Endogenous Financial Development, Growth and Volatility*

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Abstract

The paper develops a model in which both long–run growth rates and credit market development are endogenous. Agents facing idiosyncratic productivity shocks cannot perfectly commit to repay their loans, but the threat of credit market exclusion specifies endogenous debt limits preventing default in equilibrium. A growth push makes credit market participation more valuable and relaxes debt limits, reinforcing thereby the initial growth effect. Moreover, a dynamic complementarity between debt limits gives rise to multiple balanced–growth paths. A high–growth equilibrium with developed credit markets can coexist with one or two low–growth equilibria with underdeveloped credit markets. Low–growth equilibria are more volatile as they are exposed to shocks to the wealth distribution and to sunspot shocks.

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1 Introduction

Empirical studies of economic growth report the following observations: (i) a positive link between economic growth and financial development (as measured, for instance, by the ratio of private credit to GDP); (ii) a negative link between growth and the variability of growth; (iii) multiple regimes in cross–country growth dynamics that point to the existence of a "twin–peaked" pattern of income distribution.[1] While it is undisputed that financial development and growth go hand in hand, their causal relationship is a much debated issue in the empirical literature.

The purpose of this paper is to develop a simple growth model in which both long–run growth rates and financial development are endogenous. Better developed credit markets make the allocation of resources more efficient, contributing thus to higher growth rates; conversely, a growth push makes credit markets more valuable, improves financial development and reinforces the initial growth effect. Thus, the model is consistent with fact (i), incorporating a two–sided linkage between finance and growth. The model is also consistent with (ii) and (iii): there exist multiple balanced–growth paths, where the high (low) growth path is associated with a developed (underdeveloped) credit market. Moreover, low-growth equilibria are more volatile since their output growth rates react sensitively to shocks to the wealth distribution and to sunspot shocks.

We consider a model of linear endogenous growth in which a continuum of infinitely–lived agents face idiosyncratic productivity shocks. Productive agents wish to borrow from unproductive agents, but borrowers are constrained in their demand for loans whenever the value of their collateral is insufficient. We assume that only a fraction of the producer's gross output serves as collateral, and we interpret this fraction as the economy's "property rights". In addition to collateral–backed loans, we follow Kehoe and Levine (1993) and Kocherlakota (1996) by assuming that credit enforcement is based on the defaulting agents' fear of exclusion from participation in credit markets. The resulting participation constraints specify endogenous debt

[1] On (i), see King and Levine (1993) and Levine (1997); on (ii), see Ramey and Ramey (1995); on On (iii), see Durlauf and Johnson (1995) and Durlauf and Quah (1999). Features (i) and (ii) have been confirmed both in cross-country and in panel–data regressions. Empirical evidence for a negative link between financial development and output volatility has been provided by Denizer, Iyigun, and Owen (2002).

limits on all borrowers that are just tight enough to deter default. Financial development, measured as the size of the credit market relative to output, becomes endogenous and responds to changes in the underlying fundamentals.

The paper has three main conclusions. First, when innovations push the technological frontier upwards, credit markets become more valuable to borrowers who want to make use of the leading technology. Thus they get punished more severely if they are excluded from borrowing which reduces their incentives to default. Hence debt limits relax, the volume of credit goes up, and funds are more efficiently allocated. Thus besides a *direct growth effect* of the technological innovation, there is an *indirect growth effect* that comes from improved financial development. Similarly, better access to the leading technology (e.g. because of better education) makes credit market participation more valuable, improves financial development and takes both a direct and an indirect impact on growth.

Second, when property rights are weak, the economy has multiple balanced growth paths. A *high–growth equilibrium* in which interest rates are high, debt constraints are loose, and unproductive agents are inactive may coexist with one or two *low–growth equilibria* in which interest rates are low, debt constraints are tight, and unproductive agents are active. Hence, there are "convergence clubs" with some economies growing fast and others showing a poor growth performance. Whereas part of this difference in growth performance in our model can be attributed to differences in property rights, the existence of multiple equilibria shows that there can be substantial variation in cross–country income growth *without differences in the economic fundamentals*. The reason why there are multiple equilibria is a *dynamic complementarity* in the endogenously determined debt limits. Agents' expectations of credit market conditions in the future affect their incentives to default, and this in turn takes an impact on their current debt limits. If future debt constraints are tight, agents value participation in credit markets only little and their incentives to default are high. Consequently, borrowing agents face currently tight debt constraints. But for an economy with the same fundamentals, there may also be an equilibrium in which agents expect loose credit limits in the future so that participation in credit markets is desirable; in turn, agents do not default easily and current debt limits are loose.[2]

[2]It is important to note that the sole cause of multiplicity is this dynamic complementarity

Third, economies with low growth and underdeveloped financial markets tend to have higher output volatility than economies with high growth and developed financial markets. The reason is twofold. On the one hand, low–growth economies with constrained borrowers respond sensitively to shocks to the wealth distribution. If wealth shifts from lenders to borrowers, the borrowers' collateral value improves, debt limits are relaxed and output growth goes up. Such distributional shocks take no impact in a high–growth economy with unconstrained borrowers. On the other hand, we find that some of the low–growth equilibria are indeterminate, so that there exists an infinity of stochastic (sunspot) solutions in a neighborhood of these equilibria. Indeterminacy, as multiplicity of equilibria, is triggered by the dynamic complementarity discussed above.[3]

There is a substantial literature on the role of credit market frictions for economic growth (e.g. Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), Marcet and Marimon (1992), Galor and Zeira (1993), Azariadis and Chakraborty (1999)). Like these papers, the model is compatible with the view that a higher level of financial activity spurs economic growth. In contrast to most of the existing literature, however, this paper assumes perfect information and there are no exogenous frictions in the process of financial intermediation. The only friction is the inability of borrowers to perfectly commit to repay their loans. Debt limits arise endogenously without explicit costs of financial intermediation. Similar to our paper, Acemoglu and Zilibotti (1997) develop a theory of financial development which improves endogenously in the growth process. Their model is based on the idea that, due to project–size indivisibilities, security markets in less developed countries may be incomplete, preventing investment in riskier profitable projects, whereas security markets improve as the economy reaches higher stages of development. In this paper, there are no such indivisibilities, and market incompleteness is not decisive for the results.[4]

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which is based on the exclusion–based credit enforcement mechanism. With purely collateral–based enforcement, the economy corresponds to the one analyzed by Kiyotaki (1998) which turns out to have a unique determinate balanced–growth path, as we show in Section 2 of this paper.

[3]Other features which are well–known to generate multiplicity and/or indeterminacy (cf. Behabib and Farmer (1999)) are all absent in this model; there are neither complementarities in preferences, nor are there complementarities or increasing returns in production.

[4]Market incompleteness is assumed only for convenience of the analysis since it allows to focus on a single credit market instead of a large number of markets for state–contingent claims. Because all

As in this paper, there are multiple steady states in the growth models of Galor and Zeira (1993), Acemoglu and Zilibotti (1997) and Azariadis and Chakraborty (1999). In the first of these papers, the initial distribution of wealth is the sole determinant of the long–run growth performance, whereas it is the initial stock of capital in the other two papers. In our model, in contrast, neither the initial distribution of wealth, nor the initial capital endowment need to play a decisive role for the long–run growth performance. Instead, financial development can be purely a matter of self–fulfilling prophecies: when financial markets are expected to work well in the future, credit markets will be in a better condition today since agents have stronger incentives not to default. Unlike previous literature, the model in this paper has also the potential to explain regime shifts from a financially developed state to a financially underdeveloped state ("financial crisis"), and vice versa ("financial deepening").

The paper is organized as follows. Section 2 presents the model and shows that it has a unique determinate balanced growth path if debt limits are solely based on collateral. Section 3 analyzes the economy with exclusion–based credit enforcement and discusses the potential equilibrium outcomes as well as the multiplicity of growth regimes. In Section 4, we show that results are robust to weaker enforcement mechanisms where agents cannot be excluded from future lending or where the exclusion period is short. Section 5 concludes. Proofs which are not included in the main text are contained in the Appendix.

2 The model

The model we consider is a variation of Kiyotaki's (1998) growth model that is augmented by exclusion–based debt constraints in the spirit of Kehoe and Levine (1993) and Alvarez and Jermann (2000). There is a continuum $i \in [0,1]$ of agents and a single homogenous consumption/investment good. Agents are infinitely–lived and their preferences are[5]

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \ln c^i_{t+\tau} \quad .$$

results are valid even with deterministic productivity shifts, market incompleteness is not decisive.

[5]Steady states of this model can also be derived analytically for the class of CES utility functions. However, since this paper aims to analyze non–stationary equilibria as well, the analysis concentrates on the Cobb–Douglas case in which the savings rate is the constant β.

At date $t = 0$, agent $i \in [0, 1]$ is endowed with a_0^i units of the good (initial wealth), and in every period $t \geq 0$ agent i has access to an individual production technology that transforms x_t^i units of investment into $y_{t+1}^i = A_t^i x_t^i$ units of output in period $t + 1$. The agent's productivity A_t^i fluctuates stochastically between a high level A (productive state) and a low level B (unproductive state). The shifts between productive and unproductive states follow a Markov process. Specifically, the transition probability from the unproductive to the productive state is π_p, and the transition probability from the productive to the unproductive state is π_u. Productivity shifts are independent across agents.

In each period, some agents are productive and have access to the "leading technology" with productivity A; others are less productive and can only make use of some "basic technology" with productivity B. Transition probabilities determine how likely it is for producers to catch up to the leading technologies or to fall behind the leading technology. In this interpretation, a "technological innovation" is understood as an (exogenous) increase of A relative to B, or equivalently as a fall in the ratio $z \equiv B/A \in (0, 1)$, whereas "improved skills" can be interpreted as an increase in π_p/π_u which is the ratio of productive to unproductive producers.

At each date t, there is a credit market in which one unit of the good at date t is exchanged for a claim to R_t units of the good at $t + 1$.[6] For notational convenience, we define $r_t \equiv R_t/A$ and refer to r_t as the interest rate in period t. Obviously, in equilibrium, $r_t \in [z, 1]$; if r_t was smaller than z, all agents want to borrow and no agent wants to lend; if r_t was bigger than 1, all agents want to lend and no agent wants to borrow. In both cases, the credit market cannot be in equilibrium. With b_t^i denoting the stock of agent i's debt at the end of period t, this agent's budget constraint reads as

$$c_t^i + x_t^i = y_t^i - R_{t-1} b_{t-1}^i + b_t^i \ . \tag{1}$$

In each period t the timing is as follows. (i) agents repay their loans $R_{t-1} b_{t-1}^i$ or not; (ii) the current productivity shocks A_t^i are realized; (iii) the credit market opens and agents decide about borrowing/lending and consumption/investment. Since the credit market opens *after* productivity is revealed, and since debt is redeemed

[6]The absence of a complete set of security markets simplifies the exposition, but is not decisive for the results: the deterministic economy $\pi_p = \pi_u = 1$ has complete markets but all results are the same.

before the next productivity shock is revealed, no uncertainty is resolved during debt contracts.

