The Demand for Housing over the Life-Cycle under Long-Term *Gross* Debt Contracts*

Jeppe Druedahl†
University of Copenhagen

August 30, 2015

Abstract

This paper generalizes the model of individual demand for housing over the life-cycle in Attanasio, Bottazzi, Low, Nesheim and Wakefield (2012) by formulating the long-term debt contracts in *gross* terms instead of in *net* terms. This more realistic market structure have important implications for the model dynamics because it enables the households to save in financial assets instead of requiring them to save by increasing their mortgage repayments. Moreover the potential for such *precautionary balance sheet expansions* make the households able to self-insure more optimally and thus increase their ex ante expected welfare. Quantitatively the welfare gain is largest when there is no mortgage spread and no forced mortgage repayments. Qualitatively the results are robust to a substantial mortgage spread and forced repayments if just the households are impatient enough. Introducing a combination of proportional and fixed (re)financing costs does likewise not affect the central results.

Keywords: Housing, Uncertainty, Credit Constraints.

JEL-Codes: D91, R52, G11.

*I thank Matthew Wakefield, Gianluca Violante, Thomas Høgholm Jørgensen, Anders Munk-Nielsen, Patrick Kofod Mogensen, Thais Lærkholm Jensen, Alessandro Martinello, and Christian Groth for helpful comments along the way. Naturally the normal disclaimer applies.

†Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Building 26, DK-1353 Copenhagen K, Denmark. E-mail: jeppe.druedahl@econ.ku.dk. Website: http://econ.ku.dk/druedahl.

1 Introduction

Understanding the demand for housing over the life-cycle is of uttermost importance. Firstly housing services are a central component in the utility of households, and secondly housing as collateral is a key factor in determining how well households can smooth consumption in the face of e.g. income shocks.

The purpose of this paper is to generalize the model of individual demand for housing over the life-cycle in Attanasio, Bottazzi, Low, Nesheim and Wakefield (2012) (henceforth ABLNW) with a borrowing constraint in *gross* debt instead of in *net* debt. The ABLNW model framework is a good starting point because it incorporates many of the most important features of the housing choice: there is an outside option of renting, houses come in different discrete sizes, buying and selling them is subject to substantial transaction costs, and housing and consumption choices are made under uninsurable income and house price uncertainty.

Most importantly ABLNW specify mortgages as long-term debt contracts. Specifically they assume that households are subject to both a *loan-to-value (LTV)* constraint and a *loan-to-income (LTI)* constraint, but that they only need to satisfy these constraints when they originate a new or refinance an existing mortgage. ABLNW argue persuasively that this is "a novel and realistic assumption" (p. 2) because it e.g. does not force highly indebted household to deleverage sharply when house prices fall and the LTV-ratio mechanically increases.

One central limitation in the original ABLNW model, however, is that it is formulated in financial *net* worth alone. This implies that households will never hold any (gross) financial assets while they have an outstanding balance on their mortgage. This assumption is especially problematic in terms of internal consistency because ABLNW also assume that the interest rate on financial assets and mortgages are equal. Hereby it is actually cost less for a household to build up its balance sheet, and it will therefore always do so in full in order to reap the option value of a large mortgage. The reason is that households with a large gross debt are certain that they in future periods will be able to choose continuing to have a large gross debt even if house prices and their income fall; the large mortgage thus provides extra liquidity.

The central contribution in this paper is to show that modeling the long-term borrowing constraint in *gross* terms rather than *net* terms has important implications for the model dynamics because it allows for *precautionary balance sheet expansions* where financial assets are accumulated instead of mortgage debt being repaid. This amplifies the overall demand for housing, and induces a marked substitution from flats towards bigger houses. These effects are especially strong among the young, and in total the welfare gain of precautionary balance sheet expansions can be substantial for the households if the spread between the interest rate on mortgages and the return rate on financial assets is close to zero and the forced mortgage repayment rate is low. Furthermore the welfare gain remains significant even under a substantial mortgage spread and sizable forced mortgage repayments if just the households are impatient enough; the reason is that in the net stock formulation of the model impatient households have higher LTV-ratios early in life lowering the base cost of a precautionary balance sheet expansion. This increased impatience is moreover a substantial improvement in terms of matching the empirical facts on outright ownership rates and LTV-ratios in the early part of the life-cycle.

The above discussed results are robust to deviations from the baseline parameters taken from the detailed calibration in ABLNW. Finally the results are only marginally affected by introducing proportional and fixed (re)financing costs.

The paper is primarily related to a growing set of papers that aim to deepen our understanding of housing decisions in life-cycle consumption models. Apart from Attanasio, Bottazzi, Low, Nesheim and Wakefield (2012), some of the most recent contributions are Li, Liu, Yang and Yao (2015), Halket and Vasudev (2014), Chambers, Garriga and Schlagenhauf (2013), Bajari, Chan, Krueger and Miller (2013) and Chen, Michaux and Roussanov (2015).[1] None of these papers discuss the importance of modeling long-term debt in terms of gross debt and thus of allowing for *precautionary balance sheet expansions*. Balance sheet expansions are in principle allowed for in the models presented by Chambers, Garriga and Schlagenhauf (2009c), Chen, Michaux and Roussanov (2015) and Sommer, Sullivan and Verbrugge (2013), but the precautionary motive for doing so is never discussed or highlighted. Finally both Attanasio, Leicester and Wakefield (2011) and Halket and Vasudev (2014) use a mortgage setup very similar to ABLNW, and in particular also confine themselves to formulate their models in net worth alone without motivating this restrictive choice.

An additional contribution in the present paper is the novel restructuring of the

[1] Older examples are Gervais (2002), Yao and Zhang (2005), Silos (2007a,b), Campbell and Cocco (2007), Li and Yao (2007), Ríos-Rull and Sánchez-Marcos (2008), Díaz and Luengo-Prado (2008), Leth-Petersen and Ejarque (2008), Yang (2009), Chambers, Garriga and Schlagenhauf (2009a,b,c), Attanasio, Leicester and Wakefield (2011) and Fernandez-Villaverde and Krueger (2011).

state space used to considerable speed up the solution algorithm, which can generally be used in consumption-saving models with long-term debt. This speed up is absolutely necessary when solving the model in gross stocks because this basically adds both a continuous state and a continuous choice to a model which it is already very time consuming to solve.

The rest of the paper is organized as follows: Section 2 introduces the life-cycle model and shows how it generalizes the ABLNW-model. Section 3 briefly discusses the solution algorithm, and the baseline parametrization taken from ABLNW. Section 4 presents the central results on the importance of formulating the model in terms of gross stocks, which are tested for robustness in section 5. Section 6 concludes. The solution algorithm is explained in more detail in appendix A, and some further details are included in appendix B.

2 The Model

States and Choices We consider unitary households living for L periods, and retiring at the end of period T. The households are all characterized by the following four *idiosyncratic* state variables: A_{t-1}, end-of-period *financial assets*, D_{t-1}, end-of-period *mortgage debt*, H_{t-1}, end-of-period *housing status*, and finally Y_t, *non-financial income*. We let $H_{t-1} = 0$ indicate that the household was a renter in the previous period, while $H_{t-1} \in \{1, 2\}$ indicates that it respectively owned a *flat* or a *house*. Additionally the house price P_t is an *aggregate* state variable. The price of flats is given by $\kappa \cdot P_t$ with $\kappa \in [0, 1]$. The *rental price* (of flats) is given by

$$Q_t = \min\{\alpha_Y \cdot Y_t, \alpha_P \cdot \kappa \cdot P_t\}, \quad \alpha_Y, \alpha_P > 0 \tag{2.1}$$

where the first term in the minimum operator is a rent ceiling proportional to income (e.g. due to a government subsidy). In each period the households first choose their housing status, $H_t \in \{0, 1, 2\}$. If they own and do not move they can choose to keep their current mortgage, $K_t = 1$, or refinance, $K_t = 0$. Secondly they choose the size of their mortgage, $D_t \in \mathbb{R}_+$, and how much to consume, $C_t \in \mathbb{R}_+$.

