Robust Monetary Policy in the New-Keynesian Framework

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Abstract

We study the effect of model uncertainty in a New Keynesian open-economy model using robust control techniques. Due to the simple model structure, we are able to solve a version of the robust control problem analytically, and we analyze the separate effects of uncertainty concerning the determination of inflation, output and the exchange rate. We show that an increased central bank preference for robustness can make monetary policy more aggressive or more cautious, depending on the type of shock and the source of uncertainty. In a closed-economy version, however, increased robustness always makes monetary policy more aggressive.

Keywords: Knightian uncertainty, model uncertainty, policy robustness.

JEL Classification: E52, E58, F41.

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1 Introduction

Good policy design requires a good understanding of private sector behavior. Such understanding is important not only in order to identify market deficiencies and hence policy objectives, but also when trying to meet objectives in the best possible way. Recently, the New-Keynesian model as laid out by Rotemberg and Woodford (1997), Clarida et al. (1999) and others has established itself as the mainstream model for monetary policy analysis. The model captures the sluggish adjustment of prices and the intertemporal consumption decision in a model framework with optimizing households and firms. With only a very limited number of equations, the model is having a strong influence and has provided policymakers with several guiding policy principles in responding to the different disturbances in the economy (see, e.g., Clarida et al., 1999, and King, 2000).

Although the model has many attractive theoretical properties, it has been criticized by many researchers, most notably for not fitting the data well.[1] One response to such criticism is to design more complex models that are able to better capture the behavior of macroeconomic variables, following, e.g., Christiano et al. (2001). Such models gain in realism but lose in tractability. An alternative route is to acknowledge that the simple model is a misspecified description of reality, and to design policy to take this misspecification into account. In this paper we follow the second route and allow for the possibility that the model may not be the correct representation of private sector behavior. Rather, we will assume that the true model of private sector behavior lies in some neighborhood of the reference model, and we analyze how monetary policy should be designed in order to work reasonably well for all models inside this neighborhood. This problem has recently been addressed by Hansen and Sargent (2004) using "robust control" techniques. Assuming that the policymaker is unable to formulate a probability distribution over plausible models, the robust policymaker designs policy for the worst possible outcome within a pre-specified set of models.

We apply robust control techniques developed by Hansen and Sargent (2004) and Giordani and Söderlind (2003) to a New Keynesian open-economy model developed by Galí and Monacelli (2004) and Clarida et al. (2002). We generalize the standard robust control framework by allowing the policymaker's preference for robustness to differ across equations, reflecting the confidence the policymaker has in each relationship. For instance, the policymaker may be quite confident about one of

[1] See, e.g., Fuhrer (1997), Estrella and Fuhrer (2002, 2003), Rudebusch (2002).

the equations (e.g., the Phillips curve) and believe that robustness to deviations from this equation is not important, but at the same time be very uncertain about some other equation (e.g., the exchange rate relationship). This approach allows us to consider each equation in turn and ask what is the appropriate response of the robust policymaker to misspecification in this particular equation. Thus we will consider several different types of uncertainty within the model: uncertainty about firms' price-setting, uncertainty about consumer behavior, and uncertainty about the model determining the exchange rate.[2]

The ability to focus on model uncertainty in particular equations seems important. Policymakers are more confident in some relationships than in others, and so regard some model uncertainty as being more substantial than others. In open economies, monetary policymakers are particularly uncertain about the effects of the exchange rate on the economy and the effects of monetary policy on the exchange rate. Using our approach, we are able to analyze the proper response of monetary policy to such uncertainty, while keeping other sources of uncertainty fixed.

One important part of the analysis will focus on the effects of model uncertainty on monetary policy. Thus far, there is no consensus about whether increased uncertainty should lead to more aggressive or more cautious policy behavior. Following the seminal analysis of Brainard (1967), it is well-accepted that increased uncertainty about the effects of policy should lead to more cautious policy behavior, at least within a Bayesian framework. However, Söderström (2002) shows that this is not generally true for all parameters in the model: increased uncertainty about the persistence of inflation should rather make policy more aggressive. Within the robust control literature, increased uncertainty tends to lead to more aggressive policy behavior, see, e.g., Hansen and Sargent (2001, 2004), Giannoni (2002), and Giordani and Söderlind (2003).

These studies of robust control have all used numerical methods to solve for the optimal robust policy in a closed economy. Our analysis, using analytical solutions, will show that this result does not carry over to the open economy: depending on the source of uncertainty and the type of disturbance hitting the economy, optimal robust policy in an open economy can be either more aggressive or more cautious than the non-robust policy. In the closed economy version of our model, in contrast,

[2]Leitemo and Söderström (2003) study the effects of exchange rate model uncertainty on the performance of optimized simple monetary policy rules. In their framework, the central bank is uncertain about the exchange rate model, but private agents have perfect information about the exact specification of the model. In the present paper both the central bank and private agents have doubts about the true model.

robustness always makes optimal policy more aggressive.

A second set of our results will discuss the effects on the macroeconomy of the central bank's fear for model misspecification. As the central bank designs policy to do well in the worst-case scenario, this will have important consequences for the economy in other more likely outcomes. We show that the price of being robust to misspecification in the Phillips curve or the exchange rate equations comes in the form of inefficiently high output variability; whereas robustness against misspecification in the output equation comes at the cost of higher inflation variability. These results are independent of the distribution of the shocks.

