compute the following derivative:

$$\frac{\partial \varpi}{\partial \xi} = \Delta \frac{y^B}{y} \frac{\partial v}{\partial \xi} + v \frac{y^B}{y} \frac{\partial \Delta}{\partial \xi} + v \Delta \frac{\partial \left(y^B / y \right)}{\partial \xi}. \tag{38}$$

Proposition 2 shows that $\partial v / \partial \xi < 0$, implying that financial collateralization reduces the impact of a technology shift on borrowers' capital-holdings, which we know exerting a direct impact on total output through (36). We also know from *Proposition 1* that the productivity gap between borrowers and bankers shrinks as financial collateralization increases (i.e., $\partial \Delta / \partial \xi < 0$). Finally, it is immediate to prove that the last term on the right-hand side of (38) is positive, in light of greater collateralization of bank loans inducing a reallocation of capital from the bankers to the borrowers. In turn, this transfer implies both a first-order positive effect on y^B and a (milder) second-order positive impact on y, so that the overall effect on y^B / y is positive.[21]

[Insert Figure 2]

To sum up, an increase in ξ causes competing effects on ϖ. As we already know, greater financial collateralization depresses the pass-through of $\hat{\alpha}_{t-1}$ on borrowers' capital-holdings. Furthermore, raising ξ exerts two effects on the pass-through of $\hat{k}^B_{t-1}$ on $\hat{y}_t$: on one hand, bankers' marginal product of capital increases, implying a reduction of the productivity gap; on the other hand, borrowers' contribution to total production increases, as the reduction in the productivity gap reflects higher capital accumulation in the hands of the borrowers. These competing forces potentially lead to mixed results on output amplification, as captured by second-round effects of technology disturbances. To address this point, we plot ϖ as a function of ξ and μ:[22] the aim of this exercise is to examine the direction of the overall effect exerted by financial collateralization on macroeconomic volatility, rather than quantifying an empirically plausible multiplier emerging from the interaction of bankers' and borrowers' financial constraints.[23] As it emerges from Figure 2, increasing ξ compresses ϖ, at any level of μ. By contrast, increasing the income share of capital in bankers' production technology amplifies the second-round response of output. This is because μ amplifies the productivity gap through its positive effect on η.[24]

[21] Recall that total output is an increasing function of borrowers' capital. Therefore, the drop in y^I following a marginal increase in ξ is lower than the corresponding rise in y^B.

[22] The discount factors are set in accordance with our assumptions about the relative degree of impatience of the three agents in the economy and are broadly in line with existing (quarterly) calibrations involving heterogeneous agents economies: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$. We set $\rho = 0.95$, in line with the empirical evidence showing that technology shocks are generally small, but highly persistent (see, e.g., Cooley and Prescott, 1995). As for χ and ω, they are both set to 1, so as to ensure a wider set of admissible combinations of ξ and μ that correspond to positive holdings of capital for both bankers and borrowers. This is clearly displayed by the robustness evidence reported in Appendix C, which also shows that different combinations of χ and ω are close to irrelevant regarding the effect of financial collateralization on macroeconomic amplification.

[23] We leave this task for future research employing a large-scale dynamic general equilibrium model.

[24] It is important to emphasize that increasing μ may violate the condition $G^{'}(0) > \varrho > G^{'}(1)$, which ensures an interior solution as for how much capital bankers should hold in the neighborhood of the steady state. To

4.2 The Role of Leverage

To enlarge our perspective on the amplification/attenuation induced by bankers' financial collateral, we take a closer look at their balance sheet. To this end, we define bankers' equity as the difference between the value of total assets (i.e., loans and capital) and liabilities (i.e., deposits):

$$e_t^I = b_t^B + q_t k_t^I - b_t^S, \tag{39}$$

while leverage is defined as the ratio between loans and equity: $lev_t^I = b_t^B / e_t^I$.

