Interest Rate Rules, Price Determinacy and the Value of Money in a non Ricardian World

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Abstract

This article studies under which conditions interest rate rules "à la Taylor (1993)" lead to price determinacy. We scrutinize notably two famous results, which are standard in the traditional "Ricardian" model with a single dynasty of consumers: (1) a pure interest rate peg leads to nominal price indeterminacy; (2) a strong reaction (usually more than one for one) of nominal interest rates to inflation is conducive to price determinacy (the Taylor principle). This article extends the analysis to rigorous dynamic non Ricardian models. The results turn out to be quite different, since notably prices may be determinate if the interest rate responds less than one for one to inflation, and even under a pure interest rate peg.

Keywords: Interest rate rules, Price determinacy, Monetary policy rules, Taylor principle, Interest rate peg, Pigou effect, Real balance effect.

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1 Introduction

Following Taylor's (1993) stimulating article, there has been recently a very strong renewal of interest in the study of interest rate rules for monetary authorities (for a panorama of recent work, see for example McCallum 1999, Taylor 1999). In line with the recent trends in macroeconomics, several authors quite naturally investigated interest rate policies in rigorous dynamic general equilibrium RBC type models.

Most rigorous studies of optimal interest rate rules in such a maximizing framework have been cast in "Ricardian" economies populated with a single dynasty of consumers[1]. These economies have, however, as far as policy analysis is concerned, a number of particular properties, and it thus seems legitimate to extend the analysis of interest rate rules to non Ricardian economies where new agents enter in each period, and to see whether this makes a difference or not for the analysis. We shall see that it does.

In this article we shall be particularly concerned with the issue of price determinacy under various monetary rules. We shall notably scrutinize two particularly famous results.

- The first one, which originates with the article by Sargent and Wallace (1975), basically says that, under a pure nominal interest rate peg, there is nominal indeterminacy[2]. This means that, if a sequence of prices is an equilibrium, then any sequence multiple of the first one is also an equilibrium. This is no minor problem since many optimal policy packages include the famous "Friedman rule", according to which the nominal interest rate should be set equal to zero[3].

- The second one is often referred to as the "Taylor principle"[4]. The basic idea is that, in order to make prices determinate the central bank

[1] So we shall use the terminology "Ricardian" for economies with a single dynasty of consumers, and "non Ricardian" for economies where new consumers arrive in time. This terminology has its root in the "Ricardian equivalence" result (Barro, 1974), according to which, in an economy with a single dynasty of consumers, the timing of (lump sum) taxes is irrelevant as long as the government intertemporally balances its budget.

Note that this meaning of "Ricardian" is quite different from a later one which has been adopted by authors working on the "fiscal theory of the price level" (see, for example, Kocherlakota and Phelan, 1999, for a simple exposition).

[2] For a useful taxonomy of various forms of indeterminacy, see McCallum (1986).

[3] This rule originates in Friedman (1969). The intuition is that, since money costs nothing to produce, its services should be priced at zero.

[4] It should be noted that, although Taylor (1993) recommends a strong response of interest rates to inflation, this is not for the reasons explored in this article. The reasoning (see, for example, Taylor, 1998) is that if the nominal interest rate responds more than one for one to inflation, the real interest rate will respond countercyclically to inflation, which should have a stabilizing influence on the economy.

should respond "aggressively" to inflation. If interest rates respond only to inflation, a classic result is that, in order to have determinate prices, nominal interest rates should respond more than one for one to inflation[5].

We shall see that considering non-Ricardian instead of Ricardian economies dramatically modifies the answers to the two above questions[6]. Notably:

- A pure interest rate peg is fully consistent with price determinacy, provided the interest rate satisfies a very mild condition.

- Prices can be determinate even if the interest rate responds less than one for one to inflation. In fact, from the above result, it needs not even respond at all.

The rest of the article is organized as follows: Section 2 presents the model. Section 3 derives the dynamic equilibrium equations. Section 4 describes, for the sake of comparison, some traditional results in the Ricardian framework. Section 5 derives a sufficient condition for price determinacy under an interest rate peg. Section 6 gives a number of economic interpretations. Section 7 studies whether the Taylor principle is still relevant in the non Ricardian model. Section 8 extends the results to a framework with a forward looking Phillips curve. Section 9 introduces a more general fiscal policy. Section 10 concludes.