Our paper is organized as follows. In Section 2 we present the New Keynesian open-economy model and review some terminology. In Section 3 we derive the stochastic equilibrium under a robust policymaker, both in the "worst-case" model when misspecification is present and in the "approximating" model, which is the most likely outcome. Section 4 is devoted to analyzing the effects of an increased preference for robustness. Section 5 summarizes and concludes.

2 A simple New-Keynesian open-economy model

We use a very simple model of a small open economy developed by Galí and Monacelli (2004) and Clarida et al. (2002), but deviate from these authors by introducing a time-varying premium on foreign bond holdings. This enables us to analyze uncertainty concerning the model determining the exchange rate, which is an important goal of the paper. The model is a generalization of the canonical New-Keynesian model for a closed economy developed by Rotemberg and Woodford (1997) and others, and carefully examined by Clarida et al. (1999).

The world is assumed to consist of two countries: a small open home country and a large, approximately closed, foreign country. The two countries share preferences and technology and produce traded consumption goods. In the home country, firms produce domestic goods, and households consume both domestic and imported goods.

Define by π_t the rate of inflation in the domestic goods sector; by x_t the output gap in the domestic economy, i.e., the log deviation of domestic output from its flexible-price level; and by e_t the real exchange rate, defined in terms of the domestic price level as

$$e_t = s_t + p_t^f - p_t, \tag{1}$$

where s_t is the nominal exchange rate, p_t^f is the price level in the foreign economy, and p_t is the price level of domestically produced goods.[3]

The domestic inflation rate, the output gap and the real exchange rate are interrelated according to the following three equations:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + \alpha e_t + \Sigma_\pi \varepsilon_t^\pi, \quad (2)$$

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left[i_t - \mathrm{E}_t \pi_{t+1}\right] - \gamma \left[\mathrm{E}_t e_{t+1} - e_t\right] + \Sigma_x \varepsilon_t^x, \quad (3)$$

$$e_t = \mathrm{E}_t e_{t+1} - \left[i_t - \mathrm{E}_t \pi_{t+1}\right] + \Sigma_e \varepsilon_t^e. \quad (4)$$

Equation (2) is a New-Keynesian Phillips curve for the open economy, where the rate of domestic inflation depends on expected future inflation and current marginal cost, which is affected by the output gap and the exchange rate. The real exchange rate affects marginal cost through households' labor supply decision: households value their wage relative to the consumer price index (which includes prices of imported goods), so the equilibrium wage depends on the real exchange rate. The inflation shock, ε_t^π, is due to changes in the flexible-price level of the real exchange rate.

Equation (3) is an expectational IS curve, expressed in terms of the output gap, that relates the output gap to the expected future output gap, the real interest rate (as households substitute consumption over time), and the real exchange rate (as consumption is partly satisfied through imported goods). The demand shock ε_t^x reflects expected changes in the flexible-price level of output, or, equivalently, changes in the natural real interest rate.

Finally, equation (4) is a real interest parity condition, where the expected rate of real depreciation is related to the real interest rate differential (also in terms

[3]Formally, e_t defined as in equation (1) is the terms of trade, the difference between the price on imported goods (the foreign price level denominated in domestic currency) and domestic goods. A more traditional way of defining the real exchange rate would be in terms of the domestic consumer price index:

$$q_t = s_t + p_t^f - p_t^c,$$

where $p_t^c = (1-\omega)p_t + \omega(p_t^f + s_t)$, and where ω is the share of imports in domestic consumption. However, since the equation determining e_t is derived from the uncovered interest rate parity condition determining the nominal exchange rate, we will nevertheless refer to it as the real exchange rate. Our definition is not crucial to our results: the traditional real exchange rate is related to our real exchange rate by

$$q_t = (1-\omega)e_t,$$

so changes in e_t are proportionally reflected in changes in q_t.

of domestic inflation) between the domestic and foreign economies. All foreign variables are assumed to be exogenous, and therefore set to zero. The exchange rate disturbance, ε_t^e, reflects the fact that domestic households pay a premium on foreign bond holdings.

All shocks ε_t^j are assumed to be white noise with zero mean and unit variance. The Σ_j parameters determine the variance of the shocks: for instance, the variance of the supply shock $\Sigma_\pi \varepsilon_t^\pi$ will be Σ_π^2.

Appendix A shows how to derive this model from the optimizing behavior of a representative agent in a small open economy, giving a structural interpretation to all parameters in the model, following Galí and Monacelli (2004), Clarida et al. (2002) and Walsh (2003, Ch. 6.5).[4] The parameter β is the discount factor of domestic households and firms, while the parameters κ, α σ, and γ depend on "deep" parameters according to

$$\kappa \equiv \frac{(1-\theta)(1-\beta\theta)(\hat{\sigma}+\eta)}{\theta}, \tag{5}$$

$$\alpha \equiv \frac{\omega(1-\theta)(1-\beta\theta)}{\theta}, \tag{6}$$

$$\sigma \equiv \frac{\hat{\sigma}}{1-\omega}, \tag{7}$$

$$\gamma \equiv (2-\omega)\omega\delta - \frac{\omega(1-\omega)}{\hat{\sigma}}, \tag{8}$$

where θ is the probability that a firm is not able to change its price in a given period in the sticky-price model of Calvo (1983); $\hat{\sigma}$ is the elasticity of intertemporal substitution; η is the elasticity of the representative household's labor supply; ω is the share of imports in domestic consumption, i.e., the degree of openness; and δ is the elasticity of substitution across domestic and foreign goods. Clearly, the parameters κ, α, σ and β are always positive, and also γ will be positive for typical parameterizations, as $(2-\omega)\omega > (1-\omega)\omega$ and δ is typically larger than $\hat{\sigma}^{-1}$.