Figure 3 reports the response of selected variables to a one-standard deviation shock to technology.[25] As implied by (36), on impact output responds one-to-one with respect to the shock, regardless of the degree of financial collateralization. However, as ξ increases the second-round response is gradually muted. To complement our analytical insight and provide further intuition on this channel, we examine the behavior of a set of variables involved in bankers' intermediation activity. In this respect, note that deposits tend to decline at low values of ξ, while increasing as bankers can pledge a higher share of their financial assets. The reason for this can be better understood by recalling the nature of the interaction between bankers' financial and real assets. The interplay takes place on two levels: on one hand, both assets have a positive effect on savers' deposits, as embodied by (14); on the other hand, it is possible to uncover a substitution effect between the two assets, as increasing bankers' real asset-holdings exerts a negative force on lending by reducing borrowers' collateral. How do these properties affect the transmission of an expansionary technology shock? Due to the capital productivity gap between borrowers and bankers, the technology shift necessarily causes a decline of bankers' real assets, thus expanding borrowers' capital and borrowing.[26] Therefore, in equilibrium deposits are influenced by two opposite forces, namely an expansion in the amount of bankers' financial assets and a contraction in their stock of real assets. In this respect, the implied allocation of bankers' assets reflects a countercyclical flight to quality pattern (see, *inter alia*, Lang and Nakamura, 1995): during expansions (contractions), bankers increase (decrease) their holdings of the inherently riskier assets – bank loans – while decreasing (increasing) their capital-holdings, which do not bear any risk of default.

[Insert Figure 3]

see why this is the case, recall that $\mu \left(k^I\right)^{\mu-1} = \varrho$. Increasing μ inflates bankers' marginal product of capital, while leaving their user cost unaffected: Thus, as μ increases bankers are induced to hold an increasing stock of capital, so that the equality holds. An important aspect is that this effect tends to kick in earlier as ξ declines. This is because a drop in the degree of financial collateralization depresses bankers' user cost of capital. Therefore, as ξ declines and μ increases the set of steady-state allocations in which both bankers and borrowers hold capital restricts, as the condition $\varrho > G'(1)$ is eventually violated and borrowers' may virtually end up with negative capital-holdings.

[25] The baseline parameterization is the same as that employed in Figure 2. As for μ, we impose a rather conservative value, 0.4, which allows us to obtain a finite distribution of capital in the steady state.

[26] This is a distinctive feature of lender-borrower relationships involving the collateralization of a productive asset. In fact, it is possible to show that, following a positive technology shock, the major reallocation of land from the lenders to the borrowers is only attenuated by relaxing the hypothesis of zero aggregate investment – as in the KM baseline framework – while the direction of the transfer is not inverted.

How do these diverging forces translate in terms of bankers' ability to attract deposits and leverage? As ξ drops the impact of bank loans is gradually muted and deposits eventually track the dynamics of bankers' capital. In this context, the contraction of bankers' real asset overcomes the drop in deposits, so that lending expands in excess of bank equity, potentially leading to an increase in leverage. In fact, a procyclical leverage ratio is associated with a relevant degree of macroeconomic amplification, when bankers' financial assets are regarded as relatively illiquid. Figure 3 shows this tends to be the case for $\xi < 0.5$, under our baseline parameterization.

5 Welfare and Capital Adequacy Requirements

The positive analysis so far has shown that limited enforceability of deposit contracts may reduce the productivity gap between borrowers and lenders, which is the main driver of the endogenous propagation of technology shifts and other potential sources of exogenous perturbation, as embodied by (37). In light of this property, our next objective is to understand to which extent a benevolent regulator may promote a more efficient allocation of capital[27] – thus attenuating the procyclicality of bank leverage and smoothing the amplitude of fluctuations in gross output – by 'leaning against' capital misallocation, which is the key distortion in the economy. To this end, the structure of the model lends itself to the introduction of two complementary tools of regulation. First, we assume deposit insurance, which ensures that savers do not suffer a loss in the event of bankers' default.[28] A direct implication of such a measure is to shift the risk of bankers' default to the government (or a hypothetical interbank deposit protection fund), so that the renegotiation of deposit contracts is redundant and bankers' financial constraint may be discarded. However, in order to mitigate bankers' moral hazard behavior, we also assume that the regulator imposes an explicit capital adequacy requirement (Van den Heuvel, 2008). According to this regulatory constraint, equity needs to be at least a fraction θ of the loans, for bankers to be able to operate:

