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When Economic Growth is Less than Exponential

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1 Introduction

The notion of balanced growth, generally synonymous with exponential growth, has proved extremely useful in the theory of economic growth. This is not only because of the historical evidence (Kaldor's "stylized facts"), but also because of its convenient simplicity. Yet there may be a deceptive temptation to oversimplify and ignore other possible growth patterns. We argue there is a need to allow for a richer set of parameter constellations than in standard growth models and to look for a more general regularity concept than that of exponential growth. The motivation is the following:

First, when setting up growth models researchers place severe restrictions on preferences and technology such that the resulting model is compatible with balanced growth (as pointed out by Solow, 2000, Chapters 8-9). In addition, population is either assumed to grow exponentially or to be constant. This paper demonstrates that regular long-run growth, in a sense specified below, can arise even when some of the archetype restrictions are left out.

Second, standard R&D-based semi-endogenous growth models imply that the long-run per-capita growth rate is proportional to the growth rate of the labor force (Jones, 2005).[1] This class of models is frequently used for positive and normative analysis since it appears empirically plausible in many respects. And the models are consistent with more than a century of approximately exponential growth. If we employ this framework to evaluate the prospect of growth in the future, then we end up with the assertion that the growth *rate* will converge to zero. This is simply due to the fact that there must be limits to population growth, hence also to growth of human capital. The open question is then what this really implies for economic development in the future and thereby, for example, for the warranted discount rate for long-term environmental projects. This issue has not received much attention so far and the answer is not that clear at first glance. Of course, there is an alternative to the semi-endogenous growth framework, namely that of fully endogenous growth as in the first-generation R&D-based growth models of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). This approach allows of exponential growth with zero population growth. However, in spite of their path-breaking nature these models rely on the simplifying knife-edge assumption of constant returns to scale (either exactly or asymptotically) with respect to producible factors in the invention pro-

[1] Of course, if one digs a little deeper, it is not growth in population as such that matters. Rather, as Jones (2005) suggests, it is growth in human capital, but this ultimately depends on population growth.

duction function.[2] As argued, for instance by McCallum (1996), the knife-edge assumption of constant returns to scale to producible inputs should be interpreted as a simplifying approximation to the case of slightly *decreasing* returns (increasing returns can be ruled out because they have the nonsensical implication of infinite output in finite time, see Solow 1994). But the case of decreasing returns to producible inputs is exactly the semi-endogenous growth case.

A third reason for thinking about less than exponential growth is to open up for a perspective of *sustained* growth (in the sense of output per capita going to infinity for time going to infinity) in spite of the growth *rate* approaching zero. Everything less than exponential growth often seems interpreted as a fairly bad outcome and associated with economic stagnation. For instance, in the context of the Jones (1995) model with constant population, Young (1998, n. 10) states *"Thus, even if there are intertemporal spillovers, if they are not large enough to allow for constant growth, the development of the economy grinds to a halt."* However, to our knowledge, the case of zero population growth in the Jones model has not really been explored yet. We take the opportunity to let an analysis of this case serve as one of our illustrations that the usual dichotomy between either exponential growth or complete stagnation is too narrow. The analysis suggests that paths along which the rate of decline of the growth rate is proportional to the growth rate itself deserve attention. Indeed, this criterion will define our concept of *regular growth.* It turns out that exponential growth is the limiting case where the factor of proportionality, the "damping coefficient", is zero. And the "opposite" limiting case is stagnation which occurs when the "damping coefficient" is infinite.

To show the usefulness of this generalized regularity concept two further examples are provided. One of these is motivated by what seems to be a gap in the theoretical learning-by-doing literature. With the perspective of exponential growth, existing models either assume a very specific value of the learning parameter combined with zero population growth in order to avoid growth explosion (Barro and Sala-i-Martin, 2004, Section 4.3) or allow for a range of values for the learning parameter below that specific value, but then combined with exponential population growth (Arrow, 1962). There is an intermediate case, which to our knowledge has not been systematically explored. And this case leads to less-than-exponential, but sustained regular growth.

Our third example of regular growth is intended to show that the framework is easily applicable also to more realistic and complex models. As Greenwood et

[2] By "knife-edge assumption" is meant a condition imposed on a parameter value such that the set of values satisfying this condition has an empty interior in the space of all possible values for this parameter (see Growiec, 2007).

al. (1997) document, since World War II there has been a steady decline in the relative price of capital equipment and a secular rise in the ratio of new equipment investment to GNP. On this background we consider a model with investment-specific learning and embodied technical change, implying a persistent decline in the relative price of capital. When conditions do not allow of exponential growth, the same regularity emerges as in the two previous examples. We further sort out how and why the *source* of learning – be it *gross* or *net* investment – is decisive for this result.