Preferences The per-period utility function is

$$u(C_t, H_t) = \frac{C_t^{1-\rho}}{1-\rho} \cdot e^{\theta \cdot \phi(H_t)} + (\phi(H_t) - 1) \cdot \mu, \quad \rho > 1,\, \theta, \mu \geq 0 \tag{2.2}$$

where

$$\phi(H_t) = \begin{cases} 0 & \text{if } H_t = 0 \\ \phi \in [0,1] & \text{if } H_t = 1 \\ 1 & \text{if } H_t = 2 \end{cases}$$

Here ρ is a measure of risk aversion, θ is a measure of the complementarity between consumption and home ownership, μ is an absolute home ownership premium, and ϕ is a scaling factor for the utility value of owning a flat relative to owning a house. Future utility is discounted exponentially with a factor $\beta > 0$.[2]

Exogenous Processes *Income* evolves stochastically around a deterministic life-cycle profile given by

$$\mathcal{L}_t = \log(\ell_0) + \ell_1 \cdot t + \ell_2 \cdot t^2 \tag{2.3}$$

Before retirement the income (Y_t) of the households is subject to *permanent* income shocks

$$\forall t \leq T: \log Y_t = \mathcal{L}_t + \Psi_t \tag{2.4}$$

$$\Psi_t = \Psi_{t-1} + \psi_t, \quad \psi_t \sim \mathcal{N}\left(-\sigma_\psi^2/2, \sigma_\psi^2\right) \tag{2.5}$$

After retirement there is no shocks and income is determined by a fixed retirement replacement rate

$$\forall t > T: \log Y_t = \vartheta \cdot \log Y_T \tag{2.6}$$

The *house price* is modeled as an $AR(1)$ around a trend

$$\log P_t = \log(\tau_0) + \tau_1 \cdot t + \rho_P \cdot \log P_{t-1} + \xi_t, \quad \xi_t \sim \mathcal{N}\left(-\sigma_\xi^2/2, \sigma_\xi^2\right) \tag{2.7}$$

Both exogenous processes are approximated by a discrete first order Markov process with 15 states using the method in Tauchen (1986).

[2] As the model lacks both stochastic mortality and a bequest motive, it is *not* designed to fit the housing and consumption choices of the elderly.

Mortgage Constraints The mortgage constraint depends on whether the household *keeps* its current mortgage ($K_t = 1$) or *not* ($K_t = 0$). Specifically we have

$$D_t \le \begin{cases} (1-\gamma)\cdot D_{t-1} & \text{if } K_t = 1 \\ \Lambda\left(H_t, P_t, Y_t\right) & \text{if } K_t = 0 \end{cases}, \quad \gamma \in [0,1] \tag{2.8}$$

where γ is the forced repayment rate and $\Lambda\left(H_t, P_t, Y_t\right)$ is the maximum mortgage a household can take out when originating or refinancing:

$$\Lambda\left(H_t, P_t, Y_t\right) = \min\left\{\Lambda_t^{LTV}, \Lambda_t^{LTI}\right\} \tag{2.9}$$

$$\Lambda_t^{LTV} \equiv \begin{cases} \lambda_H \cdot P_t \cdot \kappa & \text{if } H_t = 1 \\ \lambda_H \cdot P_t & \text{if } H_t = 2 \end{cases}, \quad \lambda_H > 0 \tag{2.10}$$

$$\Lambda_t^{LTI} \equiv \lambda_Y \cdot Y_t, \quad \lambda_Y > 0 \tag{2.11}$$

In the terminal period, there is a "die without debt" constraint, $D_L \le 0$.

Transaction Costs The households pay proportional *transactions costs* (F_{buy}, F_{sell}) when buying and selling flats and houses so that the total net cost of these transactions are

$$\begin{aligned} \Omega_t^H = {} & (1+F_{buy}) \cdot \left[\mathbf{1}_{H_t=2,\, H_{t-1}\neq 2} + \mathbf{1}_{H_t=1,\, H_{t-1}\neq 1} \cdot \kappa\right] \cdot P_t \\ & -(1-F_{sell}) \cdot \left[\mathbf{1}_{H_{t-1}=2,\, H_t\neq 2} + \mathbf{1}_{H_{t-1}=1,\, H_{t-1}\neq 1} \cdot \kappa\right] \cdot P_t, \quad F_{buy}, F_{sell} \ge 0 \end{aligned} \tag{2.12}$$

This accounts for both moving costs, real estate agent fees and stamp duty.

Furthermore it is also costly for the households to originate and extend mortgages due to both fees and time costs. Specifically we assume that there are *quasi-proportional (re)financing costs*, i.e.

$$\begin{aligned} \Omega_t^D = {} & \mathbf{1}_{H_t=H_{t-1},K_t=0} \cdot \left[S_D \cdot \left(\max\left\{D_t - (1-\gamma)\cdot D_{t-1}, 0\right\}\right) + S_F\right] \\ & + \mathbf{1}_{H_t\neq H_{t-1},D_t>0} \cdot \left[S_D \cdot D_t + S_F\right], \quad S_D, S_F \ge 0 \end{aligned} \tag{2.13}$$

In total, the cost-function for *non-consumption expenses* consequently is

$$\Omega_t = \Omega_t^H + \Omega_t^D + \mathbf{1}_{H_t=0} \cdot Q_t \tag{2.14}$$

End-of-Period Assets *End-of-period assets* are therefore given by

$$A_t = (1+r_a)\cdot A_{t-1} - (1+r_d)\cdot D_{t-1} + Y_t + D_t - (C_t + \Omega_t) \quad (2.15)$$

where r_a is the risk free return rate of assets and r_d is the mortgage rate; we are only interested in the case $r_d \geq r_a$.

The households do not have access to an overdraft facility or credit card so $A_t \geq 0$, and we can consequently define the maximum consumption function, $\overline{C}(\bullet)$, implicitly as the C_t implying $A_t = 0$. For future reference we also define *end-of-period financial net worth* as $N_t \equiv A_t - D_t$ and *end-of-period total net worth* as $W_t \equiv N_t + \mathbf{1}_{H_t=1}\cdot\kappa\cdot P_t + \mathbf{1}_{H_t=2}\cdot P_t$.

Recursive Form The Bellman equation of the household problem is given by

$$V_t(H_{t-1}, D_{t-1}, A_{t-1}, Y_t, P_t) = \max_{H_t,K_t,D_t,C_t} u(C_t) + \beta\cdot\mathbb{E}_t[V_{t+1}(\bullet)] \quad (2.16)$$

s.t.

$$H_t \in \{0,1,2\} \quad (2.17)$$

$$K_t \in \begin{cases}\{0,1\} & \text{if } H_t = H_{t-1}\\ \{0\} & \text{else}\end{cases} \quad (2.18)$$

$$D_t \in \begin{cases}[0,(1-\gamma)\cdot D_{t-1}] & \text{if } K_t = 1\\ [0,\Lambda(\bullet)] & \text{if } K_t = 0\end{cases} \quad (2.19)$$

$$C_t \in \left[0,\overline{C}(\bullet)\right] \quad (2.20)$$

$$A_t = \overline{C}(\bullet) - C_t \quad (2.21)$$

The model can also be reformulated as the *upper envelope* of a series of discrete choice specific value functions as discussed in appendix A.