If agents can perfectly commit to repay their loans, unproductive agents would lend all their savings to productive agents at the interest rate $r_t = 1$ and the economy grows at the constant rate $A\beta$. However, agents cannot commit to repay when their collateral value is insufficient. We assume, as Kiyotaki (1998), that only a fraction $1-m$ of the producer's gross output serves as collateral so that a defaulting borrower can keep the fraction $m \in [0,1]$ of his net output. We interpret $1-m$ as the economy's "property rights" although there are certainly other, purely technological, reasons why capital is not fully alienable.[7] To allow for intertemporal trades that go beyond collateral–backed loans, we assume that a borrower who defaults on his loan will be excluded from participation in credit markets for the rest of his life, following Kehoe and Levine (1993). Thus, a credit agency keeps the name of a defaulting agent on a black list, and the agency is able to deny the defaulting agent any future loans and to seize all future asset holdings of this agent.[8] This enforcement mechanism specifies endogenous debt constraints on each agent and at each point in time.[9]

Consider now a productive agent at date t, and suppose the agent's savings ("equity") are e_t^i (i.e. $e_t^i = y_t^i - R_{t-1}b_{t-1}^i - c_t^i$). Whenever $r_t < 1$, the productive agent must be debt constrained in period t (otherwise he could invest an infinite amount and earn an infinite return in the next period). It is convenient to write these constraints in the form $b_t^i \leq \theta_t^i e_t^i$ where θ_t^i is the maximum debt–equity ratio imposed on agent i at date t. It turns out below (due to homotheticity of preferences) that the endogenously determined constraint θ_t^i is independent of the wealth of agent i. Thus all productive agents face the same constraint on their debt–equity ratio, $\theta_t^i = \theta_t$. Since $r_t < 1$, the debt constraint must be binding, and the productive agent invests $x_t^i = (1+\theta_t)e_t^i$ and ends up next period with wealth $a_{t+1}^i \equiv Ax_t^i - R_t b_t^i = R_t^+ e_t^i$

[7]For instance, investment may be irreversible or the usage of capital may require producer–specific skills so that it is impossible for other producers to make efficient use of installed capital.

[8]It is shown in Section 4 that the results can be extended to weaker enforcement mechanisms where agents can be excluded only from borrowing but not from lending, or where agents can be excluded for a short period only.

[9]We follow Alvarez and Jermann (2000) by considering a decentralized equilibrium with endogenously specified debt constraints. This formulation permits to derive a recursive equation in *debt constraints*, as opposed to a recursive identity in *promised utility levels* as is the case, for instance, in Krueger and Perri (2001).

where $R_t^+ \equiv A(1+\theta_t(1-r_t))$ is the return on equity in period t. Note that the return on equity exceeds the productivity of capital when $r_t < 1$ and $\theta_t > 0$ because of the leverage effect of debt. Because of logarithmic utility, all agents (productive and unproductive) save a fraction β of their wealth in each period.[10] Thus agent i consumes $c_t^i = (1-\beta)a_t^i$ and his wealth evolves according to $a_{t+1}^i = R_t \beta a_t^i$ if he is unproductive in t, and $a_{t+1}^i = R_t^+ \beta a_t^i$ otherwise.

Consider now how the debt constraints θ_t are determined. Let $V(a_{t+1}, (r^{t+1}, \theta^{t+1}))$ denote the expected utility of an agent who was productive in period t, enters period $t+1$ with wealth a_{t+1}, and expects future interest rates $r^{t+1} \equiv (r_{t+\tau})_{\tau=1}^{\infty}$ and debt constraints $\theta^{t+1} \equiv (\theta_{t+\tau})_{\tau=1}^{\infty}$. Further, let $\overline{V}(a_{t+1})$ denote expected utility of an agent who was productive in t, enters period $t+1$ with wealth a_{t+1} and is excluded from intertemporal trades for the rest of his life. That is, the excluded agent's return on savings is zA if he is unproductive (instead of $R_t \geq zA$) and A if he is productive (instead of $R_t^+ \geq A$). The excluded agent thus forgoes higher returns both as a borrower and as a lender.[11] Now consider a productive agent in period t with wealth a_t^i. If the agent does not default on his loan, he enters next period with wealth $R_t^+ \beta a_t^i$, but if he defaults, his wealth next period is $mA(1+\theta_t)\beta a_t^i$. The agent has no incentive to default iff

$$V(R_t^+ \beta a_t^i, (r^{t+1}, \theta^{t+1})) \geq \overline{V}(mA(1+\theta_t)\beta a_t^i) \ .$$

Homothetic utility implies that this inequality is scale invariant. That is, it does not depend on the agent's individual wealth a_t^i, and also the constant factors $A\beta$ cancel out on both sides. In particular, the same debt constraint θ_t can be imposed on all productive agents in period t. Further, the debt constraint should not be too tight in the sense that it should just enforce participation. Hence, the participation constraint is satisfied with equality whenever borrowers exhaust their debt limit.[12]

[10] In fact, the budget constraints (1) can be rewritten as $a_{t+1}^i = \tilde{R}_t^i(a_t^i - c_t^i)$ where $\tilde{R}_t^i = R_t$ if the agent is unproductive in t and $\tilde{R}_t^i = R_t^+$ otherwise. Since returns $\tilde{R}_t$ are known when the agent decides about consumption and saving in t, it follows from the Euler equations and the transversality constraint that the savings rate is a constant β.

[11] Because of the timing assumption, both V and $\overline{V}$ denote expected utility of the agent *before* the productivity shock in the current period $t+1$ is realized. Thus debt is served before the realization of the productivity shock. The alternative would however not alter the main conclusions.

[12] Borrowers must exhaust their debt limit whenever $r_t < 1$. If $r_t = 1$, the debt limit θ_t is still

Definition: θ_t prevents default and is not too tight if, and only if,

$$V(1+\theta_t(1-r_t),(r^{t+1},\theta^{t+1})) = \overline{V}(m(1+\theta_t)) \ . \tag{2}$$

Consider next the evolution of wealth in this economy. Let W_t denote aggregate wealth (=output) in period t and let W_t^p denote aggregate wealth of productive agents. Then the assumptions on productivity shifts and the law of large numbers[13] yield

$$\begin{aligned} W_{t+1} &= \beta R_t^+ W_t^p + \beta R_t (W_t - W_t^p) \ , \\ W_{t+1}^p &= (1-\pi_u)\beta R_t^+ W_t^p + \pi_p \beta R_t (W_t - W_t^p) \ . \end{aligned}$$

From these two identities one obtains the evolution of the wealth share of productive agents, $\alpha_t \equiv W_t^p / W_t$;

$$\alpha_{t+1} = \frac{(1-\pi_u)(1+\theta_t(1-r_t))\alpha_t + \pi_p r_t (1-\alpha_t)}{(1+\theta_t(1-r_t))\alpha_t + r_t(1-\alpha_t)} \ . \tag{3}$$

Finally, consider the equilibrium in the credit market. Let ϑ_t denote the *actual* debt/equity ratio of productive agents in period t. Thus, $\vartheta_t \leq \theta_t$ if productive agents are unconstrained ($r_t = 1$) and $\vartheta_t = \theta_t$ otherwise ($r_t < 1$). The credit market clears if loans of productive agents, $\beta W_t^p \vartheta_t$, do not exceed savings of unproductive agents, $\beta(W_t - W_t^p)$. Moreover, unproductive agents do not invest themselves if $r_t > z$ in which case their savings must equal the productive agents' demand for loans. Therefore,

$$\beta W_t^p \vartheta_t \leq \beta (W_t - W_t^p) \text{ with equality if } r_t > z \ , \text{ and}$$

$$\vartheta_t \leq \theta_t \text{ with equality if } r_t < 1 \ .$$

Hence, the credit market is in equilibrium if

$$\begin{aligned} \text{either,} \quad \theta_t &\geq \frac{1-\alpha_t}{\alpha_t} \quad \text{and} \quad r_t = 1 \ , \\ \text{or,} \quad \theta_t &= \frac{1-\alpha_t}{\alpha_t} \quad \text{and} \quad r_t \in (z,1) \ , \\ \text{or,} \quad \theta_t &\leq \frac{1-\alpha_t}{\alpha_t} \quad \text{and} \quad r_t = z \ . \end{aligned} \tag{4}$$

defined by equation (2), but the actual participation constraint may be slack since borrowers may not exhaust their debt limit $b_t/e_t \leq \theta_t$.

[13] As Judd (1985) proved, there is no law of large numbers for a continuum of independent random variables. Alós-Ferrer (2002) showed, however, that a law of large numbers can be reestablished if the independence assumption is relaxed.

Now we are ready to define a perfect–foresight equilibrium with limited commitment.

Definition: An equilibrium with limited commitment is a list of interest rates, debt constraints, and wealth shares of productive agents, $(r_t, \theta_t, \alpha_t)_{t=0}^{\infty}$, where $\alpha_0 \in [0, 1]$ is given, such that for all $t \geq 0$

(i) debt constraints prevent default and are not too tight, i.e. (2) is satisfied,

(ii) wealth evolves according to (3),

(iii) the credit market is in equilibrium, i.e. (4) holds.

The output growth rate in period $t + 1$ is

$$W_{t+1}/W_t = \beta A\Big((1 + \theta_t(1 - r_t))\alpha_t + r_t(1 - \alpha_t)\Big) \ . \tag{5}$$

When $r_t = 1$, productive agents are unconstrained and output growth is βA. We label such a situation an *unconstrained* equilibrium. When $r_t \in (z, 1)$, productive agents are constrained, but still all funds are invested at productive agents, so that output growth is again βA. The economy is production efficient but not consumption efficient since individual consumption is not perfectly correlated with aggregate consumption. We label this situation a *constrained and production efficient* equilibrium. Finally, when $r_t = z$, unproductive agents are active as well and output growth falls short of the efficient growth rate βA. The economy is *constrained and production inefficient*.

A key feature of the specification of debt constraints via the threat of market exclusion is that future expectations of credit market conditions (r^{t+1}, θ^{t+1}) determine the incentives to default and thereby the current debt limit θ_t. In particular, there is a dynamic complementarity between future and current debt constraints: when future debt constraints are tight, exclusion from credit markets is only a weak punishment and current debt constraints must be tight as well to enforce agents not to default. As will be seen below, this complementarity gives rise to multiple, and possibly indeterminate balanced–growth paths.

It is valuable to point out that such complementarities and multiple equilibria are absent in the same economy when debt constraints are *solely* based on collateral as in Kiyotaki (1998). In this case, the agent has no incentive to default whenever his debt repayment does not exceed the value of his collateral, i.e.

$$R_t \theta_t \beta a_t^i \leq (1-m) A (1+\theta_t) \beta a_t^i \ .$$

This participation constraint is enforced by the following debt constraint which is "not too tight":

$$\theta_t = \frac{1-m}{r_t + m - 1} \ . \qquad (6)$$

An equilibrium in which debt constraints are only based on collateral is described again by the above definition where (2) is replaced by (6). In particular, there is no impact of future credit market conditions on current debt limits, and the equilibrium turns out to be unique. Indeed, the following result is straightforward (a proof is available from the authors on request).