We also note that when $\omega = 0$ (so $\alpha = \gamma = 0$) the model reduces to the standard closed-economy model analyzed in detail by Clarida et al. (1999). Thus, although our main interest is in the consequences of model uncertainty in an open economy, we will also be able to derive results for the closed economy.

To model misspecification we follow Hansen and Sargent (2004), and introduce

[4] Galí and Monacelli (2004) and Clarida et al. (2002) eliminate the exchange rate from the model using the UIP condition (4) with $\varepsilon_t^e = 0$, thus reaching a formulation of the open-economy model that is isomorphic to the closed-economy model. We are particularly interested in model uncertainty concerning the UIP condition, and therefore include the time-varying premium ε_t^e. Therefore we cannot eliminate the exchange rate from the system.

in each equation a second type of disturbance, denoted v_t^j, which is controlled by a fictitious "evil agent", who represents the policymaker's worst fears concerning misspecification. Thus, the misspecified model is given by

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + \alpha e_t + \Sigma_\pi \left[v_t^\pi + \varepsilon_t^\pi \right], \tag{9}$$

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma} \left[i_t - \mathrm{E}_t \pi_{t+1} \right] - \gamma \left[\mathrm{E}_t e_{t+1} - e_t \right] + \Sigma_x \left[v_t^x + \varepsilon_t^x \right], \tag{10}$$

$$e_t = \mathrm{E}_t e_{t+1} - \left[i_t - \mathrm{E}_t \pi_{t+1} \right] + \Sigma_e \left[v_t^e + \varepsilon_t^e \right]. \tag{11}$$

The specification errors v_t^j will be allowed to feed back from the state variables, so although the errors enter the model as additive shocks, they may well disturb the model in the same way as multiplicative parameter uncertainty (see Hansen and Sargent, 2004).[5] The central bank will design policy for the worst possible outcome of the model, where the evil agent chooses the amount of misspecification v_t^j optimally, given some constraints (to be specified below). This model will be referred to as the *worst-case model*, and is the outcome that the central bank fears the most and therefore wants policy to be robust against. The most likely outcome of the model, on the other hand, is one where the central bank sets policy and agents form expectations to reflect misspecification in the worst-case model, but there is no such misspecification in practice (so all v_t^j are zero). We will refer to this model as the *approximating model*.

The amount of misspecification, measured by v_t^j, is scaled by the parameter Σ_j, which determines the volatility of the shock in equation j. Intuitively, the specification error is disguised by the disturbance term ε_t^j, so if the disturbance has no variance, the specification error would be detected immediately. The larger is the variance of the disturbance, the larger can the specification error be without being detected.

3 Robust monetary policy

3.1 Setting up the control problem

The central bank is assumed to minimize a standard objective function which is quadratic in deviations of inflation and the output gap from their zero target levels. To design the robust policy, the central bank takes into account a certain degree

[5]Onatski and Williams (2003) stress that the Hansen-Sargent approach to robustness does not capture all types of parameter uncertainty, and that the "robust" rules may be fragile to certain sources of uncertainty that are not captured by the robust control approach.

of model misspecification by minimizing its objective function in the worst possible model within a given set of plausible models. Depending on its preference for robustness, the central bank allocates a budget η_j to the evil agent, which is used to create misspecification in equation j. In contrast to Hansen and Sargent (2004) and Giordani and Söderlind (2003), we will distinguish between different sources of model misspecification, by allowing the evil agent to have different budget constraints for the different controls. Thus the budget constraints are

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t (v_t^\pi)^2 \leq \eta_\pi, \tag{12}$$

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t (v_t^x)^2 \leq \eta_x, \tag{13}$$

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t (v_t^e)^2 \leq \eta_e. \tag{14}$$

In a standard control problem we would have $\eta_j = 0$ for all j, while the standard robust control problem would have $\eta_j = \eta$ for all j. Here, in addition to analyzing the general effects of misspecification, letting all η_j be positive, we can also analyze specification error in one equation at a time by setting one $\eta_j > 0$ and the other two to zero.

Following Hansen and Sargent (2004) the robust monetary policy is obtained by solving the minmax problem

$$\min_{\{i_t\}} \max_{\{v_t^j\}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda x_t^2\right] \tag{15}$$

subject to the misspecified model (9)–(11) and the evil agent's budget constraints (12)–(14). The central bank thus sets the interest rate to minimize the value of its intertemporal loss function, while the evil agent sets its controls to maximize the central bank's loss, given the constraints on misspecification.