Comparison to Attanasio, Bottazzi, Low, Nesheim and Wakefield (2012) The ABLNW-model differs from the model presented here in the central aspect that the households are subject to the following unmotivated restriction on their choice set

$$\text{if } D_t > 0 \text{ then } C_t = \overline{C}(\bullet) \quad (2.22)$$

Conditional on a strictly positive debt choice, the households are thus forced to consume everything, implying that $A_t = 0$ if $D_t > 0$. Consequently *end-of-period financial net worth* is $N_t = -D_t$ if $D_t > 0$ and $N_t = A_t$ if $D_t = 0$. Therefore the whole model can be reformulated in terms of N_{t-1} alone (instead of both D_{t-1} and A_{t-1}), where specifically the borrowing constraint in (2.8) becomes

$$N_t \geq \begin{cases} \min\{(1-\gamma)\cdot N_{t-1}, 0\} & \text{if } K_t = 1 \\ -\Lambda_t & \text{if } K_t = 0 \end{cases} \tag{2.23}$$

Finally the ABLNW-model is formulated under the parametric restrictions that $\gamma = 0$, $S_D = S_F = 0$ and $r_d = r_a$.

3 Solution and Calibration

Solution Algorithm The *non-convexities* introduced by both the discrete housing choice and the various transaction costs, imply that time iteration methods are not applicable. Instead we have to rely on value function iterations to solve the model, but as it has two continuous states (A_{t-1}, D_{t-1}), three discretized states (H_{t-1}, P_t, Y_t) and both discrete (H_t, K_t) and two continuous choices (C_t, D_t), this is generally very time consuming. One important novel speed-up trick is to introduce *beginning-of-period financial net worth* defined as[3]

$$M_t \equiv (1+r_a)\cdot(A_{t-1}-D_{t-1}) - (r_d - r_a)\cdot D_{t-1} \tag{3.1}$$

The model can then be written with M_t as a state variable instead of A_{t-1}. As shown in more detail in appendix A, this implies that the optimal choices when *buying* and *renting* are all *independent* of D_{t-1}. Moreover this restructuring of the state space is helpful in proving some properties of the optimal debt choice function because M_t is now a fixed dimension. For example, it implies that D_{t-1} only have an impact on the choice set of D_t but not on the value of any choice (because its effect on net worth has been netted out in M_t); a decrease in D_{t-1} will therefore in some cases only remove non-optimal choices which cannot change the optimal choice. See appendix A for further details on the solution algorithm.

Calibration We begin from the exact same parametrization as in ABLNW and with a period length of one year. Table 3.1 provides the parameters for the exogenous income and house price processes (see figure 3.1 for the implied trends and grid nodes). For the income process we use their estimates for high education

[3] A similar trick is used in Druedahl and Jørgensen (2015).

Table 3.1: Parameters I

Parameter	Value
Housing	
ρ_P	0.94
σ^2_ψ	0.008
τ_0	4.67
τ_1	0.0232
κ	0.60
α_Y	0.50
α_P	0.01
Income	
ρ_Y	1.00
ν	0.70
$\sigma^2_{\xi,H}$	0.035
$\ell_{0,H}$	1.00
$\ell_{1,H}$	0.042
$\ell_{2,H}$	-0.00082
$\sigma^2_{\xi,L}$	0.044
$\ell_{0,L}$	0.80
$\ell_{1,L}$	0.022
$\ell_{2,L}$	-0.00037

H: High education.
L: Low education.

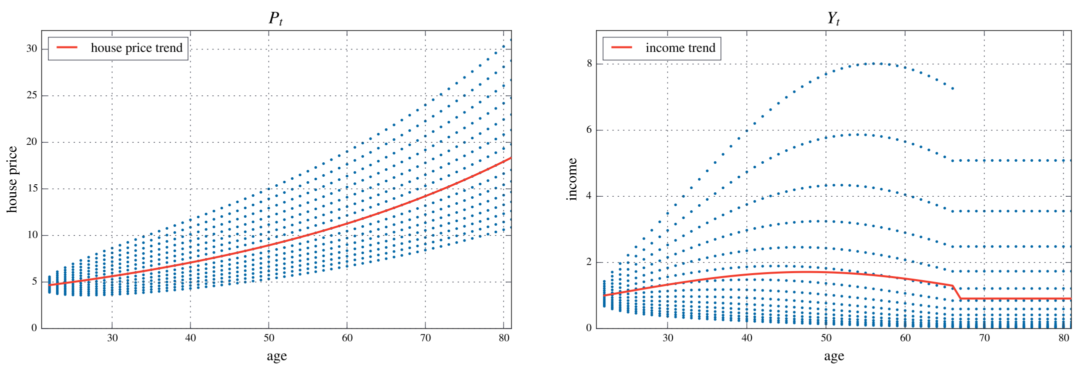
Table 3.2: Parameters II

Parameter	Value
Demographics	
L	60
T	45
Preferences	
β	1.02^{-1}
γ	1.430
θ	0.115
ϕ	0.90
μ	0.26
Borrowing and saving	
r_a	0.018
r_d	0.018
λ_H	0.9
λ_Y	3.0
Transaction costs	
F_{buy}	0.05
F_{sell}	0.05
S_D	0.00
S_F	0.00

households (A-levels or higher). The house price process is estimated using data from the *Office of the Deputy Prime Minister* (ODPM) on national and regional house prices in the UK in years 1969-2000. The income parameters are estimated using data from the *British Household Panel Survey* (BHPS) for the years 1991-2002. The 2000 wave of the BHPS is also used to set the initial distribution of financial assets (see appendix table B.1), while households are assumed to have zero housing endowments at age 22.

Table 3.2 provides the remaining parameters for the demographics, preferences,

Figure 3.1: Exogenous Trends and Grids: P_t and Y_t



Notes: See table 3.1 for the underlying parameters. The discretization is based on the method in Tauchen (1986) using 15 nodes.

borrowing and saving, and the various transaction costs. The preference parameters related to owning (μ, ϕ and θ) and the symmetric fixed transaction cost ($F = F_{buy} = F_{sell}$) were calibrated by ABLNW to fit data on home ownership rates in *The Family Expenditure Survey* (FES) years 1991-2000. The interest rate used is the average 90 day UK Treasury Bill discount rate in years 1968-1997.

One apparent questionable aspect of the baseline parametrization is that there is *no mortgage spread* ($r_a = r_d$), and *no forced mortgage repayments* ($\gamma = 0$). We will there also consider parametrizations with a mortgage spread of 0.5 percentage points[4] (i.e. $r_d = 0.023$), and a strictly positive repayment rate ($\gamma = 0.035$) implying a mortgage half-life of about 20 years, which is similar to a standard 30-year mortgage. Note, however, that if the model included a full portfolio choice with bonds and stocks, the households would be able to get a substantial higher mean return on their gross assets if they were willing to take on some risk.[5] Consequently a zero mortgage spread may actually be the best approximation to reality when we for computational reasons cannot include a full portfolio choice.

Another important aspect of the baseline parametrization is the interaction between the relatively high retirement replacement rate (high η) and a low discount rate (high β). ABLNW see their retirement replace rate of 70 percent as only covering *state* pensions. Loosely matching the model to data on non-housing net wealth closely before retirement they consequently include *private* pensions in their empirical moments. Banks, O'Dea and Oldfield (2010) (table 5), however, arrive at a median replacement rate of 70 percent for the full population including both state and private pensions. Crawford and O'Dea (2014) further show that on quintiles of life-time income the split between state and private pensions is fifty-fifty for the third quintile, while the share of state pensions falls to 40 percent for the fourth quintile and 25 percent for the highest quintile. In conclusion a case can be made for considering parametrizations which will imply less accumulation of financial assets for the median household than seen in the baseline; we will therefore also consider values of β smaller than 1.02^{-1}, which is also often seen in the precautionary savings literature.

[4] See Miles (2005) for a discussion of mortgage rate spreads in the UK.

[5] See e.g. the model with long-term debt in Alan, Crossley and Low (2012).