Proposition 1: When debt constraints are only based on collateral (condition (6) instead of (2)), the equilibrium is unique for any distribution of initial wealth α_0 and converges to a unique steady state equilibrium. The steady state equilibrium is

(a) unconstrained if $m \leq \frac{\pi_p}{\pi_u + \pi_p}$,

(b) constrained and production efficient if $m \in \left(\frac{\pi_p}{\pi_u + \pi_p}, \frac{1 + z(\pi_p - 1)}{1 + z(\pi_p + \pi_u - 1)} \right]$,

(c) constrained and production inefficient if $m > \frac{1 + z(\pi_p - 1)}{1 + z(\pi_p + \pi_u - 1)}$.

Clearly, the long–run equilibrium in this economy depends crucially on the value of collateral. When the collateral value is low enough (m is high), the balanced–growth path of this economy is production inefficient and shocks affecting the wealth distribution will permanently (and potentially persistently) affect growth rates.[14] Such shocks have no effect in an unconstrained equilibrium. This conclusion turns out

[14]Kiyotaki (1998) assumes besides a low enough collateral value also that transition probabilities are low ($\pi_u + \pi_p < 1$). Under this assumption, convergence to the constrained and production–inefficient steady state equilibrium is monotonic: a temporary productivity shock generates a persistent effect on output growth since idiosyncratic productivity shocks are persistent. Under the alternative assumption $\pi_u + \pi_p > 1$, there are trend–reverting oscillations.

to be similar when debt limits are not only based on collateral, but are determined from the borrowers' fear of market exclusion. However, equilibrium need not be unique anymore so that there can be *substantial variations in output growth without underlying differences in fundamentals such as property rights*. Furthermore, low–growth equilibria can be indeterminate; hence sunspot shocks contribute to making such equilibria more volatile.

3 Growth regimes

We consider now the economy in which debt limits are determined by the participation constraint (2). Characterizing the global set of dynamic equilibria is difficult, so we analyze first separately the two situations in which agents are either unconstrained in all periods or in which agents are constrained in all periods. Equilibria switching between these two regimes are discussed at the end of this section.

Unconstrained equilibrium

Consider first a situation where borrowers are unconstrained for all periods $t \geq 0$. When $r_t = 1$, (3) simplifies to

$$\alpha_{t+1} = \pi_p + \alpha_t(1 - \pi_u - \pi_p) \ . \tag{7}$$

Unless $\pi_u = \pi_p = 1$, the wealth share of productive agents converges to the unique stationary value $\alpha^* = \pi_p/(\pi_u + \pi_p)$. If an agent enters period $t+1$ with wealth $a = 1$ and expects not to be constrained in the future, his consumption path is $c_{t+1+\tau} = (1-\beta)(A\beta)^\tau$, $\tau \geq 0$. Hence his expected utility is

$$V(1, (r^{t+1}, \theta^{t+1})) = \sum_{\tau=0}^{\infty} \beta^\tau \ln((1-\beta)(A\beta)^\tau) = \frac{\ln(1-\beta)}{1-\beta} + \frac{\beta \ln(\beta A)}{(1-\beta)^2} \ .$$

On the other hand, a simple recursive calculation (see Lemma 1 in the Appendix) shows that the utility of a defaulting agent is

$$\overline{V}(a) = \frac{\ln a}{1-\beta} + \overline{V} \ ,$$

where the constant $\overline{V}$ is defined in Lemma 1 and independent of wealth a. Thus, from (2), debt constraints are constant, $\theta_t = \theta^*$, and satisfy

$$\frac{\ln(1-\beta)}{1-\beta} + \frac{\beta \ln(\beta A)}{(1-\beta)^2} = \frac{\ln(m(1+\theta^*))}{1-\beta} + \overline{V} \ .$$

From (4), the wealth share of productive agents must satisfy $\alpha_t \geq 1/(1+\theta^*)$. Using the definition of $\overline{V}$ this becomes

$$\ln \alpha_t \geq \ln z \cdot \frac{\beta \pi_u}{(1-\beta)(1-\beta(1-\pi_u-\pi_p))} + \ln m \; . \tag{8}$$

Hence, whenever (8) is satisfied along the sequence α_t defined by (7), there is an equilibrium with limited commitment in which debt constraints are $\theta_t = \theta^*$ and productive agents are unconstrained for all $t \geq 0$. Since α_t converges to the stationary value $\pi_p/(\pi_u + \pi_p)$, one can reformulate this result as follows.

Proposition 2: There is an unconstrained equilibrium for an appropriate initial distribution of wealth α_0 if, and only if,

$$z \leq z_0 \equiv \Big(\frac{1}{m}\frac{\pi_p}{\pi_p+\pi_u}\Big)^{\frac{(1-\beta)(1-\beta(1-\pi_u-\pi_p))}{\beta\pi_u}} \; . \tag{9}$$

The equilibrium converges to the steady state equilibrium $(r^* = 1, \theta^*, \alpha^* = \frac{\pi_p}{\pi_p+\pi_u})$, and the steady state equilibrium is locally determinate.[15]

Condition (9) states circumstances that are favorable for an unconstrained balanced–growth path. On the one hand, as in Proposition 1, strong enough property rights establish a first–best unconstrained equilibrium. On the other hand, a larger productivity differential (lower z), a larger share of productive agents (higher π_p/π_u), and more patient agents (since z_0 is decreasing in β) are beneficial for financial development. Intuitively, if productivity differences are larger, if agents have better access to the leading technology, or if they are more patient, they benefit more from future credit market participation. Thus, the threat of market exclusion is strong enough to prevent large debtors from default; there are no credit market frictions and output growth is at the highest possible level.

Binding constraints

Consider now the more interesting situation of binding debt constraints. Lemma 2 in the Appendix shows that participation constraints (2) can be expressed by the

[15]To show local determinacy of the equilibrium, one must preclude equilibria $(r_t, \theta_t, \alpha_t)$ with $r_t < 1$ that converge to $(1, \theta^*, \alpha^*)$. The proof of Proposition 4 below will show that such equilibria cannot exist: interest rates close to one must be decreasing over time provided that $z \leq z_0$.

following *recursive participation constraints* that have to be satisfied for all $t \geq 0$:

$$\ln(m(1+\theta_t)) - \ln(1+\theta_t(1-r_t)) = \beta\pi_u(\ln r_{t+1} - \ln z) + \beta(2-\pi_p-\pi_u)\ln(m(1+\theta_{t+1}))$$
$$-\beta(1-\pi_p)\ln(1+\theta_{t+1}(1-r_{t+1})) + \beta^2(\pi_p+\pi_u-1)\ln(m(1+\theta_{t+2})) \ . \qquad (10)$$

This equation balances the gains from default in period $t+1$ (the left–hand side) to the losses from exclusion from credit markets (the right–hand side). This interpretation becomes transparent in the special case of deterministic productivity shifts where shifts between productive and unproductive states follow a two–periodic cycle ($\pi_u = \pi_p = 1$). In this case the loss from market exclusion consists of only two terms: the loss from exclusion from savings in period $t+1$ (when the agent is unproductive) which is $\beta(\ln r_{t+1} - \ln z)$. The second term is the loss from exclusion from borrowing in period $t+2$ when the agent is productive again: the excluded agent forgoes the possibility of default in period $t+3$ which would pay off $\beta^2 \ln(m(1+\theta_{t+2}))$.

Another simplification of (10) obtains in the special case of i.i.d. productivity shifts where $\pi_p = 1 - \pi_u \equiv \pi$ is the probability of high productivity for each agent and at each point in time. In this case (10) is rewritten as

$$\ln(m(1+\theta_t)) - \ln(1+\theta_t(1-r_t)) = \beta(1-\pi)\Big[\ln r_{t+1} - \ln z\Big] + \beta\pi\Big[\ln(1+\theta_{t+1}(1-r_{t+1})) - \ln(1)\Big]$$
$$+\beta\Big[\ln(m(1+\theta_{t+1})) - \ln(1+\theta_{t+1}(1-r_{t+1}))\Big] \ . \qquad (11)$$

The loss from market exclusion now consists of three terms: the loss from exclusion from saving in period $t+1$ (with probability $(1-\pi)$), the loss from exclusion from borrowing in period $t+1$ (with probability π), and the discounted losses from all future market exclusions after period $t+2$ which must be equal to the gains from default in period $t+2$. Equation (11) also highlights the *dynamic complementarity* that is immanent in this model and that is the reason for the multiplicity of equilibria: both the left–hand side and the right–hand side are increasing functions of θ. When the debt limit in period $t+1$ is higher, defaulters' punishment is stronger and this in turn raises the debt limit in period t.

Consider first equilibria which are *production inefficient* in all periods; thus the interest rate is $r_t = z$ and unproductive agents are active. Equation (10) becomes now an implicit equation in $(\theta_t, \theta_{t+1}, \theta_{t+2})$. A steady state solution of this equation must satisfy

$$m(1+\theta) = (1+\theta(1-z))^{\phi} \ , \qquad (12)$$

where

$$\phi \equiv \frac{1-\beta(1-\pi_p)}{(1-\beta)(1-\beta(1-\pi_p-\pi_u))} > 1 \ .$$

Clearly, when there are no property rights ($m = 1$), there is a steady state equilibrium where the credit market shuts down ($\theta = 0$). In this autarky equilibrium, defaulters expect no penalty because collateral value is zero and market participation does not benefit them; consequently, debt limits must be zero. This extreme form of market failure obtains also in pure–exchange economies with endogenous debt constraints (e.g. Alvarez and Jermann (2000), Azariadis and Lambertini (2003), Azariadis and Kaas (2002)).[16] However, autarky is not always robust to values of m below unity. In fact when $m < 1$, the left–hand side of (12) shifts down and there fails to be steady state equilibrium near autarky if the right–hand side of (12) has a slope greater than or equal to one at $\theta = 0$ which is the case if $\phi(1-z) \geq 1$. On the other hand, if $\phi(1-z) < 1$, there exists a steady state equilibrium $\overline{\theta}_1 > 0$ when m is close to unity which collapses to the autarky equilibrium for $m = 1$. Thus, a steady state equilibrium near autarky exists if, and only if,

$$z > z_1 \equiv \frac{\beta(\pi_p + (1-\beta)(1-\pi_p-\pi_u))}{1-\beta(1-\pi_p)} \ .$$

When this inequality is satisfied and m is not too small, there is also a second solution to (12) with a larger volume of credit, $\overline{\theta}_2 > \overline{\theta}_1$. For this solution to be an equilibrium with limited commitment, one has to make sure that the credit market clears: the funds of unproductive agents must be large enough to cover the demand for loans of productive agents.[17] As the following proposition shows, this is the case whenever $z < z_2$ where $z_2 \in (z_1, 1)$. Hence, for an open set of parameters, there exists a second constrained and production inefficient equilibrium.