2 The model

In order to have a non Ricardian stucture, we shall use a model adapted from that of Weil (1987, 1991)[7], and assume that new "generations" of households are born each period. Call N_t the number of households alive at time t. We shall assume that $N_{t+1} \geq N_t$. We will actually mainly work below with the case where the population grows at the constant rate $n \geq 0$, so that $N_t = (1+n)^t$.

Consider a household j (i.e. a household born in period j). We denote by c_{jt} and m_{jt} his consumption and money holdings at time $t \geq j$. This

[5] Early results in this direction on price determinacy and monetary rules are found in Leeper (1991).

[6] A few contributions have sought to modify the "traditional" results on interest pegging or the Taylor principle. For example McCallum (1981) advocates linking interest rates to a "nominal anchor" like the price level. Bénassy (2000) introduces the non-Ricardian insight into the issue of interest pegging using a different uniqueness criterion, and so obtains different results. Benhabib, Schmitt-Grohé and Uribe (2001) show the importance of how money enters the utility and production functions. Roisland (2003) shows that capital income taxation modifies the Taylor principle.

[7] The difference is that Weil includes money in the utility function, whereas we use a cash in advance constraint.

household receives in periods $t \geq j$ an endowment y_{jt} and maximizes in each period $t \geq j$ the following utility function:

$$U_{jt} = \sum_{s=t}^{\infty} \beta^{s-t} Log\, c_{js} \tag{1}$$

Household j is submitted in period t to a "cash in advance" constraint:

$$P_t c_{jt} \leq m_{jt} \tag{2}$$

Household j enters period t with a financial wealth ω_{jt}. Transactions occur in two steps. First the bond market opens, and the household can lend at the nominal interest rate i_t an amount b_{jt} (of course b_{jt} can be negative if the household borrows to obtain liquidity). The rest is kept under the form of money m_{jt}, so that:

$$\omega_{jt} = m_{jt} + b_{jt} \tag{3}$$

Then the goods market opens, and the household sells his endowment y_{jt}, pays taxes τ_{jt} in real terms and consumes c_{jt}, subject of course to the cash constraint (2). Consequently, the budget constraint for the household is:

$$\omega_{jt+1} = (1 + i_t)\, \omega_{jt} - i_t m_{jt} + P_t y_{jt} - P_t \tau_{jt} - P_t c_{jt} \tag{4}$$

Aggregate quantities are obtained by summing the various individual variables. Since there are $N_j - N_{j-1}$ agents in cohort j, these aggregates are equal to:

$$Y_t = \sum_{j \leq t} (N_j - N_{j-1})\, y_{jt} \qquad C_t = \sum_{j \leq t} (N_j - N_{j-1})\, c_{jt} \tag{5}$$

$$T_t = \sum_{j \leq t} (N_j - N_{j-1})\, \tau_{jt} \qquad \Omega_t = \sum_{j \leq t} (N_j - N_{j-1})\, \omega_{jt} \tag{6}$$

$$M_t = \sum_{j \leq t} (N_j - N_{j-1})\, m_{jt} \qquad B_t = \sum_{j \leq t} (N_j - N_{j-1})\, b_{jt} \tag{7}$$

To be complete we have to describe how endowments and taxes are distributed among households. We assume that all households have the same income and taxes, so that:

$$y_{jt} = y_t = \frac{Y_t}{N_t} \qquad \tau_{jt} = \tau_t = \frac{T_t}{N_t} \tag{8}$$

Now the other important part of the model is the government. The households' aggregate financial wealth Ω_t has as a counterpart an identical amount Ω_t of financial liabilities of the government. The evolution of these liabilities is described by the government's budget constraint:

$$\Omega_{t+1} = (1 + i_t) B_t + M_t - P_t T_t = (1 + i_t) \Omega_t - i_t M_t - P_t T_t \tag{9}$$

3 The dynamic equilibrium

It is shown in the appendix that the following dynamic equation holds:

$$P_{t+1} Y_{t+1} = \beta (1 + n) (1 + i_t) P_t Y_t - (1 - \beta) n \Omega_{t+1} \tag{10}$$

We note that the non-Ricardian character of the economy appears through the last term, which disappears if $n = 0$, as accumulated financial wealth now matters (we shall see an explanation in section 6).