4 Results

4.1 Baseline

The central simulation results across the different parametrizations and the two formulations of the model in respectively net stocks (as in ABLNW) and gross stocks are presented in table 4.1.[6] To facilitate comparison with ABLNW, their simulation results are repeated in the left most column of table 4.1. Overall the results in the "Base" parametrization in net stocks match those from ABLNW fairly well.[7]

Comparing the second and third column of table 4.1 we see that the effect of shifting from a formulation in net stocks to one in gross stocks, i.e. of allowing for precautionary balance sheet expansions, is large under the "Base" parametrization. The overall home ownership rate increases from 62 percent to 68 percent, and as seen in figure 4.1a it is the whole ownership rate life-cycle profile which is shifting upwards; the largest increase is seen for the young, where the home ownership rate increases from 65 percent to 73 percent. Simultaneously there is also substantial substitution towards houses away from flats; in line with this, the total number of "buys per life" falls from 1.5 to 1.3 indicating that fewer households first buy a flat and then later adjust upwards to a house.

An extreme implication of the zero mortgage spread is that the rate of outright ownership drops to zero[8] as seen in figure 4.1b, and the median LTV-ratio hits the upper bound as seen in figure 4.1d.

All in all the implications of introducing precautionary balance sheet expansions is that the households can move more resources forward in life without putting themselves in a too risky position; in figure 4.2a we therefore see that mean consumption

[6] We simulate the model for 100.000 households with independent draws of both the income and house price process.

[7] The remaining discrepancy is probably a purely technical issue. Apart from different implementations of grids and interpolation procedures, two central technical issues are: 1) The ABLNW figures are an average over only 40 realizations of the house price process. 2) ABLNW impose the constraint that a household can never be technically insolvent in the sense that its debt is larger than the sum of its discounted future *minimum* income plus the *minimum* discounted sale value of its home *tomorrow* (see e.g. equatdefn.f90 line 99 and 170 in their code-files). I do *not* impose this constraint because the trend in the house price process implies that a household can be technically insolvent in a given period without being so at the end of life *even under worst case outcomes*.

[8] Here we are disregarding that some rich households with no liquidity problems may actually be *indifferent* between owning outright, and having any feasible balance sheet expansion.

Table 4.1: Gross vs. Net Stocks

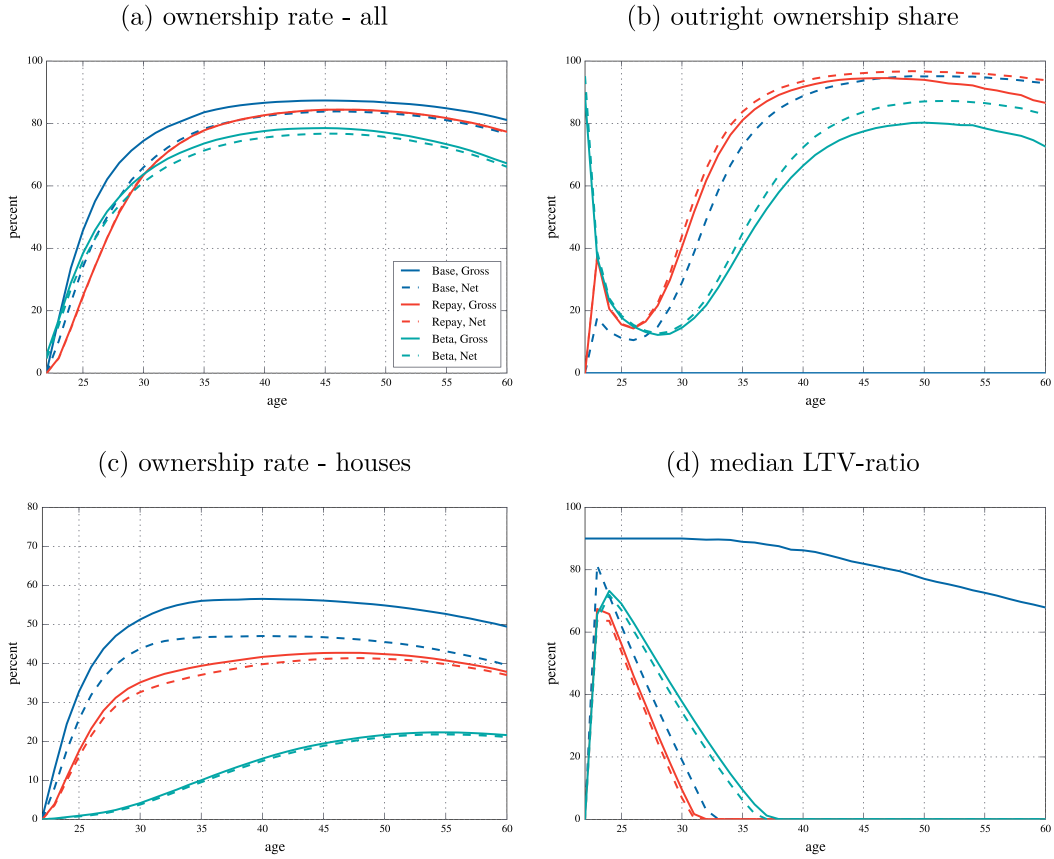
	ABLNW	Base		Spread		Repay		Beta	
	Net	Net	Gross	Net	Gross	Net	Gross	Net	Gross
				percent					
Ownership rate	62	62.3	67.7	61.7	62.3	61.7	61.7	54.9	56.3
- flats	30	28.0	24.4	31.5	29.7	31.9	30.9	42.0	43.0
- houses	32	34.3	43.3	30.1	32.6	29.7	30.8	12.8	13.2
Ownership rate (age 26-35)	58	64.9	73.2	61.8	62.2	61.7	61.6	60.4	62.9
- flats	19	22.5	22.8	28.9	25.5	29.9	27.5	55.7	57.8
- houses	39	42.3	50.3	32.9	36.6	31.7	34.0	4.7	5.1
outright share		37.6	0.0	48.8	44.1	49.8	47.4	21.1	19.6
median LTV		11.5	89.0	0.7	4.7	0.3	1.4	27.8	32.2
Ownership rate (age 36-50)	80	82.6	86.6	82.9	83.4	83.0	83.0	75.6	77.5
- flats	38	36.0	30.6	42.1	39.4	42.8	41.1	58.7	60.0
- houses	42	46.5	56.0	40.8	43.9	40.2	41.8	16.8	17.4
outright share		87.9	0.0	91.7	85.0	92.1	90.3	71.7	66.3
median LTV		0.0	83.6	0.0	0.0	0.0	0.0	0.0	0.0
Ownership rate (age 51-60)	86	80.3	84.3	80.9	81.4	80.9	81.0	71.3	72.4
- flats	46	37.7	32.1	41.2	39.6	41.6	40.7	49.7	50.4
- houses	40	42.6	52.1	39.6	41.8	39.3	40.3	21.5	22.0
outright share		91.9	0.0	93.1	77.3	93.2	89.2	82.1	75.4
median LTV		0.0	72.1	0.0	0.0	0.0	0.0	0.0	0.0
				mean (relative to median Y_t) (age 51-60)					
Total Net Worth (W_t)		15.7	15.3	15.7	15.7	15.7	15.7	10.0	9.9
Financial Net Worth (N_t)	9.0	9.0	7.9	9.1	8.9	9.1	9.0	4.7	4.6
Gross Debt (D_t)		0.0	5.3	0.0	0.3	0.0	0.0	0.0	0.2
				median (relative to median Y_t) (age 51-60)					
Total Net Worth (W_t)		12.5	12.0	12.5	12.4	12.5	12.5	7.9	7.8
Financial Net Worth (N_t)	6.3	6.5	5.5	6.5	6.4	6.5	6.5	3.2	3.1
Gross Debt (D_t)		0.0	5.4	0.0	0.0	0.0	0.0	0.0	0.0
				transactions					
Buys per life		1.5	1.3	1.5	1.4	1.5	1.5	1.3	1.4
- flats		0.9	0.6	1.0	0.9	1.0	0.9	1.1	1.1
- houses		0.6	0.6	0.5	0.5	0.5	0.5	0.2	0.2

Base parameters: see table 2.1 and 2.2.
Spread parameters: see table 2.1 and 2.2, except $r_d = 0.023$.
Repay parameters: see table 2.1 and 2.2. except $r_d = 0.023$, $\gamma = 0.035$.
Beta parameters: see table 2.1 and 2.2 except $r_d = 0.023$, $\gamma = 0.035$, $\beta = 0.96$.

increases with about 1.5-2.5 percent for households under age 35 compared to the net stock formulation.[9] Figure 4.2b further shows that the cross-sectional standard deviation of utility consequently increases a bit initially, but then fall massively, indicating a stronger ability to self-insure.