$$e_t^I \geq \theta b_t^B, \quad \theta \in [0,1], \tag{40}$$

[27] In his 2015 remit and recommendations to the Governor of the Bank of England, the Chancellor of the Exchequer indicates that the Financial Policy Commettee (FPC) "should support the Government's economic objectives by acting in a way that, where possible, facilitates the supply of finance for productive investment provided by the UK's financial system" (HM Treasury, 2015). In light of this, pursuing an efficient allocation of resources through the financial system is explicitly seen as one of the targets that the FPC has to aim for when implementing its macroprudential policy (Bank of England, 2016).

[28] As in Van den Heuvel (2008), deposit insurance is left unmodeled, though it is argued that it generally improves banks' ability to extend credit (see Diamond and Dybvig, 1983).

where θ denotes the so called capital-to-asset ratio. Under this setting the equilibrium loan rate is[29]

$$R^B = \frac{R^S - (1-\theta)\left(1-\beta^I R^S\right)}{\beta^I}. \tag{41}$$

Notably, (41) is isomorphic to (22). Intuitively, a higher leverage (lower capital) ratio implies a riskier exposure of the financial intermediary. This translates into greater transaction costs savers would have to bear in the event of bankers' default, so as to seize their financial assets. In light of this property, to minimize the spread between the loan and the deposit rate and lean against the distortion, the optimal policy is to set $\theta = 0$. Along with minimizing the fraction of bank assets that can be financed by issuing deposit liabilities, this value contracts Δ, thus ensuring a more efficient allocation of capital between bankers and borrowers. Figure 4 reports the implied response of the economy to a positive technology shock under this policy.

[Insert Figure 4]

While inducing a major compression in the response of bank leverage, setting $\theta = 0$ is not enough to neutralize the endogenous propagation channel stemming from capital misallocation, as stated by the next proposition.

Proposition 3 *The gap between bankers' and borrowers' marginal product of capital (Δ) cannot be closed by setting the capital-to-asset ratio (θ) within the range of admissible values.*

Proof. *See Appendix A.* ■

In fact, it can be proved that attaining $\Delta = 0$ would require to set θ below zero, so as to reduce the spread between the loan and the deposit rate further and allow bankers to extend as much credit as possible. In Figure 4 this level of the capital-to-asset ratio is denoted with $\theta_{\Delta=0}$. At this value the endogenous propagation of the shock is switched off, so that gross output tracks the dynamics of the productivity shifter and leverage is completely acyclical. This result may also be rationalized by combining (40) with (39) to obtain

$$b_t^S \leq q_t k_t^I + (1-\theta)\, b_t^B, \tag{42}$$

which suggests that, through a negative capital-to-asset ratio, the regulator actually pushes for an 'hyper-collateralization' of bank loans. In turn, this may eventually induce bankers to set a loan rate below the interest rate on deposits, which amounts to subsidizing borrowers' capital investment. To dig deeper into this mechanism, Figure 5 maps the spread between the loan and the deposit rate (y-axis) and the productivity gap (x-axis), for different values of the capital-to asset ratio (θ) and three different values of the loan-to-asset ratio applying to the borrowers (ω). As we move down along each line, θ decreases from its upper bound to the value consistent with $\Delta = 0$. The color of a given locus switches from green to blue when θ drops below zero.

[29] Appendix B reports bankers' optimization problem under (40).