Since the model is non-Ricardian, the dynamics will depend on the actual tax policy, as it will notably influence the dynamics of Ω_t. In order to simplify the dynamics below, we shall assume that the tax policy of the government consists in balancing the budget period by period[8]. Taxes will thus cover exactly interest payments on bonds:

$$P_t T_t = i_t B_t \tag{11}$$

We may immediately note, using the government's budget constraint (9), that under this balanced budget policy total financial wealth will remain constant:

$$\Omega_t = \Omega \qquad \text{for all } t \tag{12}$$

The dynamic equation (10) then becomes:

$$P_{t+1} Y_{t+1} = \beta (1 + n) (1 + i_t) P_t Y_t - (1 - \beta) n \Omega \tag{13}$$

We may note that taking N_t constant, or $n = 0$, we obtain the "traditional" equation:

$$P_{t+1} Y_{t+1} = \beta (1 + i_t) P_t Y_t \tag{14}$$

4 Traditional results

We shall now, using equation (14), briefly review some traditional results on price determinacy under interest rate rules in the Ricardian setting.

[8] A more general policy is considered in section 9.

4.1 Interest rate pegging

Let us start with interest rate pegging, and assume that the nominal interest rate is pegged at the value i_0. Equation (14) can be rewritten:

$$\frac{P_{t+1}}{P_t} = \beta\left(1 + i_0\right)\frac{Y_t}{Y_{t+1}} \tag{15}$$

In this formula Y_t and Y_{t+1} are exogenous. So formula (15) shows most clearly that setting nominal interest rates determines the ratios between intertemporal prices P_{t+1}/P_t, but not at all their absolute level. There is thus nominal indeterminacy, as was pointed out by Sargent and Wallace (1975).

4.2 The Taylor principle

Let us now consider more general interest rate rules of the form:

$$i_t - i_0 = a\left(\pi_t - \pi_0\right) \qquad a \geq 0 \tag{16}$$

where π_0 is the long run rate of inflation. The "Taylor principle" suggests that, for prices to be determinate, the coefficient a should be greater than 1.

Let us loglinearize equation (14), assuming that the (exogenous) output grows at the rate n. This yields:

$$\pi_{t+1} = Log\beta + i_t - n \tag{17}$$

Inserting (16) into (17), we obtain:

$$\pi_{t+1} = Log\beta + i_0 + a\left(\pi_t - \pi_0\right) - n \tag{18}$$

which can be rewritten as (we omit the constant terms):

$$\pi_t = \frac{\pi_{t+1}}{a} \tag{19}$$

Clearly the inflation rate will be determinate if $a > 1$ (the Taylor principle). Since the previous price is predetermined, a determinate inflation rate also means a determinate price.

5 Determinacy under an interest rate peg

We now revert to the more general non-Ricardian framework, and consider the first problem we mentioned, that of a pure interest rate peg. As above, we shall study the Walrasian version of the model. Let us thus assume that the interest rate is pegged at the value i_0. The dynamic system (13) is written:

$$P_{t+1}Y_{t+1}=\beta\left(1+n\right)\left(1+i_0\right)P_tY_t-\left(1-\beta\right)n\Omega \tag{20}$$

In what follows it will be convenient to use nominal income X_t as our working variable:

$$X_t=P_tY_t \tag{21}$$

so that (20) is rewritten:

$$X_{t+1}=\beta\left(1+n\right)\left(1+i_0\right)X_t-\left(1-\beta\right)n\Omega \tag{22}$$

Applying the conditions of Blanchard-Kahn (1980), we see that there is a unique solution in X_t provided that:

$$\theta=\beta\left(1+n\right)\left(1+i_0\right)>1 \tag{23}$$

and this solution is given by:

$$X_t=X_0=\frac{\left(1-\beta\right)n\Omega}{\beta\left(1+n\right)\left(1+i_0\right)-1} \tag{24}$$

6 Economic interpretations

We just found that prices will be determined if $n>0$ and condition (23), i.e. $\theta>1$, is satisfied. Since we shall encounter these conditions throughout the rest of the paper, it is time to give a few economic interpretations. There are actually several aspects.