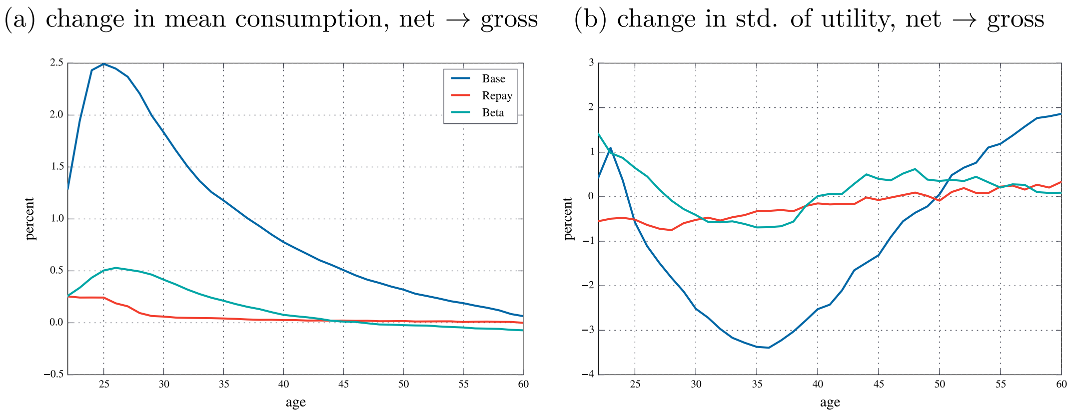
[9] The change in mean consumption never turns negative even at older ages because the steep trend in house prices implies that households save life time resources by buying earlier.

Figure 4.1: Simulated Life-Cycle Profiles I



Samples: Simulations of 100,000 households with independent draws of both the income and house price process.

Figure 4.2: Simulated Life Cycle Profiles II



Samples: Simulations of 100,000 households with independent draws of both the income and house price process.

4.2 Spread ($r_d = 0.023$) and Repay (also $\gamma = 0.035$)

The introduction of a small mortgage spread in the "Spread" parametrization ($r_d = 0.023$) makes it more expensive to have a large mortgage debt, and looking across the net stock formulations we therefore also see both a fall in the overall demand for homes, a substitution towards flats and an increase in the outright ownership rate. More importantly it also implies that the differences between the net stock and gross stock formulations become much smaller. The overall homeowner ship rate thus now only increases by 0.6 percentage points when allowing for precautionary balance sheet expansions; among the young households the increase is only 0.4 percentage points. Further also introducing forced mortgage repayments in the "Repay" parametrization ($\gamma = 0.035$) the differences between the net and gross formulations disappears almost completely with only a minor substitution towards houses left.

At first, it may seem surprising that a small 0.5 percentage points mortgage rate spread have such big effects. However, it implies that the cost of balance sheet expansions becomes linearly increasing in the extra gross debt accumulated and saved in financial assets. On the other hand the benefit of a precautionary balance sheet expansion only becomes substantial once the gross debt stock is so large that it relaxes the borrowing constraint of the households in future periods under not too unlikely decreases in income and house prices. If the households optimal LTV-ratio is low under the net stock formulation then the *base cost* of a precautionary balance sheet expansion is thus very high, and the benefits are initially small or even zero. In figure 4.1b we precisely see that for the "Rapay" parametrization in net stocks, the median LTV-ratio hits zero already before age 35. The median LTV-profile shifts somewhat upwards under gross stocks, but in conclusion the linear costs of precautionary balance sheet expansions heavily outweighs the benefits when a strictly positive mortgage spread and forced mortgage repayments are added to the baseline parametrization.

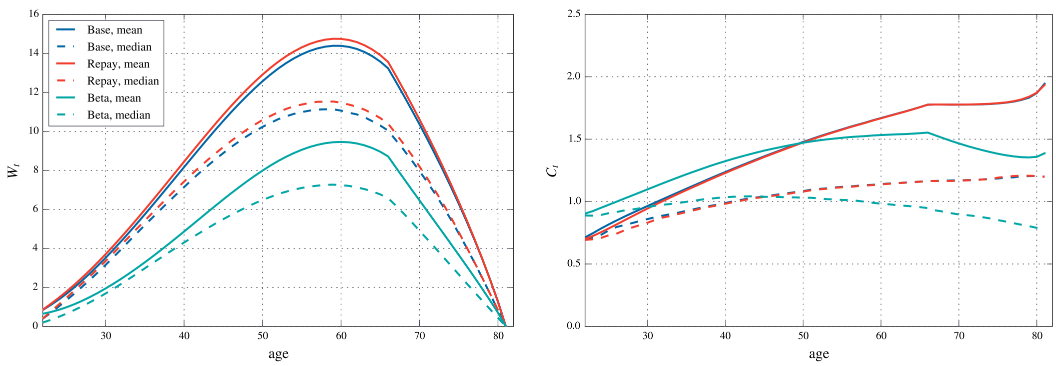
4.3 Outright Ownership

In subsection 3 we discussed the pros and cons of including a strictly positive mortgage spread when our model does not include a full portfolio choice. One apparent common problem with both parametrizations, however, is that they imply very large outright ownership rates and very low median LTV-ratios relatively early in life. Figure 4.1b shows that about 90 percent of age 40 home owners own their flat or house outright. This is unrealistically high. Crossley and O'Dea (2010) (table 3.5) e.g. show that for the full population age 40-45 only 18 percent of home owners own their house outright[10], and this only increases to 75 percent at age 60-64.

Outright ownership is mostly a function of the households non-retirement net wealth. The life-cycle profile of total net worth is shown in figure 4.3a for the different parametrizations (in gross stocks), while figure 4.3b shows the corresponding life-cycle profiles of consumption. Hereby we can see that the households in the baseline parametrization are assumed to be so patient that they achieve a continuing increase in consumption during retirement.

Figure 4.3: Simulated Life Cycle Profiles III (Only Gross)

(a) Total Net Worth, W_t (b) Consumption, C_t



Samples: Simulations of 100,000 households with independent draws of both the income and house price process.

4.4 Beta ($\beta = 0.96$)

Making the households more impatient with $\beta = 0.96$ in the "Beta" parametrization (keeping the same mortgage spread and forced mortgage repayment rate as before) naturally implies a substantial fall in the overall ownership rate, massive substitution towards flats and a much steeper increase in the home ownership rate for the young (again see table 4.1). Centrally, however, it also implies that allowing for precautionary balance sheet expansions again have important effects; e.g. it implies a 2.5 percentage points increase in the home ownership rate among the young, and a bit of relative substitution towards houses.

[10]The total home ownership rate in their sample at age 40-45 is 78 percent, only marginally lower than in our simulations.

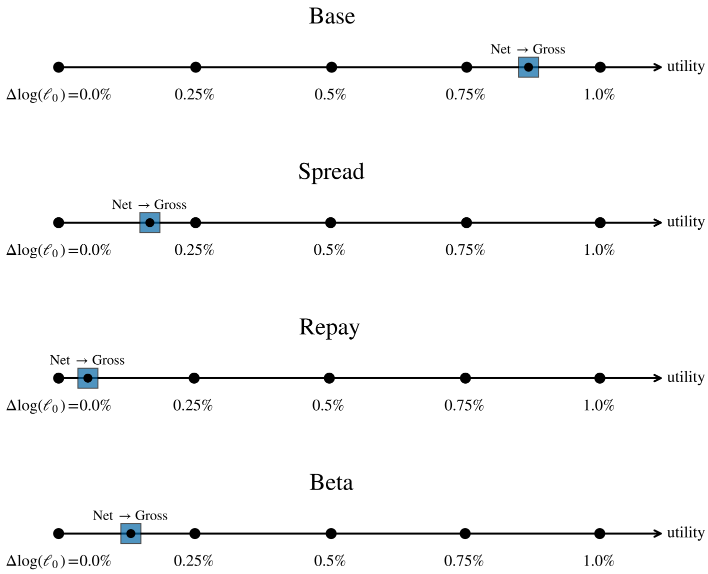
In the gross stock formulation the outright ownership rate among home owners now never increases substantially above 80 percent (see figure 4.1b), which is in line with the empirical estimates. However, it is only down to about 70 percent at age 40, which is still rather high compared to the empirical estimates. This indicates that the differences between the net stock and gross stock formulations found here is probably a *lower* bound.