The Lagrangian for this problem is given by

$$\begin{aligned} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ & \pi_t^2 + \lambda x_t^2 - \theta_\pi \left(v_t^\pi\right)^2 - \theta_\pi \left(v_t^x\right)^2 - \theta_\pi \left(v_t^e\right)^2 \\ & -\mu_t^\pi \Big[\pi_t - \beta \mathrm{E}_t \pi_{t+1} - \kappa x_t - \alpha e_t - \Sigma_\pi v_t^\pi - \Sigma_\pi \varepsilon_t^\pi\Big] \\ & -\mu_t^x \Big[x_t - \mathrm{E}_t x_{t+1} + \sigma^{-1}\left(i_t - \mathrm{E}_t \pi_{t+1}\right) \\ & +\gamma \left(\mathrm{E}_t e_{t+1} - e_t\right) - \Sigma_x v_t^x - \Sigma_x \varepsilon_t^x\Big] \\ & -\mu_t^e \Big[e_t - \mathrm{E}_t e_{t+1} + i_t - \mathrm{E}_t \pi_{t+1} - \Sigma_e v_t^e - \Sigma_e \varepsilon_t^e\Big] \Big\}, \end{aligned} \tag{16}$$

where μ_t^j are Lagrange multipliers on the constraints (9)–(11) and θ_j determine the set of models available to the evil agent that the policymaker wants to be robust against (i.e., the central bank's preference for robustness). These parameters are related to the evil agent's budget η_j: as $\eta_j \rightarrow 0$, $\theta_j \rightarrow \infty$, and the degree of misspecification approaches zero.

Throughout, we will focus on marginal amounts of model misspecification. For sufficiently large amounts of misspecification, the evil agent will be able to overturn any relationship in the model, so the approximating model (2)–(4) is not a good description of reality. We therefore want to consider reasonable degrees of model uncertainty that cannot be easily identified by the policymaker.[6] More specifically, we will analyze the effects of small increases in the preference for robustness starting from the non-robust policy, i.e., small decreases in θ_j starting from $\theta_j = \infty$.

3.2 Optimality conditions

We assume that neither the central bank nor the evil agent has access to any commitment mechanism. Consequently, we take expectations as given in the optimization and look for a discretionary equilibrium. From the first-order conditions we can derive the following optimality conditions relating inflation, output and the degree of misspecification to each other:

$$\begin{aligned} x_t &= -\left[\frac{\kappa}{\lambda} + \frac{\alpha}{(\gamma+\sigma^{-1})\lambda}\right]\pi_t, \\ &= -A\pi_t, && (17) \\ v_t^\pi &= \frac{\Sigma_\pi}{\theta_\pi}\pi_t, && (18) \\ v_t^x &= \frac{\Sigma_x}{\theta_x}\left[\lambda x_t + \kappa\pi_t\right], && (19) \\ v_t^e &= -\frac{\Sigma_e}{\sigma\theta_e}\left[\lambda x_t + \kappa\pi_t\right], && (20) \end{aligned}$$

where

$$A \equiv \frac{\kappa}{\lambda} + \frac{\alpha}{(\gamma+\sigma^{-1})\lambda}. \qquad (21)$$

[6] In numerical approaches to robust control, the amount of misspecification can be chosen such that the policymaker cannot distinguish between the approximating model and the worst-case model at reasonable statistical significance levels. See Hansen and Sargent (2004) and Giordani and Söderlind (2003).

Combining these equations we obtain

$$
\begin{aligned}
v_t^\pi &= \frac{\Sigma_\pi}{\theta_\pi}\pi_t, && (22)\\
v_t^x &= -\frac{\alpha\Sigma_x}{(\gamma+\sigma^{-1})\theta_x}\pi_t, && (23)\\
v_t^e &= \frac{\alpha\sigma^{-1}\Sigma_e}{(\gamma+\sigma^{-1})\theta_e}\pi_t. && (24)
\end{aligned}
$$

This immediately gives us our first set of results.

Proposition 1 (Optimal output–inflation trade-off)
The optimal output–inflation trade-off is not affected by the central bank's preference for robustness.

Proof See equation (17): the trade-off, measured by the coefficient A, is independent of all θ_j. □

Thus, the presence of model misspecification will not alter the central bank's optimal "targeting rule" in equation (17). However, as there is some misspecification in each equation, the optimal (reduced form) interest rate rule for the central bank will be affected by model uncertainty.

Proposition 2 (Misspecification and shocks)
Given the preference for robustness, the degree of misspecification in an equation depends positively on the variance of the shock associated with the equation.

Proof See equations (22)–(24): given θ_j, each v_j is increasing in Σ_j. □

Intuitively, the larger is the variance of a given shock, the more difficult it is for the central bank to identify misspecification in that particular equation. Therefore the central bank wants to guard against such specification errors.

Proposition 3 (Misspecification and inflation)
The degree of misspecification in all equations is larger when inflation is further away from steady state.

Proof See equations (22)–(24): all v_j increase (in absolute value) in π_t. □

The central bank fears all shocks that have inflationary effects as these force the central bank to reduce the output gap further to achieve the desired trade-off between inflation and the output gap. The evil agent adds to such shocks through misspecification in all equations. In the worst-case model, misspecification in the Phillips curve will increase inflation further when inflation is already high. Misspecification in the output equation forces output down when inflation is high, increasing the cost of counteracting an already high inflation rate. (See Walsh, 2004a, 2004b, for similar results.) The final misspecification, in the exchange rate equation, induces an exchange rate depreciation when inflation is high, leading to higher inflation and larger costs of achieving the desired trade-off between inflation and output.

From equations (22)–(24) we see that the Phillips curve is subject to misspecification in most parameterizations of the model. As long as $\Sigma_\pi > 0$ and the budget is non-zero ($\theta_\pi < \infty$), the evil agent will allocate misspecification to this equation. Indeed, in the closed-economy version of the model (when $\alpha = \gamma = 0$), the central bank will only fear misspecification in the inflation equation:

Proposition 4 (Misspecification in the closed economy)
In the closed-economy model, the policymaker only fears misspecification in the Phillips curve.