Proposition 3: Suppose that $z > z_1$ and that m is not too small (i.e. the productivity differential is not too big and property rights are not too strong). Then there exists a constrained and production inefficient steady state equilibrium ($\overline{r}_1 = z, \overline{\theta}_1, \overline{\alpha}_1$)

[16] In the pure–exchange economies of Alvarez and Jermann (2000) and Azariadis and Kaas (2002), autarky fails to be an equilibrium if agents can trade a productive asset. Equilibrium is unique and determinate under fairly general assumptions.

[17] This requirement is always fulfilled at the smaller equilibrium because the volume of credit becomes arbitrarily small when m is close to one.

which collapses to autarky when $m = 1$. Furthermore, there exists $z_2 > z_1$ such that for all $z \in (z_1, z_2)$ there is another steady state equilibrium $(\overline{r}_2 = z, \overline{\theta}_2, \overline{\alpha}_2)$ that is constrained and production inefficient. The equilibrium at $\overline{\theta}_1$ is determinate, and the equilibrium at $\overline{\theta}_2$ is indeterminate.

Proof: Appendix.

Finally, consider equilibria that are *constrained and production efficient.* These equilibria "connect" the equilibrium sets of Proposition 2 and Proposition 3. Indeed, it turns out that a small increase of z above z_0 leads to a constrained equilibrium that is close to the unconstrained equilibrium of Proposition 2 in the sense that r is close to 1 and θ is close to θ^*. This equilibrium turns out to be determinate. On the other hand, a small increase of z above z_2 leads to an indeterminate constrained equilibrium with efficient production that is close to the indeterminate equilibrium $\overline{\theta}_2$ of Proposition 3. These two equilibria collapse when $z = z_3$ where z_3 is larger than both z_0 and z_2.

Proposition 4: Suppose that m is not too small. Then there exists $1 > z_3 > \max(z_0, z_2)$ such that

(a) If $z \in (\min(z_0, z_2), \max(z_0, z_2)]$ or $z = z_3$, there is a unique steady state equilibrium (r, θ, α) that is constrained and production efficient. If $z_0 < z_2$, this equilibrium is locally determinate, otherwise it is indeterminate.

(b) If $z \in (\max(z_0, z_2), z_3)$, there are two steady state equilibria that are constrained and production efficient. The equilibrium with the lower interest rate is indeterminate.

Proof: Appendix.

Propositions 2, 3 and 4 characterize all steady state equilibria (balanced–growth paths) with limited commitment, to be summarized as follows.

Theorem: Suppose that property rights are not too strong, i.e. m is sufficiently large. Then there exist $z_i \in (0, 1)$, $i = 0, 1, 2, 3$, with $z_1 < z_2 < z_3$ such that,

(a) If $z \leq z_1$, the unique steady state equilibrium is a determinate equilibrium with efficient production.

(b) If $z_1 < z < z_3$, there are three steady state equilibria: (i) a determinate equilibrium with inefficient production near autarky, (ii) a determinate equilibrium with efficient production, (iii) an indeterminate constrained equilibrium that has inefficient production if $z < z_2$ and efficient production otherwise.

(c) If $z \geq z_3$, the unique steady state equilibrium is a determinate constrained equilibrium with inefficient production near autarky.

Moreover, the determinate equilibrium with efficient production in (a) and (b) is unconstrained if $z \leq z_0$, and constrained otherwise.

Figure 1 shows how steady state interest rates depend on z which is a measure of the size of the economy's productivity differential. When the productivity differential is large (z is small) agents have a strong desire not to be excluded from credit markets. They do not default easily and the unique equilibrium is an unconstrained equilibrium. When productivity differences are small (z is close to 1), agents do not benefit much from credit markets and the threat of market exclusion is not strong enough to establish an equilibrium with positive loans: an equilibrium near autarky is the outcome. For the intermediate range $z \in (z_1, z_3)$, however, multiple equilibria emerge: an equilibrium with efficient production and high growth coexists with one or two equilibria with inefficient production and low growth.

Note that one cannot generally conclude whether z_0 is smaller or bigger than z_1 or z_2, respectively. To give a numerical example, suppose that productivity shifts are i.i.d. with probability of high productivity equal to $\pi = 0.1$ and that $m = 1$. When $\beta = 0.5$, one has $z_0 \approx 0.077 < z_1 \approx 0.091 < z_2 \approx 0.210 < z_3 \approx 0.311$. When $\beta = 0.9$, however, one has $z_1 \approx 0.474 < z_2 \approx 0.738 < z_0 \approx 0.753 < z_3 \approx 0.787$. In both cases, the range of values for z for which there are multiple equilibria is quite large.

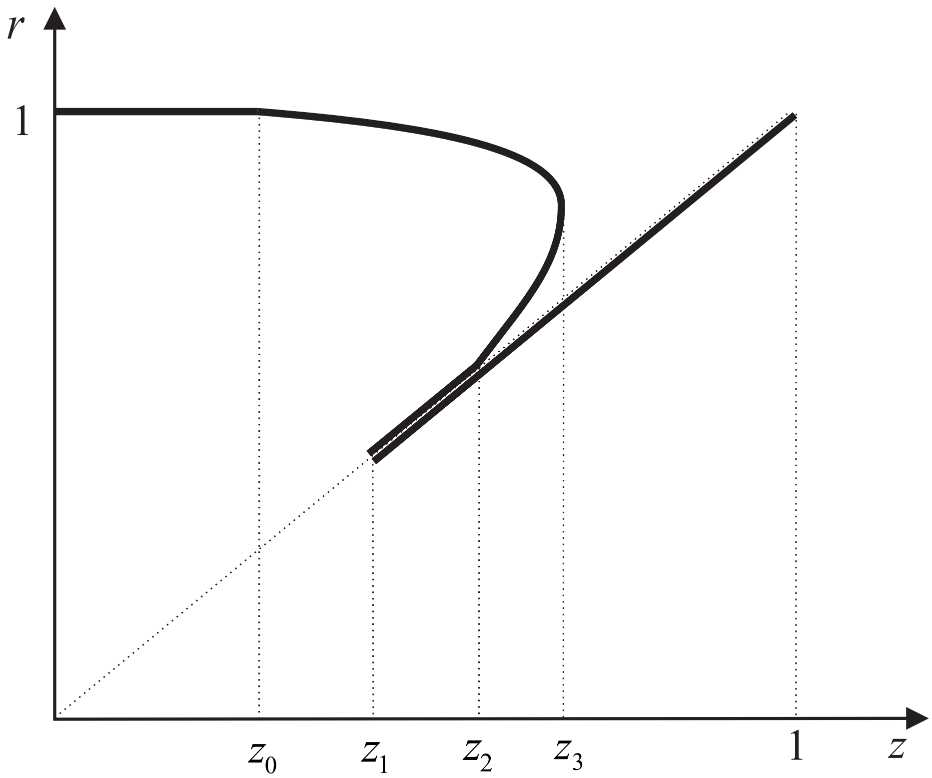

Figure 1: The bold curves show robust steady state equilibrium interest rates as z increases from 0 to 1.

4 Implications

What are the implications of the above results for the relation between financial development and long–run growth rates? First of all, the theorem states that lower values of z lead, ceteris paribus, to less frictions in the credit market. It is plausible to assume that a technological innovation raises the "leading technology" parameter A by more than the "basic technology" parameter B. Thus the parameter $z = B/A$ falls in response to an innovation shift which may in turn lead to a jump from an equilibrium with inefficient production to one with efficient production. In an economy with full commitment, the technological innovation raises the growth rate merely to the higher level of βA. In an economy with limited commitment, however there is besides this direct effect also an indirect effect of the innovation: improved credit market development makes the allocation of funds more efficient and pushes growth further upwards. Better financial development reinforces the initial growth effect.

Similarly, if skills in the economy improve (e.g. because of better education), the share of agents who are able to employ the best technology increases, and this in turn takes a positive impact on financial development. Agents are in a greater need of the credit market when they are more productive, and this relaxes debt limits. Formally, z_1 is increasing in π_p and decreasing in π_u, and numeric experiments reveal that the effect on the other critical bound z_3 are similar; hence it is more likely that the economy ends up in an equilibrium with efficient production. Again, there is both a direct and an indirect effect on growth. The direct effect is evident from (3) and (5): when π_p increases and π_u falls, the wealth share of productive agents in the steady state, $\bar{\alpha}$, increases and so does the growth rate (if the economy is in a regime of inefficient production). The indirect effect takes place because z_1 and z_3 increase, and when z falls below any of these threshold levels, the economy may jump from an equilibrium with inefficient production to one with efficient production.

Obviously, because improved property rights help to relax debt constraints, equilibria with inefficient production cease to exist when m falls below a certain threshold level, as can be seen formally from (12). This effect is essentially the same as in the economy where debt limits are only based on collateral (Proposition 1). However, as we argued above, property rights are only part of the reason why the producers' net output is not fully alienable; there may be technological limitations as well.

There are other factors that explain why the level of financial development may differ between economies. Obviously, a more patient economy not only saves more, but it also has better developed financial markets since agents care more about future market participation; default becomes a minor problem. Again, there is a direct and an indirect effect of an increase in the savings rate on growth. Further, more persistent productivity shocks are favorable for financial development.[18]

There are two reasons why low–growth equilibria are more volatile than high–growth equilibria. On the one hand, sunspots may induce output fluctuations near an

[18] It can be shown that, if $\pi_u = \pi_p = \pi$, then z_1 is increasing in $1 - \pi$ which is a measure of persistence. Numeric experiments show that the effect on the other bound z_3 are similar.

indeterminate equilibrium with inefficient production.[19] Such sunspot fluctuations cannot affect output when the indeterminate equilibrium has efficient production (i.e. if $z_2 < z < z_3$). In this case, sunspots only affect debt limits and interest rates, but the growth rate stays constant at $A\beta$. In the other case ($z_1 < z < z_2$), however, sunspots induce fluctuations in growth rates which now depend explicitly on debt limits. On the other hand, output growth in a production inefficient equilibrium reacts sensitively to shocks to the wealth distribution and to the distribution of productivities across agents. For instance, a shock to transition probabilities or to the wealth distribution changes α_t and thus, via (5), the output growth rate in an equilibrium with inefficient production. On the other hand, such shocks do not alter the growth rate in any equilibrium with efficient production – in these equilibria growth fluctuates only because of shocks to the productivity frontier (A) or to saving rates (β).

We conclude this section by a number of remarks. First, there are decentralized equilibria that are Pareto–dominated by other equilibria when $z_1 < z < z_3$; hence the first welfare theorem does not hold. To see this, suppose that productivity shocks are i.i.d. and consider a given steady state equilibrium. Then the expected utility of agent i in period $t = 0$ and before the realization of the first productivity shock is $V_0^i = \ln(a_0^i)/(1-\beta) + V_0$. The constant V_0 is the same for all agents (because productivity shocks are i.i.d.) and it depends only on the particular equilibrium under consideration (see the proof of Lemma 1 and Lemma 2). Since this constant is smallest at the equilibrium near autarky, and highest at the first–best equilibrium, steady–state equilibria can be Pareto–ranked.