In a full re-calibration of the model focused on also fitting the empirical outright ownership rates, the LTV-ratios of the households would have to increase, which would lower the base cost of precautionary balance sheet expansions making them more attractive, and thus more important to account for.

4.5 Welfare

Another approach to measuring the importance of allowing for precautionary balance sheet expansions is to consider their welfare implications. As a welfare criterion we look at the *ex ante expected discounted utility* seen from just before the beginning of life (i.e. before the draws of the initial wealth distribution). This welfare measure can be calculated as an ex post average over all the households in our simulation. To further clarify the quantification of the welfare gain of shifting from the net stock to the gross stock formulation, we compare it to the welfare gains in the net stock formulation implied by upward shifts in the life-cycle profile of income (i.e. increases in ℓ_0, see equation (2.3)).

Figure 4.4: Welfare Gain of Net → Gross



Notes: A value of e.g. 1.0 percent shows that the welfare gain is equivalent to the welfare gain from a 1.0 percent increase in the trend of life-cycle income profile (calculated under the net formulation of the model).

Figure 4.4 reports the results of our investigation. As expected the welfare gain is largest in the "Base" parametrization, where a household would rather have access to precautionary balance sheet expansions than get a 0.75 percent increase in their life-cycle profile of income. The welfare gain is much smaller in the "Spread" parametrization and almost disappears completely in the "Repay" parametrization. In the "Beta" parametrization, with a higher degree of impatience, the welfare gain increases a bit again, but is still relatively small and the equivalent income increase is below 0.25 percent.

5 Robustness

5.1 Different Parametrizations (μ, ϕ, θ and F)

Table 5.1 shows the simulation results when changing each single calibration parameter (μ, ϕ, θ and F) such that it induces more home ownership building on top of the "Beta" parametrization, where the home ownership rate was markedly too low. In all cases the differences between the net stock and gross stocks formulations remain approximately the same or become even larger indicating that the results are parametrically robust. Note that this also holds true in the cases where respectively θ is lowered to 0.100 and μ is increased to 0.32 implying higher home ownership rates than in the original "Base" parametrization.

Table 5.1: Parametric Robustness

	$\mu = 0.32$		$\theta = 0.100$		$\phi = 0.80$		$F = 0.025$	
	Net	Gross	Net	Gross	Net	Gross	Net	Gross
					percent			
Ownership rate	65.8	67.4	63.5	65.1	56.2	57.6	57.2	58.5
- flats	49.7	51.0	49.0	50.4	40.5	41.7	36.1	36.6
- houses	16.1	16.4	14.5	14.7	15.6	15.8	21.1	21.8
Ownership rate (age 26-35)	83.2	85.6	78.9	81.7	63.6	66.4	66.3	68.6
- flats	76.7	78.9	73.6	76.1	57.8	60.4	52.6	53.7
- houses	6.4	6.7	5.3	5.5	5.7	6.0	13.7	14.8
outright share	15.8	14.6	17.2	15.9	19.7	18.2	14.8	13.7
median LTV	36.1	41.8	34.3	40.0	29.0	33.7	34.5	39.2
Ownership rate (age 36-50)	84.6	86.5	83.2	85.1	77.3	79.2	76.8	78.3
- flats	62.8	64.4	63.8	65.4	56.6	58.2	45.3	45.6
- houses	21.7	22.1	19.3	19.6	20.6	20.9	31.4	32.7
outright share	66.3	60.8	68.0	62.4	70.6	65.3	59.9	55.1
median LTV	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Ownership rate (age 51-60)	77.2	78.3	76.2	77.3	72.2	73.4	71.9	72.8
- flats	50.3	51.2	51.9	52.7	46.3	47.2	38.1	38.3
- houses	26.8	27.1	24.3	24.6	25.8	26.1	33.8	34.5
outright share	79.5	71.8	80.6	72.8	81.4	74.4	74.3	67.7
median LTV	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
					mean (relative to median Y_t) (age 51-60)			
Total Net Worth (W_t)	10.4	10.2	10.3	10.2	10.1	10.0	10.1	10.0
Financial Net Worth (N_t)	4.4	4.2	4.5	4.3	4.5	4.4	4.3	4.2
Gross Debt (D_t)	0.0	0.3	0.0	0.2	0.0	0.2	0.1	0.3
					median (relative to median Y_t) (age 51-60)			
Total Net Worth (W_t)	8.2	8.0	8.1	8.0	7.9	7.8	8.0	7.9
Financial Net Worth (N_t)	2.9	2.7	3.0	2.9	3.1	3.0	2.8	2.7
Gross Debt (D_t)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
					transactions			
Buys per life	1.6	1.6	1.5	1.5	1.4	1.4	1.9	1.9
- flats	1.3	1.3	1.2	1.2	1.1	1.1	1.4	1.4
- houses	0.3	0.3	0.2	0.2	0.3	0.3	0.4	0.4

Parameters: see table 2.1 and 2.2, except for $r_d = 0.023$, $\beta = 0.96$, $\gamma = 0.035$.

The optimal robustness test would naturally be to re-calibrate the model to fit the original home ownership moments and e.g. add the life-cycle profile of outright ownership as a set of new moments to fit. This task is, however, beyond the scope of this paper. Furthermore it is not obvious that it will at all be possible to fit these moments because the increased impatience implies that the home ownership profile becomes too steep both in the sense of a too fast increase for the young households and a too strong decrease for households approaching retirement. Consequently it would be necessary to extend the model to make housing in general and owning in particular less valuable for younger households. An exogenous taste shifter justified by e.g. changes in family size and composition would be the simplest solution. A more endogenous mechanism could be a strong incentive for young households to remain geographically mobile for both family and career reasons; this reduces home ownership among the young households because of the large transaction costs.[11]

5.2 (Re)financing Costs

Table 5.2 shows the simulation results when (re)financing costs are introduced. The fixed cost is set to one week of income for the young, i.e. $S_F = \frac{1}{52}$, and the proportional cost is assumed to be 1 percent, i.e. $S_D = 0.01$. In themselves these financing costs naturally reduce the home ownership rate, but their effect on the difference between the net and gross stocks formulations is more or less negligible. The total home ownership rate now increases by 5.1 percentage points in the "Base" parametrization and 1.6 percentage points in the "Beta" parametrization, while the original effects were 5.4 and 1.4 percentage points (see table 4.1). Underlying this, however, we see that allowing for precautionary balance sheet expansions now, especially in the "Base" parametrization, has a relatively smaller effect on the home ownership rate for the young households; naturally this is counter-weighted by a relatively larger positive effect for the older households. In total the results in this paper does *not* depend on assuming zero (re)financing costs.

[11] See for e.g. Halket and Vasudev (2014) for a thorough investigation of these isuues.