Proof Setting $\alpha = \gamma = 0$ in equations (22)–(24) gives $v_t^x = v_t^e = 0$. □

In the closed economy, the policymaker is able to counteract any specification errors in the output equation by an appropriate adjustment of the interest rate. As interest rate movements do not influence central bank loss independently, the central bank does not fear such specification errors.

In the open economy, however, the central bank cannot directly offset output shocks by changing the interest rate, as this would affect the exchange rate and therefore inflation (see Walsh, 1999). Thus, the existence of an exchange rate channel makes the output equation more prone to misspecification, and the policymaker will fear that output is low when inflation is high. This would make the central bank lower the interest rate, leading to an exchange rate depreciation that increases inflation even further.

The stronger is the effect of the interest rate on output (the smaller is σ), the more prone is the exchange rate equation to misspecification. When inflation is positive, the central bank fears that a real exchange rate depreciation will further increase inflation. In order to curb the effects on inflation, the interest rate would need to be increased, which would reduce output. This is particularly costly for the policymaker if the interest rate has a strong effect on output.

These effects of output and exchange rate misspecification are stronger when the exchange rate has a strong effect on inflation (so α is large). The central bank therefore fears such specification errors more when α is large. On the other hand, if the exchange rate has a sufficiently strong impact on output (so γ is large), the central bank will fear less misspecification of the output or exchange rate equations. The reason is that the exchange rate depreciation (caused by higher inflation) would offset some of the negative impact of higher interest rates on output. As γ approaches infinity, only misspecification in the inflation equation has consequences for central bank loss.

3.3 Solving the model

As there is no persistence in the model, the only state variables are the three shocks, $\varepsilon_t^\pi, \varepsilon_t^x$ and ε_t^e. We will thus look for a solution for the endogenous variables π_t, x_t, e_t, the central bank's control i_t, and the evil agent's controls v_t^π, v_t^x, v_t^e in terms of the three shocks. The solution of the *worst-case model* will be of the form

$$\begin{bmatrix} \pi_t \\ x_t \\ e_t \end{bmatrix} = \begin{bmatrix} a_\pi & a_x & a_e \\ b_\pi & b_x & b_e \\ c_\pi & c_x & c_e \end{bmatrix} \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \\ \varepsilon_t^e \end{bmatrix}, \tag{25}$$

the worst possible degree of misspecification will be given by

$$\begin{bmatrix} v_t^\pi \\ v_t^x \\ v_t^e \end{bmatrix} = \begin{bmatrix} \hat{a}_\pi & \hat{a}_x & \hat{a}_e \\ \hat{b}_\pi & \hat{b}_x & \hat{b}_e \\ \hat{c}_\pi & \hat{c}_x & \hat{c}_e \end{bmatrix} \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \\ \varepsilon_t^e \end{bmatrix}, \tag{26}$$

and the policy rule will be

$$i_t = d_\pi \varepsilon_t^\pi + d_x \varepsilon_t^x + d_e \varepsilon_t^e. \tag{27}$$

Finally, the approximating model, where policy is conducted according to (27), but there is no misspecification (so all v_t^j are zero), will be given by

$$\begin{bmatrix} \pi_t \\ x_t \\ e_t \end{bmatrix} = \begin{bmatrix} \bar{a}_\pi & \bar{a}_x & \bar{a}_e \\ \bar{b}_\pi & \bar{b}_x & \bar{b}_e \\ \bar{c}_\pi & \bar{c}_x & \bar{c}_e \end{bmatrix} \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \\ \varepsilon_t^e \end{bmatrix}. \tag{28}$$

To find these solutions, we begin by looking for the worst-case solution for π_t, x_t, e_t in (25) and the worst possible degree of misspecification in (26). Here we note that equation (17) and (22)–(24) imply that

$$b_j = -Aa_j, \tag{29}$$

$$\hat{a}_j = \frac{\Sigma_\pi}{\theta_\pi} a_j, \tag{30}$$

$$\hat{b}_j = -\frac{\alpha\Sigma_x}{(\kappa+\sigma^{-1})\theta_x} a_j, \tag{31}$$

$$\hat{c}_j = \frac{\alpha\sigma^{-1}\Sigma_e}{(\kappa+\sigma^{-1})\theta_e} a_j, \tag{32}$$

so we need only to solve for the coefficients a_j, c_j, d_j. Second, we will find the optimal policy rule (27). Third, we will find the solution for the approximating model (28) by using the optimal policy rule in the original model given by (2)–(4).

Note that we allow the evil agent only to respond to the same variables as the policymaker. This differs from the setup of Hansen and Sargent (2004) and Giordani and Söderlind (2003), where the evil agent is allowed to respond also to lagged state variables, thus introducing persistence in the shocks. In our setup, the evil agent is not allowed to introduce serial correlation in the shocks, as there is no such persistence from the outset. This assumption is mainly for tractability, but is also consistent with the assumption in both approaches that the evil agent is not allowed to introduce additional state variables to increase the degree of serial correlation in the endogenous variables.