The paper is structured as follows. Section 2 introduces proportionality of the rate of decline of the growth rate and the growth rate itself as defining "regular growth". It is shown that this regularity concept nests, inter alia, exponential growth, arithmetic growth, and stagnation as special cases. Sections 3, 4, and 5 present our three economic examples which, by allowing for a richer set of parameter constellations than in standard growth models, give rise to growth patterns satisfying our regularity criterion, yet being non-exponential. Asymptotic stability of the regular growth pattern is established in all three examples. Finally, Section 6 summarizes the findings.

2 Regular Growth

Growth theory explains long-run economic development as some pattern of regular growth. The most common regularity concept is that of exponential growth. Occasionally another regularity pattern turns up, namely that of arithmetic growth. Indeed, a Ramsey growth model with AK technology and CARA preferences features arithmetic GDP per capita growth (e.g., Blanchard and Fischer, 1989, pp. 44-45). Similarly, under Hartwick's rule, a model with essential, non-renewable resources (but without population growth, technical change, and capital depreciation) features arithmetic growth of capital (Solow, 1974; Hartwick, 1977). In similar settings, Mitra (1983), Pezzey (2004), and Asheim et al. (2007) consider growth paths of the form $x(t) = x(0)(1 + \mu t)^{\omega}$, $\mu, \omega > 0$, which, by the last-mentioned authors, is called "quasi-arithmetic growth". In these analyses the quasi-arithmetic growth pattern is associated with exogenous quasi-arithmetic growth in either population or technology. In this way results by Dasgupta and Heal (1979, pp. 303-308) on optimal growth within a classical utilitarian framework with non-renewable resources, constant population, and constant technology are extended. Hakenes and Irmen (2007) also study exogenous quasi-arithmetic growth paths. Their angle is to evaluate the plausibility of equations of motion for technology on the basis of the ultimate forward-looking as well as backward-looking behavior of the implied path.

In our view there is a rationale for a concept of regular growth, subsuming exponential growth and arithmetic growth as well as the range between these two. Also some kind of less-than-arithmetic growth should be included. We label this general concept *regular growth*, for reasons that will become clear below. The example we consider in Section 3 illustrates that by varying one parameter (the elasticity of knowledge creation with respect to the level of existing knowledge), the whole range between complete stagnation and exponential growth of the knowledge stock is spanned. Furthermore, the example shows how a quasi-arithmetic growth pattern for knowledge, capital, output, and consumption may arise *endogenously* in a two-sector, knowledge-driven growth model. The second and third example, discussed in Section 4 and 5, respectively, show that also models of learning by doing and learning by investing may endogenously generate quasi-arithmetic growth.

To describe our suggested concept of regular growth, a few definitions are needed. Let the variable $x(t)$ be a positively-valued differentiable function of time t. Then the growth rate of $x(t)$ at time t is:

$$g_1(t) \equiv \frac{\dot{x}(t)}{x(t)},$$

where $\dot{x}(t) \equiv dx(t)/dt$. We call $g_1(t)$ the first-order growth rate. Since we seek a more general concept of regular growth than exponential growth, we allow $g_1(t)$ to be time-variant. Indeed, the regularity we look for relates precisely to the way growth rates change over time. Presupposing $g_1(t)$ is strictly positive within the time range considered, let $g_2(t)$ denote the second-order growth rate of $x(t)$ at time t, i.e.,

$$g_2(t) \equiv \frac{\dot{g}_1(t)}{g_1(t)}.$$

We suggest the following criterion as defining *regular growth*:

$$g_2(t) = -\beta g_1(t) \quad \text{for all } t \geq 0, \tag{1}$$

where $\beta \geq 0$. That is, the second-order growth rate is proportional to the first-order growth rate with a non-positive factor of proportionality. The coefficient β is called the *damping coefficient*, since it indicates the rate of damping in the growth process.

Let x_0 and α denote the initial values $x(0) > 0$ and $g_1(0) > 0$, respectively. The unique solution of the second-order differential equation (1) may then be expressed as:

$$x(t) = x_0 \left(1 + \alpha\beta t\right)^{\frac{1}{\beta}}. \tag{2}$$