6.1 The Pigou, or real balance effect

When one looks at the dynamic equations (10) and (13), it appears clearly that a feature that drives most of the results is the presence of accumulated financial assets Ω_t in the dynamic equations. This is indeed a "nominal anchor" which is intrumental in tying down the value of prices. This presence of accumulated financial assets in various behavioral equations, and notably in the consumption function, has a history in the literature under the names of "Pigou effect" (Pigou, 1943, 1947) or "real balance effect" (Patinkin, 1965).

Why, in a world of rational expectations, does this Pigou effect occur when $n>0$, and not for $n=0$, has been very well explained by Weil (1991). Because of the intertemporal governement budget constraint, when $n=0$ the value of financial assets is exactly matched by discounted future tax liabilities, so that the effect on intertemporal wealth is zero. Now if instead

$n > 0$ part of future taxes will be paid by yet unborn generations, so that part of Ω_t represents real wealth for the households alive.

6.2 Determinacy and the return on financial assets

Now $n > 0$ creates a real balance effect. But this not the end of the story. Clearly this effect will be really operative only if the agents actually want to hold money and financial assets. And this is where condition (23) comes in. In order to interpret it, let us rewrite (23) under the following form:

$$(1+n)(1+i_0) > \frac{1}{\beta} \tag{25}$$

The left hand side is the real rate of return on bonds. Indeed since $P_t Y_t = X_t$ is constant, and real ressources grow at the rate $1+n$, in the steady state prices decrease at the rate $1+n$, and therefore the real rate of interest is $(1+n)(1+i_0)$.

Now $1/\beta$ on the right hand side of (25) is the real rate of return that would prevail in a constant endowment economy with a single dynasty and a discount rate β, which is sometimes called the "autarchic" rate of return.

So conditions (23) or (25) essentially say that the real rate of return of bonds must be superior to the autarchic rate of return. We see that the above condition is very much similar to that found by Wallace (1980) for the viability of money in the traditional Samuelsonian (1958) overlapping generations model. There is an important difference, though: in Wallace (1980) the only financial store of value is money, so the rate of return condition concerns the return on money. Here this condition concerns the return on bonds, and accordingly the nominal interest rate plays an important role.

6.3 Suboptimality, price determinacy and the value of money

A theme that appears in the literature (and which is related to the previous one) is that money is valuable, and prices determinate, when the economy without money is Pareto suboptimal. This is notably studied in Wallace (1980) in the framework of the traditional Samuelsonian (1958) OLG model. Then, if the quantity of money stays constant, money will be valued and prices determinate if the "autarchic" equilibrium without money is Pareto suboptimal.

Here the relation between the suboptimality of the autarchic equilibrium and price determinacy becomes somewhat less clearcut. Indeed the condition for Pareto suboptimality of the autarchic equilibrium, is[9] $\beta(1+n) > 1$. Now the condition for price determinacy is $\beta(1+n)(1+i_0) > 1$, so the two conditions are not the same. In particular the above inequalities suggest that there are cases where the autarchic equilibrium is Pareto optimal and nevertheless prices are determinate. This will occur when:

$$\beta(1+n) < 1 \qquad \text{and} \qquad \beta(1+n)(1+i_0) > 1 \tag{26}$$

One can also find cases (section 9 below) where the autarchic equilibrium is Pareto suboptimal, and prices nevertheless are not determinate.

So, although there is a clear conceptual connection between the two, there is not a one to one relationship between suboptimality and price determinacy.