Second, equilibria need not be dynamically efficient, in the sense that the interest rate is bigger than the growth rate so that the present value of future output tends to zero as time goes to infinity. Indeed, in the case of i.i.d. productivity shocks it turns out that the extreme equilibria (the ones in (a) and (c) of the Theorem) are dynamically efficient, but the middle equilibrium in case (b) is dynamically inefficient: the interest rate is smaller than the growth rate, and the present value of future output and consumption is unbounded. Note, however, that the agents still

[19] The existence of sunspot equilibria follows from standard results since this model, using the recursive participation constraint (10), falls into the temporary equilibrium framework of Chiappori and Guesnerie (1991).

satisfy their intertemporal budget constraint, because the return on equity exceeds the return on loans so that constrained borrowers discount at a higher rate.

Third, since there is a large set of intertemporal equilibria, one might expect that there are not only equilibria converging to the indeterminate steady state, but that there may also be equilibrium sequences that fluctuate *permanently* between the different regimes. To analyze the global dynamics in full generality is difficult since there is one predetermined variable (α_t) and two jump variables (r_t and θ_t). In the special case of i.i.d. productivity shifts, however, the predetermined variable is a constant, $\alpha_t = \pi$, and the dynamics of (r_t, θ_t) follows equation (11) which has only one lag. Together with the market–clearing conditions, the equilibrium dynamics is only one–dimensional. It turns out that this dynamics must be monotonic and cannot produce deterministic cycles. Thus, when $z \in (z_1, z_3)$, all deterministic equilibria converge to the indeterminate equilibrium.

Proposition 5: Suppose that productivity shocks are i.i.d. and that there is a non–stationary equilibrium with limited commitment $(r_t, \theta_t, \alpha_t = \pi)$. Then this equilibrium must converge to the indeterminate steady state equilibrium of the Theorem.

Proof: Appendix.

Despite the non–existence of deterministic equilibrium cycles, the multiplicity of equilibria still implies that *small* shocks to economic fundamentals may have *large* effects on the volume of credit and on economic growth. For instance, when z falls below z_3 from a value above z_3, an economy with a poor credit market may experience a *financial deepening* as the economy moves from a unique determinate equilibrium near autarky with low credit volume towards an indeterminate equilibrium with efficient production and a high credit volume. Conversely, when z increases above z_1 from a value below z_1, the economy may experience a *financial crisis* as the economy moves from a unique determinate equilibrium with efficient production towards an indeterminate equilibrium with inefficient production and tight constraints.

5 Alternative enforcement

In this section we explore the question whether the multiplicity of balanced–growth paths is an artifact of the assumptions that defaulters can be excluded from both borrowing *and lending* and that exclusion lasts *infinitely long*. We show, however, that this is not the case; the basic conclusions remain valid when (i) defaulters can only be excluded from borrowing but they can still save at the market rate r_t and (ii) defaulters are only excluded for one period.

For the ease of exposition, assume that productivity shifts are i.i.d. so that the wealth share of productive agents is constant, $\alpha_t = \pi$, where π is the probability of high productivity for each agent and at each point in time. We also abstract from property rights by setting $m = 1$.

5.1 No exclusion from lending

Until now, it was assumed that the assets of defaulters can be seized easily so that they cannot earn the market rate of return r_t but must earn their autarky return z when they are unproductive. This is reflected by the expression $(\ln r_{t+1} - \ln z)$ in the recursive participation constraint (11) which stands for the loss from exclusion from saving in period $t+1$. One might argue that defaulting agents should be able to find alternative means of saving that cannot be seized by creditors; it is relatively easy to hold assets anonymously, but rather difficult to obtain anonymous loans.

In an international economics context, Bulow and Rogoff (1989) argued that the threat of denial of future credit is not sufficient to establish an international loan market. In the environment of this paper, in contrast, the loan market does not shut down when denial of credit is the only punishment. Although, without collateral, there cannot be an equilibrium with unconstrained agents, there are still equilibria with positive loans between productive and unproductive agents, and for a certain range of parameters there are again multiple and indeterminate equilibria.

When agents can be excluded from borrowing but not from saving, the term $(\ln r_{t+1} - \ln z)$ disappears from the recursive participation constraint (11) so that one has

$$\ln(1+\theta_t) - \ln(1+\theta_t(1-r_t)) = \beta\pi\Big[\ln(1+\theta_{t+1}(1-r_{t+1})) - \ln(1)\Big]$$

$$+\beta\Big[\ln(1+\theta_{t+1})-\ln(1+\theta_{t+1}(1-r_{t+1}))\Big]\ . \tag{13}$$

Now the only punishment of defaulters is the exclusion from borrowing in period $t+1$ (which occurs with probability π) and the discounted loss from all future borrowing exclusions.

The first observation is that there cannot be an *unconstrained equilibrium.* When $r_t = 1$ there is no leverage effect so that productive agents do not gain from access to the credit market. They receive the same return on their equity no matter whether they are excluded from borrowing or not. Therefore exclusion from borrowing is no punishment and, given that there are no property rights, no creditor would grant a loan to a productive agent. Formally, the only solution of equation (13) with $r_t = r_{t+1} = 1$ is $\theta = 0$ which cannot be an equilibrium.[20]

A second observation is that *constrained and production inefficient equilibria* are the same as before. When $r_t = z$, unproductive agents do not gain from access to savings so that it makes no difference whether they are excluded from savings or not. Formally, equation (13) is the same as equation (11) when $r_t = r_{t+1} = z$. Hence, Proposition 3 remains unchanged when there is no exclusion from lending.

Lastly, consider *constrained and production efficient* equilibria. The only change is that the term $(\ln r_{t+1} - \ln z)$ disappears from equation (21) in the proof of Proposition 4. The steady state equilibrium interest rate is thus independent of z and can be shown to be equal to z_2. Hence, there exists a unique constrained and production efficient steady state equilibrium with $r = z_2$ provided that $z \leq z_2$. Moreover, it can be checked easily that $(dr_{t+1})/(dr_t)|_{r=z_2} > 1$ so that this equilibrium is determinate.

These results are summarized in Proposition 6 and Figure 2.[21]

Proposition 6: Suppose that defaulting agents are excluded from borrowing forever but not from lending. Then,

[20]Explosive solutions satisfying $\ln(1+\theta_t) = \ln(1+\theta_0)\beta^{-t}$ cannot be an equilibrium either since they violate the transversality condition of Lemma 2.

[21]The results (and Figure 2) generalize to Markovian productivity shifts that are not i.i.d. A variation of the proof of Proposition 4 shows that $\ln z$ cancels out in the modified equation (25) so that the steady state debt limit θ is independent of z. By (23), the steady state interest rate does not depend on z either and it must be equal to z_2.

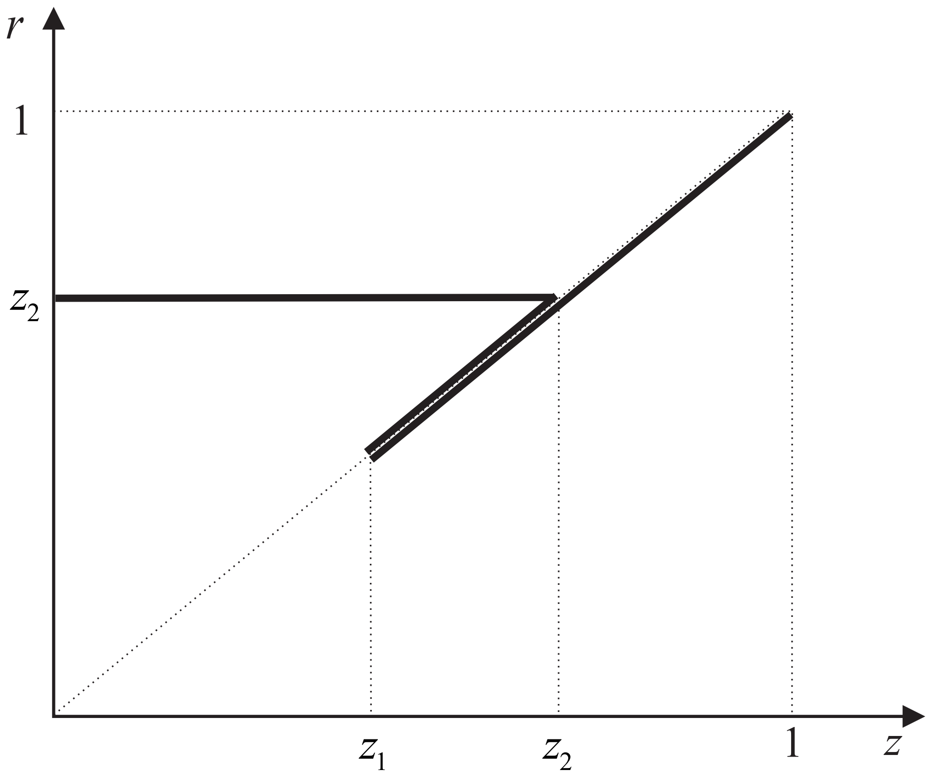

Figure 2: Robust steady state equilibria without exclusion from lending.

(a) If $z \leq z_1$, the unique steady state equilibrium is a determinate constrained and production efficient equilibrium with $r = z_2$.

(b) If $z_1 < z < z_2$, there are three steady state equilibria: (i) a determinate autarky equilibrium, (ii) a determinate constrained and production efficient equilibrium, (iii) an indeterminate constrained and production inefficient equilibrium.

(c) If $z \geq z_2$, the unique steady state equilibrium is a determinate autarky equilibrium.

5.2 Short exclusion length

Do the results depend on the assumption that credit market exclusion lasts forever? The answer is again negative. When agents can be excluded for only one period, the last term in equation (11) which stands for the losses from exclusion after period $t+1$ disappears so that one obtains

$$\ln(1+\theta_t)-\ln(1+\theta_t(1-r_t)) = \beta(1-\pi)\Big[\ln r_{t+1}-\ln z\Big]+\beta\pi\ln(1+\theta_{t+1}(1-r_{t+1}))\ . \quad (14)$$

The steady state solutions to this equation are closely related to the steady state solutions of equation (11). Indeed, a straightforward extension of the previous analysis shows that all previous results survive. The only change is that the critical values z_i are smaller; with weaker enforcement, it is more likely that agents are constrained (thus z_0 becomes smaller) and it is more likely that autarky is the unique equilibrium (thus z_3 becomes smaller). The other critical values are smaller as well. Hence, the Theorem extends as follows.