3.3.1 The worst-case model

First, to find an expression for the interest rate, solve the output equation (10) for the interest rate i_t and substitute for x_t and v_t^x using the optimal trade-off in (17) and the worst possible output misspecification in (23), yielding

$$i_t = (1-\sigma A)\,\mathrm{E}_t\pi_{t+1} + \sigma B\pi_t - \sigma\gamma \mathrm{E}_t\Delta e_{t+1} + \sigma\Sigma_x\varepsilon_t^x, \tag{33}$$

where

$$B \equiv A - \frac{\alpha\Sigma_x^2}{(\gamma+\sigma^{-1})\theta_x} >_\theta 0, \tag{34}$$

where we use the notation $z >_\theta 0$ to denote that the coefficient z is positive when the degree for robustness approaches zero, i.e., $\lim_{\theta\to\infty} z > 0$, and vice versa for $z <_\theta 0$. Equation (33) is not a true reaction function due to the presence of non-predetermined variables (i.e., π_t and e_t) on the right-hand side, but is an optimal implicit instrument rule, using the terminology of Giannoni and Woodford (2003), although obtained under discretion rather than under commitment from a timeless perspective. In the closed-economy case, this rule is independent of the preference for robustness, similar to the results in Walsh (2004a,b). However, in the open economy this is no longer true, as the central bank also fears misspecification in the output equation. However, to derive the true policy reaction function we must therefore first solve for the forward-looking variables as functions of the underlying shocks.

Using the policy trade-off from (17) and the evil agent's control v_t^π from (22) in the Phillips curve (9), we obtain

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - \kappa A \pi_t + \alpha e_t + \frac{\Sigma_\pi^2}{\theta_\pi} \pi_t + \Sigma_\pi \varepsilon_t^\pi, \tag{35}$$

and collecting terms we get

$$C \pi_t = \beta \mathrm{E}_t \pi_{t+1} + \alpha e_t + \Sigma_\pi \varepsilon_t^\pi, \tag{36}$$

where

$$C \equiv 1 + \kappa A - \frac{\Sigma_\pi^2}{\theta_\pi} >_\theta 0. \tag{37}$$

Likewise, using the interest rate from (33) and the expression for v_t^e from (24) in the UIP condition (11) yields

$$(1 + \sigma\gamma) e_t = (1 + \sigma\gamma) \mathrm{E}_t e_{t+1} + \sigma A \mathrm{E}_t \pi_{t+1} - D \pi_t - \sigma \Sigma_x \varepsilon_t^x + \Sigma_e \varepsilon_t^e, \tag{38}$$

where

$$D \equiv \sigma B - \frac{\alpha \sigma^{-1} \Sigma_e^2}{(\gamma + \sigma^{-1}) \theta_e} >_\theta 0. \tag{39}$$

Note that B is decreasing in output uncertainty (increasing in θ_x), C is decreasing in inflation uncertainty, and D is decreasing in both output and exchange rate uncertainty.

To find the reduced form for inflation and the exchange rate, first write equations (36) and (38) as

$$\pi_t = a_1 \mathrm{E}_t \pi_{t+1} + a_2 e_t + a_3 \varepsilon_t^\pi, \tag{40}$$

$$e_t = \mathrm{E}_t e_{t+1} + c_1 \mathrm{E}_t \pi_{t+1} + c_2 \pi_t + c_3 \varepsilon_t^x + c_4 \varepsilon_t^e, \tag{41}$$

where

$$
\begin{aligned}
a_1 &\equiv \frac{\beta}{C}, \quad a_2 \equiv \frac{\alpha}{C}, \quad a_3 \equiv \frac{\Sigma_\pi}{C}, && (42)\\
c_1 &\equiv \frac{\sigma A}{1+\sigma\gamma}, \quad c_2 \equiv -\frac{D}{1+\sigma\gamma}, \quad c_3 \equiv -\frac{\sigma\Sigma_x}{1+\sigma\gamma}, \quad c_4 \equiv \frac{\Sigma_e}{1+\sigma\gamma}, && (43)
\end{aligned}
$$

and we seek a solution of the form

$$
\begin{aligned}
\pi_t &= a_\pi\varepsilon_t^\pi + a_x\varepsilon_t^x + a_e\varepsilon_t^e, && (44)\\
e_t &= c_\pi\varepsilon_t^\pi + c_x\varepsilon_t^x + c_e\varepsilon_t^e, && (45)
\end{aligned}
$$

where the coefficients remain to be determined.

Noting that $\mathrm{E}_t\pi_{t+1} = \mathrm{E}_t e_{t+1} = 0$ and combining (40)–(41) with (44)–(45) we can solve for the reduced-form coefficients as

$$
\begin{aligned}
a_\pi &= \frac{(1+\sigma\gamma)\Sigma_\pi}{E} >_\theta 0, && (46)\\
a_x &= -\frac{\sigma\alpha\Sigma_x}{E} <_\theta 0, && (47)\\
a_e &= \frac{\alpha\Sigma_e}{E} >_\theta 0, && (48)\\
c_\pi &= -\frac{D\Sigma_\pi}{E} <_\theta 0, && (49)\\
c_x &= -\frac{\sigma C\Sigma_x}{E} <_\theta 0, && (50)\\
c_e &= \frac{C\Sigma_e}{E} >_\theta 0, && (51)
\end{aligned}
$$

where

$$
\begin{aligned}
E &\equiv (1-a_2c_2)(1+\sigma\gamma)C\\
&= (1+\sigma\gamma)C + \alpha D >_\theta 0, && (52)
\end{aligned}
$$

and E is decreasing in all three types of uncertainty. See Appendix B for details.