6.4 A discontinuity

A particularly striking aspect of the results is the discontinuity in the determinacy conditions that arises when going from $n = 0$ to $n > 0$. An intuition is the following: just as OLG type models, the Weil model has 2 types of long-run equilibria: the ones where financial assets have real value (these are the ones studied in this article) and the ones where, either because prices are infinite or $\Omega = 0$, financial assets have no real value. These last ones are called "autarchic" because no value is transmitted across generations through the assets. There should be a continuity result between the Ricardian model and the "autarchic" equilibria of the non Ricardian model, because they have a similar structure. But such is not the case for the equilibria with positively valued financial assets, so that the discontinuity needs not surprise us in this case.

7 The Taylor principle

Let us continue with the non-Ricardian model and turn to the Taylor principle. Loglinearizing equation (13), we obtain the following equation (we eliminate the constants):

$$p_{t+1} = \theta p_t + \gamma i_t \tag{27}$$

with:

$$\theta = \beta(1+n)(1+i_0) \qquad \gamma = \beta(1+n) \tag{28}$$

[9] This is derived in Weil (1989) for the continuous time version of the model. A direct proof for the discrete time version we use here is available from the author.

Combining with the equation giving the interest rate we obtain:

$$p_{t+1} = \theta p_t + \gamma a \pi_t \tag{29}$$

This can actually be rewritten as a two dimensional dynamic system in inflation and the price level:

$$p_t = \pi_t + p_{t-1} \tag{30}$$

$$\pi_{t+1} = (\theta - 1) p_t + \gamma a \pi_t = (\theta - 1 + \gamma a) \pi_t + (\theta - 1) p_{t-1} \tag{31}$$

or in matrix form:

$$\begin{bmatrix} \pi_{t+1} \\ p_t \end{bmatrix} = \begin{bmatrix} \theta - 1 + \gamma a & \theta - 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ p_{t-1} \end{bmatrix} \tag{32}$$

The characteristic polynomial is:

$$\Phi(\mu) = \gamma a (1 - \mu) + \mu (\mu - \theta) \tag{33}$$

We have one predetermined variable (the past price) and one non predetermined (inflation). So there will be a determinate solution if the polynomial $\Phi(\mu)$ has one root of modulus smaller than 1, and the other greater than 1. So we compute:

$$\Phi(0) = \gamma a \geq 0 \tag{34}$$

$$\Phi(1) = 1 - \theta \tag{35}$$

We see that, if $\theta > 1$, we have one root between zero and one, and the other greater than one. So $\theta > 1$ is again a sufficient condition for determinacy, and whether a is above or below 1 is not important anymore.

8 The Taylor principle with a Phillips curve

So far we have studied the issue of price determinacy under the assumption of full market clearing. But the issue of price determinacy under interest rate rules has been very often studied in models with non clearing markets where output is demand determined and prices adjust partially according to a forward looking "Phillips curve" of the type:

$$\pi_t = \frac{1}{\alpha} E_t \pi_{t+1} + \delta y_t \qquad \alpha > 1 \qquad \delta > 0 \tag{36}$$

We want to show now that the results we obtained above in a Walrasian economy extend to this framework as well.

Phillips curves such as (36) are most often derived from a framework of contracts à la Calvo (1983)[10]. Clearly the rigorous derivation of such a Phillips curve in our setting, mixing overlapping contracts and heterogeneous cohorts, would take us much too far. So we shall simply take the Phillips curve (36) as given, and show that going from a Ricardian to a non-Ricardian framework leads again to major changes.

Again the monetary authority uses an interest rate rule of the Taylor type:

$$i_t - i_0 = a\left(\pi_t - \pi_0\right) \tag{37}$$

In order to better highlight the differences, let us now begin with the Ricardian version of the model.