Table 5.2: Refinancing Costs

	Base		Beta	
	Net	Gross	Net	Gross
	percent			
Ownership rate	61.3	66.4	53.0	54.6
- flats	30.5	23.7	41.9	42.8
- houses	30.8	42.6	11.0	11.8
Ownership rate (age 26-35)	62.6	69.9	56.4	58.9
- flats	26.3	20.8	53.3	55.1
- houses	36.3	49.1	3.0	3.8
outright share	45.1	1.5	24.8	22.2
median LTV	3.7	79.5	24.3	29.3
Ownership rate (age 36-50)	81.8	85.7	73.1	75.7
- flats	40.7	30.2	59.1	60.5
- houses	41.1	55.5	13.9	15.1
outright share	93.5	2.1	81.5	70.7
median LTV	0.6	73.1	0.3	0.0
Ownership rate (age 51-60)	79.9	83.7	69.8	71.6
- flats	40.9	32.1	50.8	51.5
- houses	39.0	51.6	19.0	20.0
outright share	96.8	0.4	92.2	81.0
median LTV	0.0	67.5	0.4	0.9

Parameters: see table 2.1 and 2.2, except for $S_F = 1/52$, and $S_D = 0.01$.

6 Conclusions

The central contribution in this paper has been to show that modeling the long-term borrowing constraint in *gross* terms rather than *net* terms has important implications for the model dynamics. We have shown that letting households have access to precautionary balance sheet expansions boosts the overall demand for housing and induces a substitution from flats towards bigger houses. The further analysis showed that these results were robust to introducing both a substantial mortgage spread and marked forced mortgage repayments if just the households were impatient enough. The required level of impatience was moreover shown to make the model better match the outright ownership rates found empirically. Finally we concluded that changing individual parameters or adding refinancing costs did if anything rather amplify than dampen the importance of allowing households to do precautionary balance expansions.

A full re-calibration of the model in gross stocks was left for future work, but our results indicate that matching outright ownership rates will be central for such an exercise. Combining such a re-calibration with a discussion of the model's ability to explain the response of non-durable consumption to both house price shocks and income shocks is an interesting topic; both when looking at it in the aggregate and across different stages of the life-cycle. Adding a stochastic interest rate process would further open up for matching the model to data on the actual refinancing behavior of households and thereby a discussion of the full welfare costs of the refinancing frictions.

Finally the whole setup is naturally only a first step in building a full general equilibrium model with endogenous house prices where allowing the households to survive being underwater and doing precautionary balance sheet expansions could be central for explaining why turnover rates fall so steeply in recessions freezing the housing market.

A Solution Algorithm

A.1 Restructured State Space

The Bellman equation with M_t as a state variable is given by

$$\begin{aligned}
V_t(H_{t-1}, D_{t-1}, M_t, Y_t, P_t) &= \max_{H_t, K_t, D_t, C_t} u(C_t) + \beta \cdot \mathbb{E}_t[V_{t+1}(\bullet)] && \text{(A.1)}\\
&\text{s.t.} \\
H_t &\in \{0,1,2\} && \text{(A.2)}\\
K_t &\in \begin{cases} \{0,1\} & \text{if } H_t = H_{t-1} \\ \{0\} & \text{else} \end{cases} && \text{(A.3)}\\
D_t &\in \begin{cases} [0, (1-\gamma)\cdot D_{t-1}] & \text{if } K_t = 1 \\ [0, \Lambda(\bullet)] & \text{if } K_t = 0 \end{cases} && \text{(A.4)}\\
C_t &\in \left[0, \overline{C}(\bullet)\right] && \text{(A.5)}\\
A_t &= \overline{C}(\bullet) - C_t && \text{(A.6)}\\
M_{t+1} &= (1+r_a)\cdot A_t - (1+r_d)\cdot D_t && \text{(A.7)}
\end{aligned}$$

where

$$\overline{C}(H_{t-1}, M_t, Y_t, P_t, H_t, K_t, D_t) = M_t + Y_t + D_t - \Omega(\bullet) \qquad \text{(A.8)}$$

We denote the optimal choice functions by $H_t^\star(\bullet)$, $K_t^\star(\bullet)$, $D_t^\star(\bullet)$ and $C_t^\star(\bullet)$. Alternatively the model can be reformulated as the *upper envelope* of a series of *choice specific value functions*, i.e

$$V_t(H_{t-1}, D_{t-1}, M_t, Y_t, P_t) = \max\left\{V_t^{own}, V_t^{rent}\right\} \qquad \text{(A.9)}$$

where

$$V_t^{own}(\bullet) \equiv \begin{cases} \max\left\{V_t^{buy,flat}(\bullet), V_t^{buy,house}(\bullet)\right\} & \text{if } H_t = 0 \\ \max\left\{V_t^{keep}(\bullet), V_t^{refi}(\bullet), V_t^{buy,house}(\bullet)\right\} & \text{if } H_t = 1 \\ \max\left\{V_t^{keep}(\bullet), V_t^{refi}, V_t^{buy,flat}(\bullet)\right\} & \text{if } H_t = 2 \end{cases}$$

and the choices are restricted as follows in the various cases

1. **Keep**: $H_t = H_{t-1}$, $K_t = 1$ and $D_t \in [0\,,\,(1-\gamma)\cdot D_{t-1}]$.

2. **Refi(nance)**: $H_t = H_{t-1}$, $K_t = 0$ and $D_t \in [0, \Lambda(H_t, P_t, Y_t)]$.
3. **Buy, flat**: $H_t = 1$ and $D_t \in [0\,,\,\Lambda(1, P_t, Y_t))]$.[12]
4. **Buy, house**: $H_t = 2$ and $D_t \in [0\,,\,\Lambda(2, P_t, Y_t))]$.
5. **Rent**: $H_t = 0$ and $D_t = 0$.

Interpolating the choice specific value functions *separately* (and finding the maximum) when evaluating the continuation value has a gain in terms of precision when solving the model. The gain seems to be large relative to the increased computation time.

A.2 Choice Bounds

Lemma A.1. *For all $x \in \mathbb{R}_+$ we have the following logical implication*

$$\begin{aligned} D_t^{\star,keep}(\bullet, x, \bullet) &= z \rightarrow \\ \forall D_{t-1} \in \left[\frac{z}{1-\gamma} : x\right] : D_t^{\star,keep}(\bullet, D_{t-1}, \bullet) &= z \end{aligned}$$

Proof. Decreasing D_{t-1} from x to $\frac{z}{1-\gamma} \leq x$ only (weakly) shrinks the choice set and removes non-optimal choices. This cannot change the optimal choice. □

Lemma A.2. *If $S_D = 0$ and $S_F = 0$ then if $D_{t-1} \in \left[0, \frac{1}{1-\gamma} \cdot \Lambda(H_t, P_t, Y_t))\right]$ we have*

$$V_t^{refi}(\bullet, D_{t-1}, \bullet) \geq V_t^{keep}(\bullet, 0, \bullet)$$

Proof. Given $(1-\gamma) \cdot D_{t-1} \leq \Lambda(H_t, P_t, Y_t))$ the choice set is (weakly) smaller under *keeping*, so the optimal choice cannot be any better than under *refinancing* (when refinancing is cost less, $S_D = S_F = 0$). □

Lemma A.3. *If $S_D = 0$ then for all $D_{t-1} \in \mathbb{R}_+$ we have*

$$\begin{aligned} V_t^{refi}(\bullet, D_{t-1}, \bullet) &= V_t^{refi}(\bullet, 0, \bullet) \\ D_t^{\star,refi}(\bullet, D_{t-1}, \bullet) &= D_t^{\star,refi}(\bullet, 0, \bullet) \\ C_t^{\star,refi}(\bullet, D_{t-1}, \bullet) &= C_t^{\star,refi}(\bullet, 0, \bullet) \end{aligned}$$

[12] Note that when $S_F > 0$ there is also a first order kink at $D_t > 0$ vs. $D_t = 0$, which we for simplicity have chosen not to include as a discrete choice.