Thus, for a small degree of uncertainty, inflation in the worst-case model is positively related to the inflation and exchange rate disturbances, but negatively related to the output disturbance. For the output gap, the coefficients are of the opposite sign (see equation (17)), so output is negatively related to the inflation and exchange rate disturbances, but positively related to the output disturbance. The exchange rate is positively related to the exchange rate disturbance, but negatively related to the inflation and output disturbances.

Equations (22)–(24) then imply that the worst possible degree of misspecification in the inflation and exchange rate equations is positively related to inflation and exchange rate disturbances, but negatively related to the output disturbance, while misspecification in the output equation is negatively related to inflation and exchange rate disturbances, but positively related to the output disturbance.

3.3.2 The policy rule

Using the solution for inflation and the exchange rate in the interest rate equation (33), the reduced-form solution for the interest rate is

$$\begin{aligned} i_t &= \sigma B \pi_t + \sigma \gamma e_t + \sigma \Sigma_x \varepsilon_t^x \\ &= d_\pi \varepsilon_t^\pi + d_x \varepsilon_t^x + d_e \varepsilon_t^e, \end{aligned} \quad (53)$$

where

$$d_\pi = \sigma \left[B a_\pi + \gamma c_\pi \right] >_\theta 0, \quad (54)$$

$$d_x = \sigma \left[B a_x + \gamma c_x + \Sigma_x \right] >_\theta 0, \quad (55)$$

$$d_e = \sigma \left[B a_e + \gamma c_e \right] >_\theta 0. \quad (56)$$

Thus, for small amounts of uncertainty, monetary policy responds positively to each disturbance: positive realizations of the inflation, output or exchange rate disturbances all make the central bank raise the interest rate. Again, see Appendix B for details.

The result that monetary policy is tightened after positive inflation or output disturbances is well-known from the closed-economy version of the model, see, e.g., Clarida et al. (1999). Here, policy is tightened also after a positive exchange rate disturbance: An exchange rate depreciation tends to increase domestic inflation, so by tightening policy, the central bank induces an immediate appreciation and an expected depreciation of the exchange rate, which reduces both inflation and output.

3.3.3 The approximating model

The solution for the worst-case model derived so far is the reduced form under the worst possible case of misspecification, so the evil agent uses its controls as efficiently as possible, and the policy rule and private agents' expectations reflect this misspecification. However, this is also a very unlikely model. We therefore follow Hansen and Sargent (2004) to find the most likely, or "approximating" model, where the policy rule and agents' expectations reflect the central bank's preference for robustness, but the actual misspecification is zero.

As in the worst-case model, expectations are zero. Thus we find the approximating model by using the optimal robust interest rate rule from equation (53) in the original model (2)–(4).[7] This yields

$$\begin{aligned}
\pi_t &= \kappa x_t + \alpha e_t + \Sigma_\pi \varepsilon_t^\pi, && (57)\\
x_t &= -\sigma^{-1} i_t + \gamma e_t + \Sigma_x \varepsilon_t^x, && (58)\\
e_t &= -i_t + \Sigma_e \varepsilon_t^e, && (59)
\end{aligned}$$

and the solution is

$$\begin{aligned}
\pi_t &= \bar{a}_\pi \varepsilon_t^\pi + \bar{a}_x \varepsilon_t^x + \bar{a}_e \varepsilon_t^e, && (60)\\
x_t &= \bar{b}_\pi \varepsilon_t^\pi + \bar{b}_x \varepsilon_t^x + \bar{b}_e \varepsilon_t^e, && (61)\\
e_t &= \bar{c}_\pi \varepsilon_t^\pi + \bar{c}_x \varepsilon_t^x + \bar{c}_e \varepsilon_t^e, && (62)
\end{aligned}$$

where

$$\begin{aligned}
\bar{a}_\pi &= \Sigma_\pi - \left[\alpha + \kappa(\gamma + \sigma^{-1})\right] d_\pi >_\theta 0, && (63)\\
\bar{a}_x &= \kappa \Sigma_x - \left[\alpha + \kappa(\gamma + \sigma^{-1})\right] d_x <_\theta 0, && (64)\\
\bar{a}_e &= (\alpha + \kappa\gamma)\Sigma_e - \left[\alpha + \kappa(\gamma + \sigma^{-1})\right] d_e >_\theta 0, && (65)\\
\bar{b}_\pi &= -(\gamma + \sigma^{-1}) d_\pi <_\theta 0, && (66)\\
\bar{b}_x &= \Sigma_x - (\gamma + \sigma^{-1}) d_x >_\theta 0, && (67)\\
\bar{b}_e &= \gamma \Sigma_e - (\gamma + \sigma^{-1}) d_e <_\theta 0, && (68)\\
\bar{c}_\pi &= -d_\pi <_\theta 0, && (69)\\
\bar{c}_x &= -d_x <_\theta 0, && (70)\\
\bar{c}_e &= \Sigma_e - d_e >_\theta 0. && (71)
\end{aligned}$$

Again see Appendix B for details.