8.1 The Ricardian case

Output is now endogenous. Since it is demand determined, equation (14) is still valid. Loglinearizing it we obtain:

$$y_{t+1} = y_t + Log\beta + \left(i_t - \pi_{t+1}\right) \tag{38}$$

Combining this with the interest rule (37) yields:

$$y_{t+1} = y_t + Log\beta + i_0 + a\left(\pi_t - \pi_0\right) - \pi_{t+1} \tag{39}$$

Equations (36) and (39) are rewritten, omitting again the irrelevant constants (and replacing $E_t\pi_{t+1}$ by π_{t+1}, since the model is deterministic):

$$\pi_{t+1} = \alpha\left(\pi_t - \delta y_t\right) \tag{40}$$

$$y_{t+1} = \left(1 + \alpha\delta\right) y_t + \left(a - \alpha\right)\pi_t \tag{41}$$

This is written under matrix form:

$$\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1+\alpha\delta & a-\alpha \\ -\alpha\delta & \alpha \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} \tag{42}$$

The characteristic polynomial is:

[10] They can be also derived from a model with convex costs of changing prices. See Rotemberg (1987) for an early derivation under both interpretations.

$$\Phi(\mu) = (\mu - 1)(\mu - \alpha) + \alpha\delta(a - \mu) \tag{43}$$

$$\Phi(0) = \alpha(1 + \delta a) > 0 \tag{44}$$

$$\Phi(1) = \alpha\delta(a - 1) \tag{45}$$

If $a < 1$, we have one root between 0 and 1. Since neither y_t and π_t are predetermined, this means that we have indeterminacy. On the other hand if $a > 1$ the two roots have modulus greater than 1, and we have determinacy. We thus find again that the Taylor principle holds in this Ricardian framework.

8.2 The non-Ricardian case

Let us now move to the non-Ricardian economy. Equation (13) still holds. Loglinearizing it, we find that output, inflation and prices are linked by the following equation (again we eliminate irrelevant constants):

$$y_{t+1} + p_{t+1} = \theta(y_t + p_t) + \gamma i_t \tag{46}$$

where the values of θ and γ are given in equation (28). We now express y_{t+1}, π_{t+1} and p_t as a function of the corresponding lagged variables:

$$p_t = \pi_t + p_{t-1} \tag{47}$$

$$\pi_{t+1} = \alpha(\pi_t - \delta y_t) \tag{48}$$

$$y_{t+1} = (\alpha\delta + \theta) y_t + (\gamma a - \alpha + \theta - 1)\pi_t + (\theta - 1) p_{t-1} \tag{49}$$

or in matrix form:

$$\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ p_t \end{bmatrix} = \begin{bmatrix} \alpha\delta + \theta & \gamma a - \alpha + \theta - 1 & \theta - 1 \\ -\alpha\delta & \alpha & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ p_{t-1} \end{bmatrix} \tag{50}$$

The characteristic polynomial is:

$$\Phi(\mu) = (1 - \mu)(\alpha - \mu)(\theta - \mu) + \alpha\delta\gamma a(1 - \mu) + \alpha\delta\mu(\mu - \theta) \tag{51}$$

We shall now show that $\theta > 1$ is again a sufficient condition for determinacy. There is one predetermined variable (the past price) and two non predetermined ones (output and inflation). So there will be a determinate solution if the polynomial $\Phi(\mu)$ has one root of modulus smaller than 1, and two roots of modulus greater than 1. Let us compute:

$$\Phi(0) = \alpha(\theta + \delta\gamma a) > 0 \tag{52}$$

$$\Phi(1) = \alpha\delta(1-\theta) \tag{53}$$

So there is, assuming $\theta > 1$, one root between zero and one. Now the product of the three roots is $\alpha(\theta + \delta\gamma a) > 1$. So the only possible case where the two remaining roots would not be of modulus greater than 1 would be that where we have two negative roots, one smaller than -1, one greater. In that case we would have $\Phi(-1) < 0$. This means that, together with $\theta > 1$, $\Phi(-1) > 0$ is a sufficient condition for determinacy. So we compute:

$$\Phi(-1) = 2(1+\alpha)(1+\theta) + 2\alpha\delta\gamma a + \alpha\delta(1+\theta) > 0 \tag{54}$$

So to summarize, if $\theta > 1$, we have one root between zero and one, and two roots of modulus greater than one, so that the inflation rate is determinate, and thus so is the price level.