Proposition 7: Let productivity shifts be i.i.d. and assume that defaulters are excluded from borrowing and lending for only one period after default. Then there are $\hat{z}_i < z_i$, $i = 0,1,2,3$ with $\hat{z}_1 < \hat{z}_2 < \hat{z}_3$ such that the Theorem remains valid where all z_i are replaced by $\hat{z}_i$.

Proof: Appendix.

Proposition 7 cannot be generalized to arbitrary Markovian productivity shifts. Suppose, for instance, that productivity shifts are deterministic ($\pi_p = \pi_u = 1$). Then, the recursive participation constraint (10) with one–period exclusion becomes

$$\ln(1+\theta_t) - \ln(1+\theta_t(1-r_t)) = \beta\Big[\ln r_{t+1} - \ln z\Big]\ .$$

A defaulting agent cannot save in period $t+1$ so that he forgoes the excess of the market return r_{t+1} over his own productivity z, but there is no further punishment after period $t+1$. A straightforward analysis of this equation shows that (i) autarky is the unique production–inefficient equilibrium for all $z > 0$ and it is determinate; (ii) there is a determinate unconstrained equilibrium provided that $z \leq \hat{z}_0 \equiv 2^{-1/\beta}$; (iii) there exists an indeterminate constrained and production efficient equilibrium provided that $z \leq \hat{z}_3$ where $\hat{z}_3 > \hat{z}_0$. When $z \in (\hat{z}_0, \hat{z}_3)$ there is another constrained and production efficient equilibrium. In other words, the critical values z_1 and z_2 of Theorem 1 and Figure 1 both collapse at $z = 0$, and for all $0 < z \leq z_3$ there are three steady state equilibria.

6 Conclusions

We have studied a simple growth model in which debt constraints arise endogenously by the borrowers' fear of market exclusion after default. There is a two–sided linkage between growth and credit market development: better developed credit markets make the factor allocation more efficient, and a growth push is likely to relax endogenous credit constraints.

With a standard specification of preferences and technology, multiplicity and indeterminacy of equilibrium growth paths turn out to be generic features in the sense that they occur for an open set of model parameters. This outcome does neither require that defaulters can be excluded from future lending, nor does exclusion need to last very long. The key to the multiplicity result is a dynamic complementarity that is reflected in a recursive equation linking subsequent debt constraints and interest rates. When agents expect future debt constraints to be high, they incur a large loss when they are excluded from the credit market. Therefore they do not default easily and they can be allowed to borrow more today. The paper shows that such self–fulfilling believes about financial frictions alone can give rise to multiple growth regimes. Put differently, although institutional differences play an important role in explaining differences in financial development, they may not be the only reason.[22] The paper also shows that equilibria with low growth are more volatile for two reasons: first, output growth depends on the wealth distribution and not just on technology and preference parameters; second, output growth may react to sunspot shocks.

The environment in this paper differs from previous literature that is based on self–enforcing debt constraints à la Kehoe and Levine (1993) (for example, Alvarez

[22] Modelling the emergence of credit enforcement institutions is an important task that has not received much attention so far; see however Koeppl (2002).

and Jermann (2000, 2001), Krueger and Perri (2001)). Typically, in this literature, agents face idiosyncratic *endowment shocks* and they want to borrow in order to smooth their consumption path. As has been shown by Azariadis and Kaas (2002), the equilibrium in such an economy with infinite exclusion after default is unique and determinate. On the other hand, Azariadis and Lambertini (2003) show that there are multiple equilibria in a pure–exchange overlapping–generations economy with self–enforcing debt constraints. In their model, the dynamic complementarity turns out to be stronger because the length of market exclusion is, by definition, shorter than in an infinite–horizon economy. In this paper, as in Kehoe and Perri (2002), agents are infinitely–lived and they face *productivity shocks* rather than endowment shocks. Our results show that indeterminacy of decentralized equilibria is an important phenomenon in such environments.

Appendix

Lemma 1: Expected utility of an autarkic agent who was productive in the previous period and has wealth a is $\overline{V}(a) = \ln a/(1-\beta) + \overline{V}$ where

$$\overline{V} = \frac{1}{1-\beta}\Big(\ln(1-\beta) + \frac{\beta\ln(\beta A)}{1-\beta} + \frac{\beta\pi_u \ln z}{(1-\beta)(1-\beta(1-\pi_p-\pi_u))}\Big)\ .$$

Proof: Let $\overline{V}^u(a)$ denote utility of an autarkic agent who was unproductive in the previous period and starts the current period with wealth a. Then one obtains the recursive identities

$$\begin{aligned}\overline{V}(a) &= \ln((1-\beta)a) + \beta\Big((1-\pi_u)\overline{V}(A\beta a) + \pi_u\overline{V}^u(zA\beta a)\Big)\ ,\\ \overline{V}^u(a) &= \ln((1-\beta)a) + \beta\Big((1-\pi_p)\overline{V}^u(zA\beta a) + \pi_p\overline{V}(A\beta a)\Big)\ .\end{aligned}$$

By substituting $\overline{V}(a) = \ln a/(1-\beta) + \overline{V}$ and $\overline{V}^u(a) = \ln(a)/(1-\beta) + \overline{V}^u$ into these equations, the terms $\ln(a)/(1-\beta)$ cancel out and one obtains two linear equations in $\overline{V}$ and $\overline{V}^u$ that can be solved easily. □

Lemma 2: Consider a sequence $(r_t, \theta_t)_{t=0}^{\infty}$. Then the following are equivalent.

(a) The participation constraint (2) is satisfied for all $t \geq 0$.

(b) The recursive participation constraint (10) holds for all $t \geq 0$ and the transversality conditions

$$\lim_{t\to\infty} \beta^t V^p(\theta_t, r_t) = 0 \ , \ \lim_{t\to\infty} \beta^t V^u(\theta_t, r_t, \theta_{t+1}, r_{t+1}) = 0$$

are satisfied where $V^p(\theta_t, r_t)$ and $V^u(\theta_t, r_t, \theta_{t+1}, r_{t+1})$ are defined in (17) and (18) below.

Proof: We show first that (a) implies (b). Write utility of an agent who was productive in period t and enters period $t+1$ with wealth a as $V^p_{t+1}(a) = V(a, (r^{t+1}, \theta^{t+1}))$. Similarly, $V^u_{t+1}(a)$ denotes utility of an agent in period $t+1$ who was unproductive in period t. Then one obtains for all $t \geq 0$ the recursive equations:

$$\begin{aligned} V^p_{t+1}(a) &= \ln((1-\beta)a) + \beta(1-\pi_u)V^p_{t+2}(R^+_{t+1}\beta a) + \beta\pi_u V^u_{t+2}(R_{t+1}\beta a) \ , \\ V^u_{t+1}(a) &= \ln((1-\beta)a) + \beta(1-\pi_p)V^u_{t+2}(R_{t+1}\beta a) + \beta\pi_p V^p_{t+2}(R^+_{t+1}\beta a) \ . \end{aligned}$$

Substitution of $V^p_{t+1}(a) = \ln a/(1-\beta) + V^p_{t+1}$ and $V^u_{t+1}(a) = \ln a/(1-\beta) + V^u_{t+1}$ yields two recursive equations in V^u_{t+1} and V^p_{t+1}:

$$V^p_{t+1} = \ln(1-\beta) + \beta\Big((1-\pi_u)\Big(\frac{\ln(R^+_{t+1}\beta)}{1-\beta} + V^p_{t+2}\Big) + \pi_u\Big(\frac{\ln(R_{t+1}\beta)}{1-\beta} + V^u_{t+2}\Big)\Big) \ , \tag{15}$$

$$V^u_{t+1} = \ln(1-\beta) + \beta\Big((1-\pi_p)\Big(\frac{\ln(R_{t+1}\beta)}{1-\beta} + V^u_{t+2}\Big) + \pi_p\Big(\frac{\ln(R^+_{t+1}\beta)}{1-\beta} + V^p_{t+2}\Big)\Big) \ . \tag{16}$$

Since (2) holds for all $t \geq 0$, one obtains together with Lemma 1:

$$V^p_{t+1} = \overline{V} + \frac{\ln(m(1+\theta_t)) - \ln(1+\theta_t(1-r_t))}{1-\beta} \equiv V^p(\theta_t, r_t) \ . \tag{17}$$

Plugging (17) into (15) and solving for V^u_{t+2} yields for all $t \geq 0$,

$$V^u_{t+2} = V^u(\theta_t, r_t, \theta_{t+1}, r_{t+1}) \equiv \frac{1}{\beta\pi_u}\Bigg(V^p(\theta_t, r_t) - \ln(1-\beta) \tag{18}$$

$$-\beta(1-\pi_u)\Big(\frac{\ln(A\beta(1+\theta_{t+1}(1-r_{t+1})))}{1-\beta} + V^p(\theta_{t+1}, r_{t+1})\Big) - \beta\pi_u\frac{\ln(A\beta r_{t+1})}{1-\beta}\Bigg) \ .$$

By plugging both (17) and (18) into equation (16) for $t+2$ instead of $t+1$ yields an implicit equation in $(\theta_\tau, r_\tau)_{\tau=t,t+1,t+2}$. After rearranging, one can show that all terms $\ln(1-\beta)$ and $\ln(\beta A)$ cancel out, so that one is left with equation (10). The transversality conditions must be satisfied since V^p_1 and V^u_1 are the finite expected utilities of productive and unproductive agents (with wealth $a = 1$) that can be expressed as infinite sums of expected log consumption levels. The "bubble components" in (15) and (16), $\beta^t V_t^p$ and $\beta^t V_t^u$, must go to zero as t goes to infinity.

To show that (b) implies (a), note that (10) implies that the recursive identities (15) and (16) are satisfied. Together with the transversality conditions, V_1^p and V_1^u are indeed the finite expected utility levels of productive and unproductive agents in period one. By construction of (17) and Lemma 1, the participation constraints (2) must be satisfied for all $t \geq 0$. □

Proof of Proposition 3: As has been shown above, (12) has two non–negative solutions whenever $z > z_1$ and m is big enough. Let these solutions be denoted $\overline{\theta}_1(z) \geq 0$ and $\overline{\theta}_2(z) > \overline{\theta}_1(z)$. $\overline{\theta}_2(z)$ is strictly increasing in z. To any solution θ, the corresponding stationary wealth distribution satisfies (see (3))

$$\alpha = \frac{(1-\pi_u)(1+\theta(1-z))\alpha + \pi_p z(1-\alpha)}{(1+\theta(1-z))\alpha + z(1-\alpha)} \ . \tag{19}$$

This equation has a unique solution $\alpha(\theta) \in (0,1)$. The solution satisfies the market–clearing condition (4) if $\alpha(\theta) \leq 1/(1+\theta)$. Clearly, this condition holds for $\overline{\theta}_1$ if m is big enough, and it must also hold for $\overline{\theta}_2$ when m is big enough and when z is close to z_1 (because then $\overline{\theta}_2$ is near zero). But when z is increased, the condition will eventually be violated. The critical value $z_2 > z_1$ is defined by $\alpha(\overline{\theta}(z)) = 1/(1+\overline{\theta})$. Plugging this identity into (19) yields

$$\overline{\theta}(z) = \frac{\pi_u}{\pi_p z + (1-\pi_u)(1-z)} \ ,$$

and substituting this expression into (12) yields the equation that determines z_2:

$$m(z(\pi_p + \pi_u - 1) + 1)(z(\pi_p + \pi_u - 1) + 1 - \pi_u)^{\phi - 1} = (1 - z(1-\pi_p))^{\phi} \ . \tag{20}$$

When m is close to one, this equation has a unique solution $z_2(m) \in (0,1)$ that satisfies $z_2 > z_1$.