As explained above, closing the productivity gap through a constant capital requirement proves to be infeasible for the policy maker, given that $\Delta = 0$ may only be attained at negative values of θ and $R^B - R^S$. It is interesting to see how capital requirements interact with the friction affecting the banker-borrower relationship. In this respect, setting $\theta = 0$ allows the regulator to further compress the productivity gap at $\omega = 1$, as compared with lower loan-to-value ratios. In fact, raising ω relaxes borrowers' collateral constraint, allowing them to increase their capital holdings, so that bankers' marginal product of capital is allowed to increase in equilibrium. Analogous implications can be drawn when evaluating at which values of $R^B - R^S$ the productivity gap is closed: in this case, a stronger compression of the interest rate spread is necessary as the loan-to-value ratio drops, as well as a more negative θ.

[Insert Figure 5]

5.1 A Countercyclical Capital Buffer

The normative analysis so far has shown that imposing a constant capital requirement plays a limited role in smoothing economic fluctuations. Thus, we turn our attention to an alternative regulatory measure, potentially capable of reducing gross output fluctuations by affecting the cyclicality of bankers' balance sheet. To this end, recent years have witnessed an increasing interest of the policymakers towards leaning against credit imbalances, pursuing macroeconomic stabilization through policy rules that set a countercyclical capital buffer. *De facto*, countercyclical capital regulation is a key block of the Basel III international regulatory framework for banks.[30] Based on the analysis of the transmission mechanism and the response of bank capital, we now examine the functioning of this type of policy tool within our framework. Thus, we allow for capital requirements to vary with the macroeconomic conditions (see, e.g., Angeloni and Faia, 2013, Nelson and Pinter, 2016 and Clerc *et al.*, 2015):

$$\frac{\theta_t}{\theta} = \left(\frac{b_t^B}{b^B}\right)^{\varphi}, \quad \varphi \geq 0, \tag{43}$$

where $\varphi = 0$ implies a constant capital-to-asset ratio, while $\varphi > 0$ induces a countercyclical capital buffer.[31]

By linearizing the time-varying counterpart of (41) in the neighborhood of the steady state we obtain:

$$\hat{R}_t^B = \psi\hat{\theta}_t, \tag{44}$$

[30] The regulatory framework evolved through three main waves. Basel I has introduced the basic capital adequacy ratio as the foundation for banking risk regulation. Basel II has reinforced it and allowed banks to use internal risk-based measure to weight the share of asset to be hold. Basel III has been brought in response to the 2007-2008 crisis, with the key innovation consisting of introducing countercyclical capital requirements, that is, imposing banks to build resilience in good times with higher capital requirements and relax them during bad times.

[31] According to the Basel III regime, capital regulation can respond to a wide range of macroeconomic indicators. Here we assume it to respond to deviations of b_t^B from its long-run equilibirum, b^B.

where $\psi = \frac{1-\beta^I R^S}{\beta^I R^B}\theta$ is positive, in light of assuming $\beta^I R^S < 1$. We also linearize (43), obtaining:

$$\hat{\theta}_t = \varphi \hat{b}_t^B. \tag{45}$$

After linearizing borrowers' financial constraint, we can substitute for $\hat{b}_t^B$ in (45) and plug the resulting expression into (44), so as to obtain:

$$\hat{R}_t^B = \frac{\psi\varphi}{1+\psi\varphi}\left(E_t\hat{q}_{t+1} + \hat{k}_t^B\right). \tag{46}$$

Thus, it is possible to establish a connection between the loan rate and borrowers' expected collateral value. Increasing the responsiveness of the capital-to-asset ratio to changes in aggregate lending amplifies this channel: raising φ implies that marginal deviations of b_t^B from its steady state transmit more promptly to the capital-to-asset ratio and, in turn, to the loan rate through the combined effect of (44) and (45). This induces a feedback effect on borrowers' capacity to attract external funding, as embodied by their collateral constraint: higher sensitivity of the loan rate to variations in aggregate lending (i.e., a steeper loan supply function) implies stronger discounting of borrowers' expected collateral. In the limit, as $\varphi \to \infty$ there is a perfect pass-through of $E_t\hat{q}_{t+1} + \hat{k}_t^B$ on $\hat{R}_t^B$. Therefore, as in the face of a technology shock both terms move in the same direction and by the same extent, borrowing does not deviate from its steady-state level and output displays no endogenous propagation.