9 Variable government liabilities

We shall now study a generalization of the fiscal policy (11) and assume that, instead of balancing the budget, the government engineers through taxes proportional expansions (or reductions) of its financial liabilities Ω_t (such an experiment was studied in Wallace, 1980), and we shall see how this affects the conditions for determinacy. More precisely we shall assume taxes of the form:

$$T_t = i_t B_t + \rho\Omega_t \qquad \rho < 1 \tag{55}$$

As a result the evolution of Ω_t is given by:

$$\Omega_{t+1} = (1-\rho)\Omega_t \tag{56}$$

Most of the analysis seen previously is still valid, and in particular equation (10) which we recall here:

$$P_{t+1}Y_{t+1} = \beta(1+n)(1+i_t)P_tY_t - (1-\beta)n\Omega_{t+1} \tag{57}$$

The dynamic system consists of equations (56) and (57). In view of the homogeneity properties, this is actually a system in P_t/Ω_t and Y_t. Using (56), equation (57) can be rewritten as:

$$\frac{P_{t+1}Y_{t+1}}{\Omega_{t+1}} = \frac{\beta\left(1+n\right)\left(1+i_t\right)}{1-\rho}\frac{P_tY_t}{\Omega_t} - \left(1-\beta\right)n \tag{58}$$

We can first study the determinacy conditions for a pure interest rate peg, i.e. $i_t = i_0$. Inserting this into (58), we see that the condition for determinacy is:

$$\beta\left(1+n\right)\left(1+i_0\right) > 1-\rho \tag{59}$$

or $\theta > 1-\rho$. We may first note that this equation has an interpretation very similar to that of equation (23) that we saw before. Indeed it can be rewritten:

$$\frac{\left(1+n\right)\left(1+i_0\right)}{1-\rho} > \frac{1}{\beta} \tag{60}$$

Since nominal assets are growing at the rate $1-\rho$, the long run rate of inflation is $\left(1-\rho\right)/\left(1+n\right)$, so that the left hand side is the real rate of return on financial assets, and the rest of the intuition given in section 6.2 continues to hold.

We shall now see that the "expanded" condition (60) is actually sufficient for determinacy in all the non-Ricardian cases we have been considering in sections 7 and 8. Let us indeed loglinearize equation (58). We obtain:

$$p_{t+1} + y_{t+1} - \omega_{t+1} = \frac{\theta}{1-\rho}\left(p_t + y_t - \omega_t\right) + \frac{\gamma a}{1-\rho}\pi_t \tag{61}$$

where θ and γ are the same as in (28). We see that all the analysis we carried in the previous sections will be valid provided we replace p_t by $p_t - \omega_t$ and the parameters θ and γ by $\theta/\left(1-\rho\right)$ and $\gamma/\left(1-\rho\right)$ respectively.

Now we should note two things about condition (59). The first is that it shows most clearly the tradeoffs faced by the government on fiscal and monetary policy. Indeed a strict fiscal policy (low ρ) allows to lead a less rigorous monetary policy (low i_0), and the other way around. We may note that this is not the same tradeoff as the one found in the Ricardian framework (see for example Leeper, 1991).

The consideration of policies such as (55) also weakens the link between price determinacy and suboptimality. For example this introduces the possibility that the autarchic equilibrium is Pareto suboptimal, and prices nevertheless are not determinate (this will occur for negative enough ρ). This point was already made in Wallace (1980).

10 Conclusions

We have seen that going from a Ricardian to a non Ricardian framework changes dramatically the conditions for price determinacy under interest rate rules. It is usually found in a Ricardian framework that interest rate pegging leads to nominal indeterminacy, and that a more than one to one response of interest rates to inflation leads to price determinacy.

We found instead that a strong response of the interest rate rule to inflation is not necessary for price determinacy, which can be achieved even under an interest rate peg. We identified a sufficient condition for determinacy (conditions 23 or 59), which expresses that the real rate of return on nominal assets must be superior to the real rate of return that would prevail in the corresponding economy without financial assets. This condition ensures that agents will be actually willing to hold money and financial assets in the long run, obviously a critical condition if one wants money to have value, and prices to be determinate.