Proof. When $S_D = 0$ the term with D_{t-1} disappears from (2.13). □

Lemma A.4. *If* $D_{t-1} \geq \frac{1}{1-\gamma} \cdot \Lambda\left(H_t, P_t, Y_t\right)$ *we have*

$$\begin{aligned}
V_t^{refi}\left(\bullet, D_{t-1}, \bullet\right) &= V_t^{refi}\left(\bullet, \frac{1}{1-\gamma} \cdot \Lambda\left(H_t, P_t, Y_t\right), \bullet\right) \\
D_t^{\star,refi}\left(\bullet, D_{t-1}, \bullet\right) &= D_t^{\star,refi}\left(\bullet, \frac{1}{1-\gamma} \cdot \Lambda\left(H_t, P_t, Y_t\right), \bullet\right) \\
C_t^{\star,refi}\left(\bullet, D_{t-1}, \bullet\right) &= C_t^{\star}\left(\bullet, \frac{1}{1-\gamma} \cdot \Lambda\left(H_t, P_t, Y_t\right), \bullet\right)
\end{aligned}$$

Proof. Due to the maximum operator in (2.13) increasing D_{t-1} above $\frac{1}{1-\gamma} \cdot \Lambda\left(H_t, P_t, Y_t\right)$ does not affect any choices under *refinancing*. □

A.3 Implementation

The code is written in *C* with *OpenMP 4.0* for parallelization and called from *Python 2.7.6* using the *CFFI* interface. The C-code is compiled using the *TDM-GCC 4.9.2* compiler. Only free open source languages and programs are required to run the code. The code-files are available from the author upon request.

B Further Details

B.1 Discretization

The state variables are discretized as follows:

- Y_t: Tauchen ($N_Y = 15$). All transitions with a probability less than 10^{-6} are disregarded.
- P_t: Tauchen ($N_P = 15$). All transitions with a probability less than 10^{-6} are disregarded.
- D_{t-1}: Age and H_{t-1} dependent grid (more nodes closer to zero, $N_D = 50$), where the upper bound is given recursively by

$$\begin{aligned} D_{t-1} &\in \left[0; \overline{D}\left(t-1, H\right)\right] \\ \forall t \geq 1,\ \overline{D}\left(t, H\right) &= \left(1-\gamma\right) \cdot \max\left\{\overline{D}\left(t-1, H\right), \Lambda\left(H, \overline{P}_t, \overline{Y}_t\right)\right)\right\} \\ \overline{D}\left(-1, H\right) &= 0 \end{aligned}$$

  where an over-line denotes the *upper support* of a variable.

- M_t: Age and H_{t-1} dependent grid (more nodes closer to lowest point, $N_M = 100$), the lower bound is given by

$$\underline{M}_t = -\left(1 + r_d\right) \cdot \overline{D}\left(t-1, H\right) \quad \text{(B.1)}$$

B.2 Simulation: Initial Wealth

Table B.1: Initial Wealth

Low Education		*High Education*	
Cum. Prob.	$A_{-1} = x \cdot Y_{0,L}$	Cum. Prob.	$A_{-1} = x \cdot Y_{0,H}$
33.0	0.0000	22.0	0.0000
39.7	0.0000	29.8	0.0016
46.4	0.0008	37.6	0.0093
53.1	0.0070	45.4	0.0304
59.8	0.0251	53.2	0.0708
66.5	0.0435	61.0	0.1332
73.2	0.0785	68.8	0.2186
79.9	0.1566	76.6	0.4564
86.6	0.2490	84.4	0.7217
93.3	0.6598	92.2	1.2434
100.0	1.4617	100.0	4.4281

References

Alan, S., T. Crossley and H. Low (2012): "Saving on a Rainy Day, Borrowing for a Rainy Day," IFS Working Paper W12/11.

Attanasio, O., A. Leicester and M. Wakefield (2011): "Do House Prices Drive Consumption Growth? The Coincident Cycles of House Prices and Consumption in the UK," *Journal of the European Economic Association*, 9(3), 399–435.

Attanasio, O. P., R. Bottazzi, H. W. Low, L. Nesheim and M. Wakefield (2012): "Modelling the demand for housing over the life cycle," *Review of Economic Dynamics*, 15(1), 1–18.

Bajari, P., P. Chan, D. Krueger and D. Miller (2013): "A Dynamic Model of Housing Demand: Estimation and Policy Implications," *International Economic Review*, 54(2), 409–442.

Banks, J., C. O'Dea and Z. Oldfield (2010): "Cognitive Function, Numeracy and Retirement Saving Trajectories," *The Economic Journal*, 120(548), F381–F410.

Campbell, J. Y. and J. F. Cocco (2007): "How do house prices affect consumption? Evidence from micro data," *Journal of Monetary Economics*, 54(3), 591–621.

Chambers, M., C. Garriga and D. E. Schlagenhauf (2009a): "Accounting for Changes in the Homeownership Rate," *International Economic Review*, 50(3), 677–726.

——— (2009b): "Housing policy and the progressivity of income taxation," *Journal of Monetary Economics*, 56(8), 1116–1134.

——— (2013): "Did Housing Policies Cause the Postwar Boom in Home Ownership?," *NBER Working Paper 18821.*

Chambers, M. S., C. Garriga and D. Schlagenhauf (2009c): "The loan structure and housing tenure decisions in an equilibrium model of mortgage choice," *Review of Economic Dynamics*, 12(3), 444–468.

Chen, H., M. Michaux and N. Roussanov (2015): "Houses as ATMs? Mortgage Refinancing and Macroeconomic Uncertainty," NBER Working Paper 19421.

Crawford, R. and C. O'Dea (2014): "Cash and Pensions: Have the elderly in England saved optimally for retirement," IFS Working Papers W14/22.

Crossley, T. F. and C. O'Dea (2010): "The wealth and saving of UK families on the eve of the crisis," IFS Reports R71.

Díaz, A. and M. J. Luengo-Prado (2008): "On the user cost and homeownership," *Review of Economic Dynamics*, 11(3), 584–613.

Druedahl, J. and C. N. Jørgensen (2015): "Precautionary Borrowing and the Credit Card Debt Puzzle," Working Paper.

Fernandez-Villaverde, J. and D. Krueger (2011): "Consumption and Saving over the Life Cycle: How important are durables?," *Macroeconomic Dynamics*, 15(05), 725–770.

Gervais, M. (2002): "Housing taxation and capital accumulation," *Journal of Monetary Economics*, 49(7), 1461–1489.

Halket, J. and S. Vasudev (2014): "Saving up or settling down: Home ownership over the life cycle," *Review of Economic Dynamics*, 17(2), 345–366.

Leth-Petersen, S. and J. Ejarque (2008): "Consumption and Savings of First Time House Owners," Working Paper.

Li, W., H. Liu, F. Yang and R. Yao (2015): "Housing over time and over the life cycle: a structural estimation," International Economic Review, forthcoming.

Li, W. and R. Yao (2007): "The Life-Cycle Effects of House Price Changes," *Journal of Money, Credit and Banking*, 39(6), 1375–1409.

Miles, D. (2005): "Incentives Information and Efficiency in the UK Mortgage Market," *The Economic Journal*, 115(502), C82–C98.

Ríos-Rull, J.-V. and V. Sánchez-Marcos (2008): "An Aggregate Economy with Different Size Houses," *Journal of the European Economic Association*, 6(2-3), 705–714.

Silos, P. (2007a): "Housing, portfolio choice and the macroeconomy," *Journal of Economic Dynamics and Control*, 31(8), 2774–2801.

——— (2007b): "Housing Tenure and Wealth Distribution in Life Cycle Economies," *The B.E. Journal of Macroeconomics*, 7(1).

Sommer, K., P. Sullivan and R. Verbrugge (2013): "The equilibrium effect of fundamentals on house prices and rents," *Journal of Monetary Economics*, 60(7), 854–870.

Tauchen, G. (1986): "Finite state markov-chain approximations to univariate and vector autoregressions," *Economics Letters*, 20(2), 177–181.

Yang, F. (2009): "Consumption over the life cycle: How different is housing?," *Review of Economic Dynamics*, 12(3), 423–443.

Yao, R. and H. H. Zhang (2005): "Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints," *Review of Financial Studies*, 18(1), 197–239.