[Insert Figure 6]

To assess the stabilization performance of the countercyclical capital buffer rule we set the steady-state capital-to-asset ratio to 8% – in line with the full weight level of Basel I and the treatment of non-rated corporate loans in Basel II and III – while varying φ over the support $[0, 1]$.[32] As expected, at $\varphi = 0$ (i.e., a capital-to-asset ratio kept at its steady state level) we observe the strongest amplification of the output response, while the lending rate and bank leverage are both acyclical (see Figure 6). By contrast, increasing the degree of countercyclicality of the capital buffer proves to be effective at attenuating the response of gross output to the shock, progressively compressing bank leverage. Notably, as $\varphi \to \infty$ leverage displays a strong degree of countercyclicality, while lending does not deviate from its steady-state level, as conjectured above. In turn, this results in the dynamics of gross output featuring no endogenous propagation of technology shocks.

6 Concluding Remarks

We have devised a credit economy where bankers intermediate funds between savers and borrowers, assuming that bankers' ability to collect deposits is affected by limited enforceability: as a result, if bankers default, savers acquire the right to liquidate bankers' asset-holdings.

[32] Alternative values of θ would only alter the quantitative implications of the exercise, while not affecting its key qualitative result.

In this context, we have examined the role of bank loans as a form of collateral in deposit contracts. Due to the structure of our credit economy, which may well account for different forms of financial intermediation, savers anticipate that liquidating financial assets is conditional on borrowers being solvent on their debt obligations. This friction limits the degree of collateralization of bankers' financial assets beyond that of capital. We have demonstrated three main results: i) limited enforceability of deposit contracts counteracts the effects of limited enforceability of loan contracts, thus reducing capital misallocation as it emerges in KM; ii) greater collateralization of bankers' financial assets dampens macroeconomic fluctuations by reducing the degree of procyclicality of bank leverage; iii) while imposing a fixed capital-to-asset ratio to the bankers cannot fully neutralize capital misallocation and switch off the associated endogenous propagation channel of productivity shocks, a countercyclical capital adequacy requirement proves to be rather effective at smoothing the business cycle.

Our model is necessarily stylized, though it can be generalized along a number of dimensions. For instance, a realistic extension could consist of allowing bankers to issue equity (outside equity), so as to evaluate how a different debt-equity mix may affect macroeconomic amplification over expansions – when equity can be issued frictionlessly – and contractions, when equity issuance may be precluded due to tighter information frictions. This factor should counteract the role of financial assets and help obtaining a countercyclical leverage. In connection with this point, we could also allow for occasionally binding financial constraints, so as to evaluate how the policy-maker should behave across contractions – when constraints tighten – and expansions, when constraints may become non-binding. However, as this type of extensions necessarily hinder the analytical tractability of our problem, we leave them for future research projects based on large scale models.

Figures

Figure 1: Steady-state equilibrium.