It remains to show local determinacy/indeterminacy of the steady state equilibria. We provide a proof only for the special cases of deterministic shocks ($\pi_p = \pi_u = 1$) or i.i.d. shocks ($\pi_p + \pi_u = 1$) (A proof in the general case is available from the author on request). In these both cases (10) becomes a one–dimensional difference equation in θ_t since $r_t = z$. It can be checked easily that $(d\theta_{t+1})/(d\theta_t)|_{\overline{\theta}_1 = 0} > 1$ if, and only if, $z > z_1$. Hence, the equilibrium near autarky is determinate if, and only if, $z > z_1$. And when this equilibrium is determinate, the other equilibrium is indeterminate. □

Proof of Proposition 4: Consider first the case of i.i.d. productivity shocks where $\pi_p = 1 - \pi_u \equiv \pi$ is the probability of a productive state for all agents, so that the wealth share of productive agents equals the probability of the productive state in each period: $\alpha_t = \pi$ for all $t \geq 1$. Hence, when $r_t \in (z, 1)$, (4) implies that debt constraints are constant: $\theta_t = (1-\pi)/\pi$. The participation constraint (10) yields a one–dimensional equation in interest rates:

$$\beta\pi \ln \pi + (1-\beta)\ln m - \ln(\pi + (1-\pi)(1-r_t)) =$$
$$\beta(1-\pi)\Big(\ln r_{t+1} - \ln z - \ln\Big(\pi + (1-\pi)(1-r_{t+1})\Big)\Big)\ , \tag{21}$$

whose steady state solution must satisfy

$$\psi(r) \equiv r\Big(\pi + (1-\pi)(1-r)\Big)^{\frac{1}{\beta(1-\pi)}-1}\pi^{-\frac{\pi}{1-\pi}}m^{-\frac{1-\beta}{\beta(1-\pi)}} = z\ . \tag{22}$$

The left–hand side has a unique maximum at $r = \beta$, and define $z_3 = \psi(\beta)$. z_3 is less than one provided that m is not too small. Further, $\psi(z) = z$ has the unique solution $z = z_2 < 1$, and $\psi(1) = z_0 < 1$, again provided that m is not too small. That is, for $z = z_2$, there is a steady state solution of (22) that falls together with the second steady state of Proposition 3, and for $z = z_0$, there is a steady state solution of (22) that coincides with the unconstrained equilibrium of Proposition 2. One can also show that $z_2 < \beta$. Figure 3 reveals that there is a unique constrained steady state $r \in (z, 1)$ if $z \in (\min(z_0, z_2), \max(z_0, z_2)]$, and there are two such steady states if $z \in (\max(z_0, z_2), z_3)$. Furthermore, differentiation of (21) shows that $dr_{t+1}/dr_t = r/\beta$ at the steady state. Hence, a steady state $r > \beta$ is determinate, and a steady state $r < \beta$ is indeterminate. When $z \leq z_0$, there is no steady state with $r > \beta$, and $1 > r_t > \beta$ must be strictly decreasing; hence the equilibrium of Proposition 2 is locally determinate.

Suppose now that productivity shifts are not i.i.d., i.e. $\pi_u + \pi_p \neq 1$. From (4), $\alpha_t = 1/(1+\theta_t)$. Substitution of α_t and α_{t+1} in (3) gives

$$\frac{1}{1+\theta_{t+1}} = (1-\pi_u)(1 - \frac{r_t\theta_t}{1+\theta_t}) + \pi_p\frac{r_t\theta_t}{1+\theta_t}\ .$$

Solving for r_t yields

$$r_t = \frac{(1+\theta_t)(\pi_u - \theta_{t+1}(1-\pi_u))}{(1+\theta_{t+1})\theta_t(\pi_p + \pi_u - 1)}\ . \tag{23}$$

One also obtains from this equation

$$1 + \theta_t(1-r_t) = \frac{(1+\theta_t)(\pi_p(\theta_{t+1}+1) - 1)}{(1+\theta_{t+1})(\pi_p + \pi_u - 1)}\ . \tag{24}$$

Assume first $\pi_u + \pi_p > 1$. Then substitution of (23) and (24) into (10) yields a one–dimensional dynamic equation in θ_t:

$$(1-\beta)\ln(1+\theta_{t+1}) + \beta(1-\beta)(\pi_u + \pi_p - 1)\ln(1+\theta_{t+2}) + \beta\pi_u \ln\theta_{t+1}$$

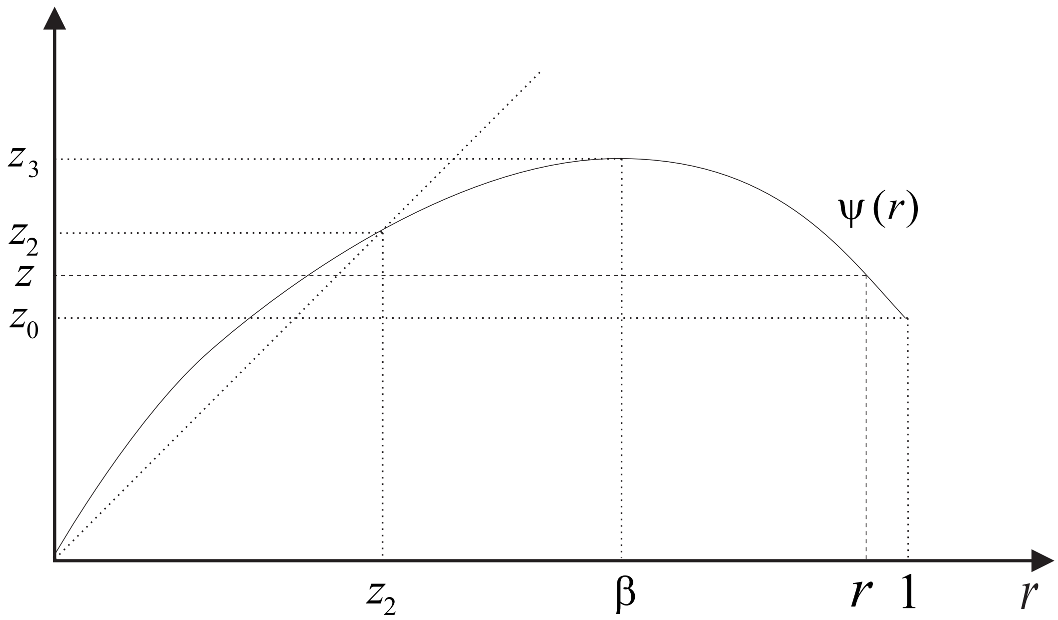

Figure 3: The constrained equilibrium with efficient production satisfies $\psi(r) = z$ and $z < r < 1$.

$$+(1-\beta(1-\pi_u-\pi_p))\ln(\pi_p+\pi_u-1)+\beta\pi_u \ln z+(1-\beta)(1-\beta(1-\pi_p-\pi_u))\ln m = \quad (25)$$
$$\ln\Big(\pi_p(1+\theta_{t+1})-1\Big)-\beta(1-\pi_p)\ln\Big(\pi_p(1+\theta_{t+2})-1\Big)+\beta\pi_u\ln\Big(\pi_u-(1-\pi_u)\theta_{t+2}\Big)\ .$$

The solution to this equation requires that $\theta_{t+1} \in ((1-\pi_p)/\pi_p, \pi_u/(1-\pi_u))$ for all t (otherwise (23) and (24) are not positive). A steady state solution satisfies

$$\psi(\theta) \equiv (1+\theta)^{-\frac{(1-\beta)(1-\beta(1-\pi_u-\pi_p))}{\beta\pi_u}}\theta^{-1}(1-(1-\pi_u)(1+\theta))$$
$$\cdot(\pi_p(1+\theta)-1)^{\frac{1-\beta(1-\pi_p)}{\beta\pi_u}}(\pi_p+\pi_u-1)^{\frac{-1+\beta(1-\pi_u-\pi_p)}{\beta\pi_u}}m^{-\frac{(1-\beta)(1-\beta(1-\pi_p-\pi_u))}{\beta\pi_u}} = z\ .$$

The first three terms of ψ are decreasing in θ and the fourth term is increasing. Further $\psi(\theta) = 0$ if $\theta = \pi_u/(1-\pi_u)$ and $\theta = (1-\pi_p)/\pi_p$. In between there is a unique maximum $\theta_{\max}$, and define $z_3 = \psi(\theta_{\max})$. z_3 is less than one when m is not too small. From (23), $r = z$ iff $\theta = \theta_2 \equiv \pi_u/((1-\pi_u)(1-z)+\pi_p z)$, and $r = 1$ if $\theta = \theta_0 \equiv \pi_u/\pi_p$. One finds that $\psi(\theta_2) = z_2$ and $\psi(\theta_0) = z_0$. Hence, similar to Figure 3 and the argument above, there is no equilibrium if $z < \min(z_0, z_2)$ and there is a unique equilibrium if $z \in [\min(z_0, z_2), \max(z_0, z_2))$. If $z \in [\max(z_0, z_2), z_3)]$ there are again two solutions $\theta_1 < \theta_{\max} < \theta_2$.