11 Appendix

We shall derive in this appendix the dynamic equation (10). Consider the household's budget equation (4). We assume that i_t is strictly positive, so the household will always want to satisfy the "cash in advance" equation exactly. We thus have $m_{jt} = P_t c_{jt}$ and the budget constraint is rewritten:

$$\omega_{jt+1} = (1 + i_t)\,\omega_{jt} + P_t y_t - P_t \tau_t - (1 + i_t)\,P_t c_{jt} \tag{62}$$

In what follows we shall repeatedly aggregate discounted values. It is convenient to compute in monetary terms, and we shall thus use the following interest factors:

$$R_t = \prod_{s=0}^{t-1} \frac{1}{1 + i_s} \qquad R_0 = 1 \tag{63}$$

Applying the interest factors (63) to the budget constraint (62), it becomes:

$$R_{s+1}\omega_{js+1} = R_s \omega_{js} + R_{s+1} P_s\,(y_s - \tau_s) - R_s P_s c_{js} \tag{64}$$

If we now aggregate all budget constraints (64) from time t to infinity, and assume that $R_s \omega_{js}$ goes to zero as s goes to infinity (this is the usual transversality condition), we obtain the intertemporal budget constraint of the household:

$$\sum_{s=t}^{\infty} R_s P_s c_{js} = R_t \omega_{jt} + \sum_{s=t}^{\infty} R_{s+1} P_s \left(y_s - \tau_s\right) \tag{65}$$

Now maximizing utility function (1) subject to the intertemporal budget constraint (65) yields the following consumption function for a household j:

$$R_t P_t c_{jt} = (1-\beta) \left[R_t \omega_{jt} + \sum_{s=t}^{\infty} R_{s+1} P_s \left(y_s - \tau_s\right) \right] \tag{66}$$

Summing this across the N_t agents alive in period t, we obtain the aggregate consumption C_t:

$$R_t P_t C_t = (1-\beta) \left[R_t \Omega_t + N_t \sum_{s=t}^{\infty} R_{s+1} P_s \left(y_s - \tau_s\right) \right] \tag{67}$$

In equilibrium we have $C_t = Y_t$, so the equilibrium equation is:

$$R_t P_t Y_t = (1-\beta) \left[R_t \Omega_t + N_t \sum_{s=t}^{\infty} R_{s+1} P_s \left(y_s - \tau_s\right) \right] \tag{68}$$

Let us divide both sides by N_t and use $Y_t = N_t y_t$:

$$R_t P_t y_t = (1-\beta) \left[\frac{R_t \Omega_t}{N_t} + \sum_{s=t}^{\infty} R_{s+1} P_s \left(y_s - \tau_s\right) \right] \tag{69}$$

Let us rewrite this equation for $t+1$ and subtract it from (69). We obtain:

$$R_t P_t y_t - R_{t+1} P_{t+1} y_{t+1} = (1-\beta) \left[\frac{R_t \Omega_t}{N_t} - \frac{R_{t+1} \Omega_{t+1}}{N_{t+1}} + R_{t+1} P_t \left(y_t - \tau_t\right) \right] \tag{70}$$

Now let us multiply the government's budget equation (9) by R_{t+1}/N_t:

$$\frac{R_t \Omega_t}{N_t} = \frac{R_{t+1} \Omega_{t+1}}{N_t} + R_{t+1} P_t \tau_t + \left(R_t - R_{t+1}\right) P_t y_t \tag{71}$$

Insert this into equation (70):

$$R_{t+1} P_{t+1} y_{t+1} = \beta R_t P_t y_t - (1-\beta) \left(\frac{1}{N_t} - \frac{1}{N_{t+1}} \right) R_{t+1} \Omega_{t+1} \tag{72}$$

and multiplying by N_{t+1}/R_{t+1}:

$$P_{t+1} Y_{t+1} = \beta \frac{N_{t+1}}{N_t} \left(1 + i_t\right) P_t Y_t - (1-\beta) \left(\frac{N_{t+1}}{N_t} - 1 \right) \Omega_{t+1} \tag{73}$$

Assuming finally $N_{t+1}/N_t = 1 + n$, we obtain:

$$P_{t+1}Y_{t+1} = \beta\,(1+n)\,(1+i_t)\,P_tY_t - (1-\beta)\,n\Omega_{t+1} \tag{74}$$

which is equation (10).

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