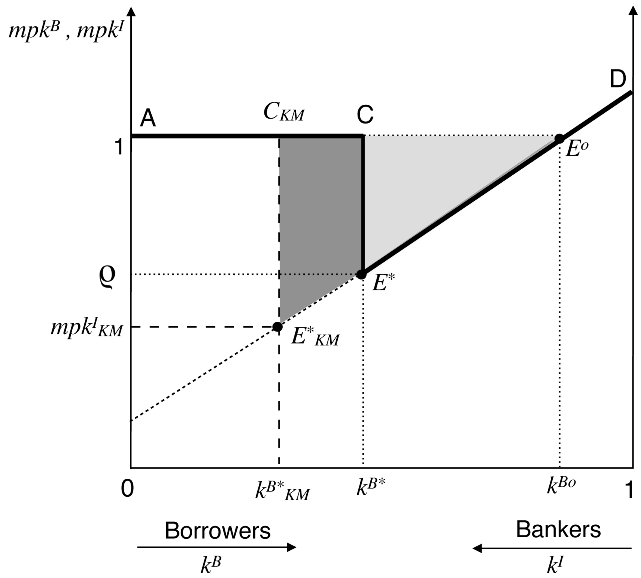
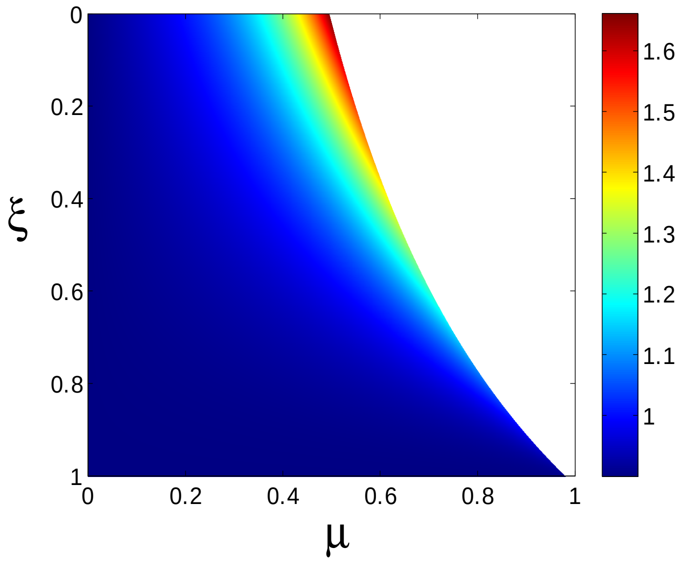
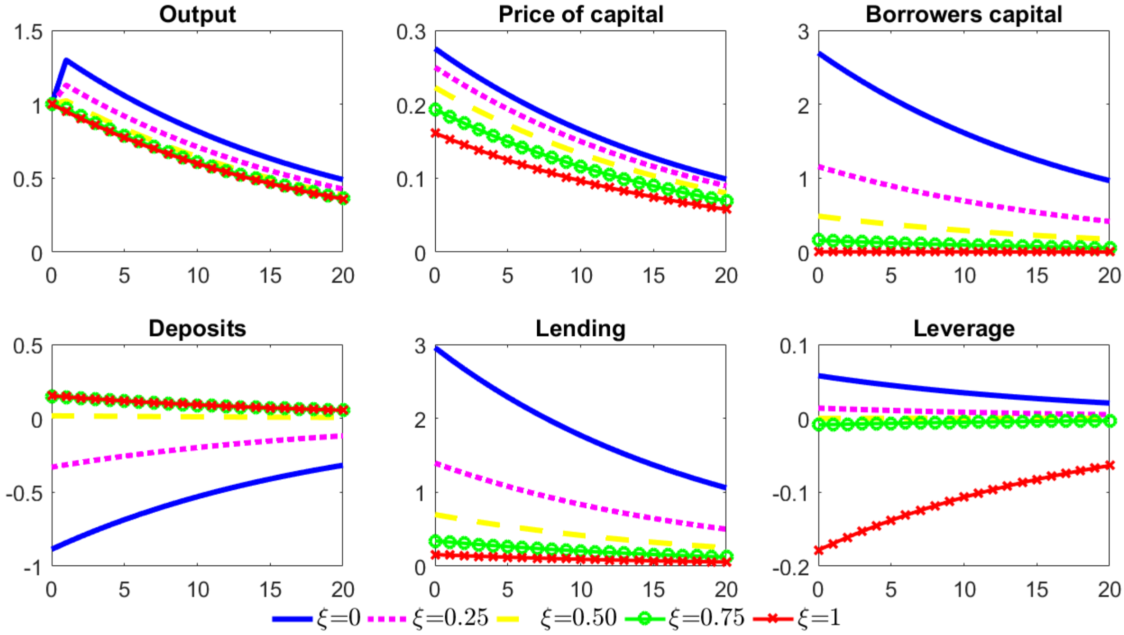


Figure 2: Business cycle amplification.