If $\pi_u + \pi_p < 1$ the analysis is very similar. Instead of (25) one obtains

$$(1-\beta)\ln(1+\theta_{t+1}) + \beta(1-\beta)(\pi_u+\pi_p-1)\ln(1+\theta_{t+2}) + \beta\pi_u \ln\theta_{t+1}$$

$$+(1-\beta(1-\pi_u-\pi_p))\ln(1-\pi_u-\pi_p) + \beta\pi_u \ln z + (1-\beta)(1-\beta(1-\pi_p-\pi_u))\ln m =$$

$$\ln(1-\pi_p(1+\theta_{t+1})) - \beta(1-\pi_p)\ln(1-\pi_p(1+\theta_{t+2})) + \beta\pi_u \ln((1-\pi_u)\theta_{t+2}-\pi_u) \ ,$$

and the steady state solution $\theta \in (\pi_u/(1-\pi_u), (1-\pi_p)/\pi_p)$ must satisfy

$$\psi(\theta) \equiv (1+\theta)^{-\frac{(1-\beta)(1-\beta(1-\pi_u-\pi_p))}{\beta\pi_u}} \theta^{-1}((1-\pi_u)(1+\theta)-1)$$

$$\cdot(1-\pi_p(1+\theta))^{\frac{1-\beta(1-\pi_p)}{\beta\pi_u}} (1-\pi_p+\pi_u)^{\frac{-1+\beta(1-\pi_u-\pi_p)}{\beta\pi_u}} m^{-\frac{(1-\beta)(1-\beta(1-\pi_p-\pi_u))}{\beta\pi_u}} = z \ .$$

The remainder of the proof is the same. □

Proof of Proposition 5: With i.i.d. productivity shifts the dynamics is described by equation (11) together with the market clearing condition (4). In each period, the equilibrium is either unconstrained (*U*), constrained and production efficient (*E*) or constrained and production inefficient (*I*). These regimes are

$$\begin{aligned} (U) &\qquad r_t = 1 \quad \text{and} \quad \theta_t \geq \hat{\theta} \equiv \frac{1-\pi}{\pi} \ , \\ (E) &\qquad z < r_t < 1 \quad \text{and} \quad \theta_t = \hat{\theta} \ , \\ (I) &\qquad r_t = z \quad \text{and} \quad \theta_t \leq \hat{\theta} \ . \end{aligned}$$

Any equilibrium that stays only in the regimes (*U*) or (*I*) can be described by a one–dimensional equation in θ_t since r_t attains only the values z or 1. From (11), the dynamics is described by four different maps, denoted $\theta_{t+1} = F_{SS'}(\theta_t)$ where $S, S' \in \{U, I\}$ are the regimes in period t and $t+1$ respectively. For instance, the map F_{UI} applies when $r_t = 1$ and $r_{t+1} = z$ (i.e. $S = U$ and $S' = I$ and thus $\theta_t \geq \hat{\theta}$ and $\theta_{t+1} \leq \hat{\theta}$). It can be checked easily that all four maps are strictly monotonically increasing (because of the dynamic complementarity) and that the following inequalities must be satisfied:

$$F_{UU} < F_{UI} \ , \ F_{II} < F_{UI} \ , \ F_{IU} < F_{II} \ , \ F_{IU} < F_{UU} \ . \tag{26}$$

In the space (θ_t, θ_{t+1}) each of these four maps applies only in one of the four quadrants that are divided by the lines $\theta_t = \hat{\theta}$ and $\theta_{t+1} = \hat{\theta}$. Equilibria leaving the regimes (*U*) or (*I*) into the regime (*E*) or leaving the regime (*E*) into the regimes (*U*) or (*I*) are on the dividing lines in the (θ_t, θ_{t+1})–diagram and they are restricted by the corresponding $F_{SS'}$ curves. For instance, if (θ_t, θ_{t+1}) leaves (*U*) into (*E*) one has $\theta_{t+1} = \hat{\theta}$ and $F_{UI}^{-1}(\hat{\theta}) < \theta_t < F_{UU}^{-1}(\hat{\theta})$ (see Figure 4 (a)). The dynamics in the regime (*E*) follow equation (21) which is a monotonic map between r_t and r_{t+1}. Thus there cannot be cycles that stay in (E) and each equilibrium must either converge to an indeterminate equilibrium in (E) or it must leave (E) into any of the other regimes.

Several graphical arguments prove now that there are no cycles between regimes and that all non–stationary equilibria must converge to the indeterminate equilibrium. These arguments are shown in the diagrams of Figure 4 in which the bold curves show the maps between θ_t and θ_{t+1} in each of four different subcases with multiple steady states. In these graphs it is assumed that $m = 1$ so that autarky, $\overline{\theta}_1 = 0$ is always a steady state equilibrium, and the other production inefficient equilibrium, if it exists, is denoted $\overline{\theta} = \overline{\theta}_2$. The extension to $m < 1$ is straightforward.

Whenever there is a (U) steady state θ^*, it is determinate (Proposition 2) and the map F_{UU} has a slope > 1 at the steady state. Explosive solutions $\theta_t \to \infty$ can be excluded since they violate the first transversality condition of Lemma 2. If there is a non–autarkic (I) steady state $\bar{\theta}$, it is indeterminate (Proposition 3) and the map F_{II} has a slope between 0 and 1 at the steady state.

(a) $z_2 > z > z_1$ and $z < z_0$. In this case there is a (U) steady state and there is an indeterminate non–autarkic steady state in (I). (26) and the fact that F_{II} has a steady state $0 < \bar{\theta} < \hat{\theta}$ imply $F_{IU}(\hat{\theta}) < F_{II}(\hat{\theta}) < \hat{\theta}$ and $F_{UI}(\hat{\theta}) > F_{II}(\hat{\theta})$. As Figure 4 (a) shows, all non–stationary equilibria must converge to the indeterminate equilibrium $\bar{\theta}$.

(b) $z_2 > z > z_1$ and $z > z_0$. There is no (U) steady state, there is a determinate steady state in (E) and there is an indeterminate non–autarkic steady state in (I). As Figure 4 (b) shows (together the dynamics of interest rates in equation (21)), all non–stationary equilibria must again converge to $\bar{\theta}$.

(c) $z_3 > z > z_2$ and $z < z_0$. There is a (U) steady state and an indeterminate (E) steady state. Figure 4 (c) and equation (21) imply convergence to the indeterminate steady state.

(d) $z_3 > z > z_2$ and $z > z_0$. There are two (E) steady states. The one with the higher interest rate is determinate, the other is indeterminate. Again, Figure 4 (d) and equation (21) imply that all non–stationary equilibria converge to the indeterminate steady state.

These arguments preclude the existence of deterministic cycles and ensure convergence to the indeterminate equilibrium. □

Proof of Proposition 7: From (14), an *unconstrained* steady state satisfies $\ln(1 + \theta) = -\beta(1 - \pi) \ln z$. Since market clearing requires that $1 + \theta \geq 1/\pi$, such a solution exists if, and only if,

$$z \leq \hat{z}_0 \equiv \pi^{1/(\beta(1-\pi))} < z_0 = \pi^{(1-\beta)/(\beta(1-\pi))} \ .$$

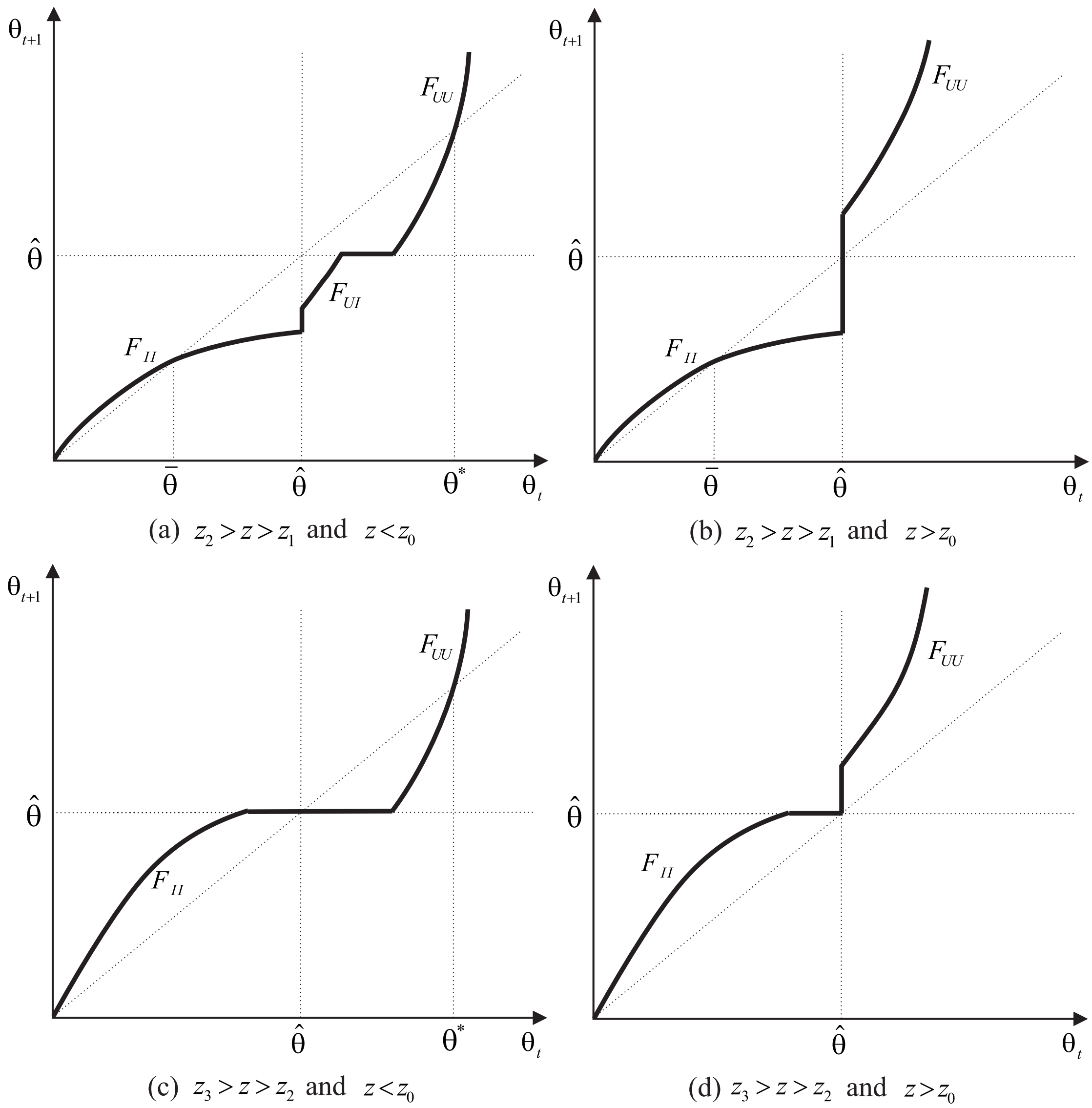

Figure 4: Equilibrium dynamics with i.i.d. productivity shocks.

A *constrained and production inefficient* steady state must satisfy instead of (12)

$$1 + \theta = (1 + \theta(1 - z))^{\hat{\phi}} ,$$

where $\hat{\phi} = 1 + \beta\pi < \phi$. Thus,

$$\hat{z}_1 \equiv \frac{\beta\pi}{1 + \beta\pi} < z_1 = \frac{\beta\pi}{1 - \beta(1 - \pi)} ,$$

$$\hat{z}_2 \equiv \frac{1-\pi^{\hat{z}_1}}{1-\pi} < z_2 = \frac{1-\pi^{z_1}}{1-\pi} \ .$$

Finally, a *constrained and production efficient* steady state interest rate $r \in (z, 1)$ must solve instead of (22)

$$\hat{\psi}(r) \equiv r\Big(\pi + (1-\pi)(1-r)\Big)^{\frac{1+\beta\pi}{\beta(1-\pi)}} \pi^{-\frac{\pi}{1-\pi}} = z \ .$$

Since $\hat{\psi}(r) < \psi(r)$, the maximum of $\hat{\psi}$ must be smaller than the maximum of ψ and this yields $\hat{z}_3 < z_3$. □

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