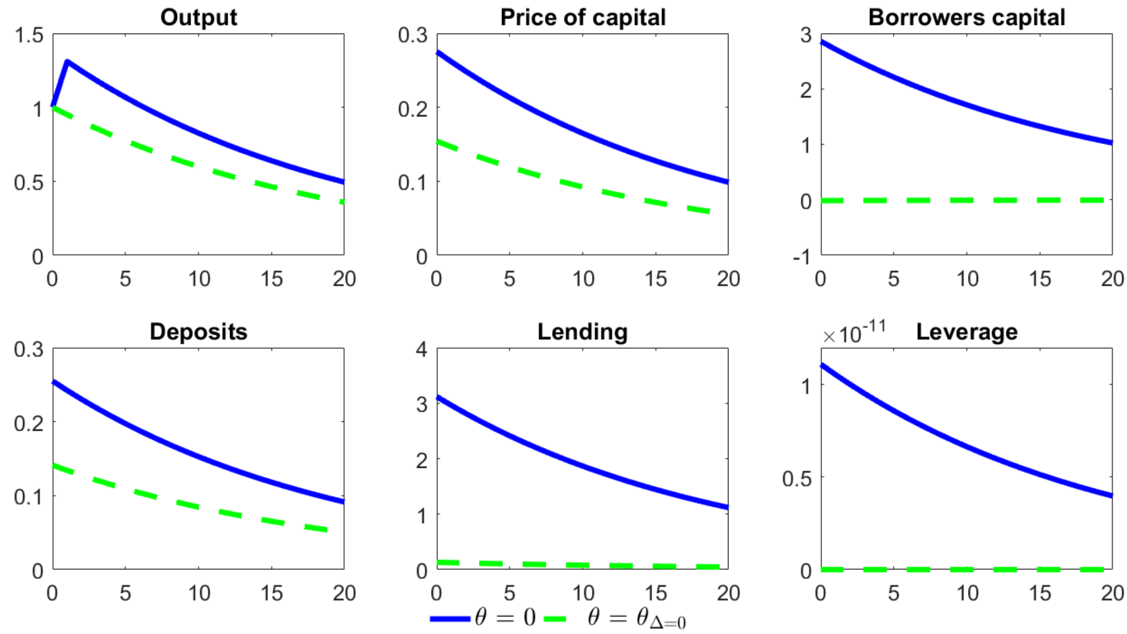
Notes. Multiplier on technology shock, ϖ, as a function of ξ and μ, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\chi = \omega = 1$. The white area denotes inadmissible equilibria where bankers' capital-holdings are virtually negative.

Figure 3: Impulse responses to a positive technology shock.



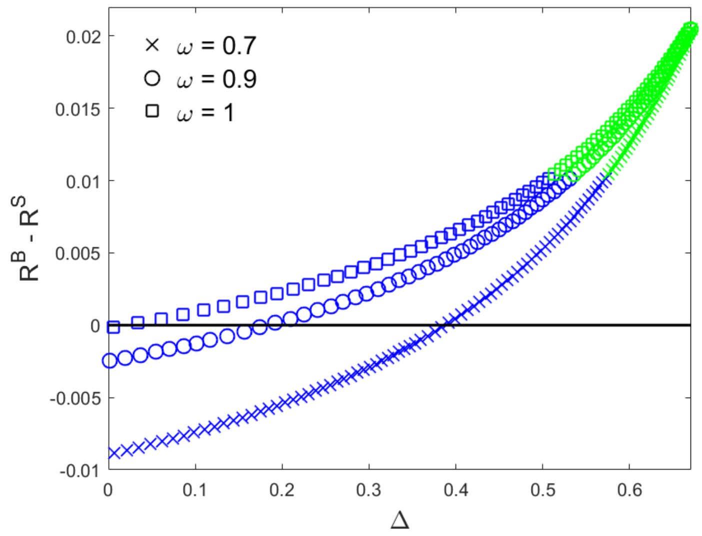
Notes. Responses of selected variables to a one-standard-deviation shock to technology, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\chi = \omega = 1$, $\mu = 0.4$.

Figure 4: Impulse responses under $\theta = 0$ and $\theta = \theta_{\Delta=0}$.



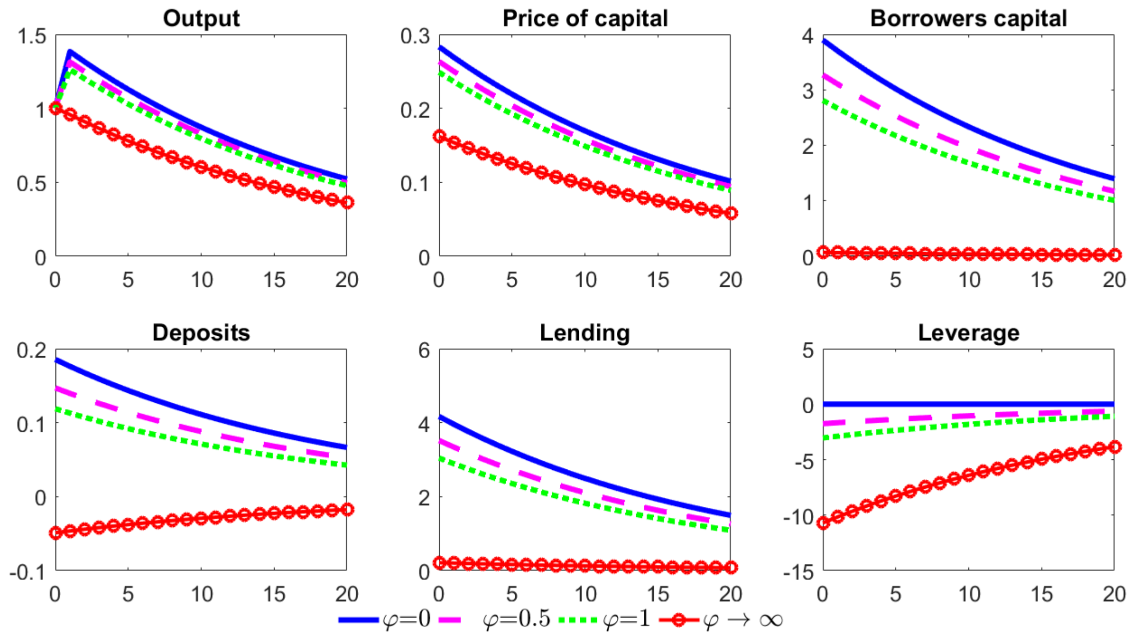
Notes. Responses of selected variables to a one-standard-deviation shock to technology, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\chi = \omega = 1$, $\mu = 0.4$.

Figure 5: Interaction between the capital-to-asset and the loan-to-value ratios.



Notes. $R^B - R^S$ (y-axis) and Δ (x-axis) for different values of θ and ω. As we move down along each line, θ decreases from its upper bound to the value consistent with $\Delta = 0$. The color of a given locus switches from green to blue when θ drops below zero. The rest of the parameters are set as follows: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\mu = 0.4$.

Figure 6: Impulse responses under different φs.



Notes. Responses of selected variables to a one-standard-deviation shock to technology, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$, $\chi = \omega = 1$, $\mu = 0.4$.

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