Thus, in this most likely outcome of the model, a positive realization of the inflation shock makes the central bank tighten policy to counteract the inflationary impulse. This reduces the output gap and makes the real exchange rate appreciate. After a positive output shock, the central bank also tightens policy, leading to a real appreciation. In a closed economy, the central bank could offset all effects of the output shock on inflation, but here the real appreciation reduces inflation, so

[7] As policy is optimal only for the misspecified model, we can no longer use the optimal output–inflation trade-off (17) to determine the output gap.

inflation and the exchange rate are negatively related to the output shock. Finally, a positive exchange rate shock tends to increase inflation, so again the central bank tightens policy to offset these effects. This reduces the output gap, but the net effect on inflation is still positive.

3.4 The closed-economy solution

As an interesting special case, we can find the reduced-form solution for the closed-economy version of our model by setting $\alpha = \gamma = 0$. In this case the solution of the worst-case model is

$$\pi_t = \frac{\Sigma_\pi}{\hat{E}}\varepsilon_t^\pi, \tag{72}$$

$$y_t = -\frac{\kappa\Sigma_\pi}{\lambda\hat{E}}\varepsilon_t^\pi, \tag{73}$$

$$v_t^\pi = \frac{\Sigma_\pi^2}{\hat{E}\theta_\pi}\varepsilon_t^\pi, \tag{74}$$

where

$$\hat{E} = 1 + \frac{\kappa^2}{\lambda} - \frac{\Sigma_\pi^2}{\theta_\pi}. \tag{75}$$

The optimal policy rule is

$$i_t = \frac{\kappa\sigma\Sigma_\pi}{\lambda\hat{E}}\varepsilon_t^\pi + \sigma\Sigma_x\varepsilon_t^x, \tag{76}$$

and the solution for the approximating model is given by

$$\pi_t = \left[1 - \frac{\kappa^2\Sigma_\pi}{\lambda\hat{E}}\right]\Sigma_\pi\varepsilon_t^\pi, \tag{77}$$

$$x_t = -\frac{\kappa\Sigma_\pi}{\lambda\hat{E}}\varepsilon_t^\pi. \tag{78}$$

As shown in Proposition 4, the closed-economy solution is affected only by misspecification in the Phillips curve (θ_π), as the central bank directly offsets any misspecification in the output gap equation. We can then immediately evaluate the effects of increased uncertainty on inflation, output and monetary policy.

Proposition 5 (Worst-case closed-economy model)
In the worst-case closed-economy model, an increased preference for robustness makes the central bank fear that inflation and output are more volatile.

Proof As uncertainty about the Phillips curve increases (so θ_π falls), $\hat{E}$ falls, which increases the response of inflation and output to the inflation disturbance in equation (72) and (73). □

As the central bank wants to be robust against a larger set of models, its worst-case scenario has more noise in the Phillips curve, thus increasing the volatility of inflation. This will force the central bank to induce more volatility also in output, which further increases central bank loss.

This also implies the following:

Proposition 6 (Monetary policy in the closed economy)
In the closed-economy model, an increased preference for robustness makes monetary policy more aggressive.

Proof As θ_π is reduced and $\hat{E}$ falls, the policy response to the inflation disturbance in equation (76) increases. □

As inflation responds more strongly to its own disturbance, the central bank will have to respond more aggressively in order to control inflation. This result is reminiscent of that in Söderström (2002). Using a Bayesian approach in a backward-looking model, he shows that increased uncertainty about the persistence of inflation makes monetary policy more aggressive, as the central bank wants to reduce the probability that inflation moves away from the target in the future. Here there is no persistence in inflation, so current inflation has no information about its future path. But as the robust central bank fears that inflation is more responsive to shocks, the central bank responds more aggressively to the inflation shock. Output disturbances, on the other hand, are perfectly offset.

This more aggressive policy behavior implies that inflation is more insulated from the disturbance, while output becomes more volatile:

Proposition 7 (Approximating closed-economy model)
In the approximating closed-economy model, an increased preference for robustness makes inflation less variable and output more variable.

Proof As uncertainty increases and $\hat{E}$ falls, the coefficient in the inflation equation (77) becomes smaller, while that in the output equation (78) becomes larger. □

In the closed-economy model, policy responds more aggressively to inflation shocks when the central bank's preference for robustness increases. This policy response dampens the effects of the shock on inflation, but at the cost of more output volatility.

Similar results are obtained by Giordani and Söderlind (2003), who analyze optimal robust policy in the standard New-Keynesian model with persistence in the shocks. Using numerical methods they show that robustness makes the central bank fear that shocks are more persistent, and therefore the optimal robust policy is more aggressive than the non-robust policy. Although our model does not include persistence, we are able to show that robustness always leads to more aggressive policy, in any parameterization of the model.

4 The effects of model uncertainty in the open economy

We now turn to the main focus of our analysis: the effects of model misspecification on optimal monetary policy and the behavior of the economy in our open-economy model. We will thus analyze the effects on the model solution of an increase in the preference for robustness, i.e., a decrease in each θ_j. For instance, for the coefficient of inflation on the inflation shock, we will evaluate the derivative

$$\frac{\partial |a_\pi|}{\partial \theta_j}, \quad j = \pi, x, e, \tag{79}$$

that is, the marginal effects on the absolute value of the coefficient of a decrease in each θ_j.

First, we will see whether inflation, output and the exchange rate in the worst-case model are more or less sensitive to shocks under model uncertainty. Second, we will analyze the consequences for the optimal policy behavior, and see whether monetary policy is more or less aggressive under model uncertainty. Finally, we will demonstrate how an increased preference for robustness affects the macroeconomy in the approximating model. Some short proofs are presented here, while more extensive proofs are relegated to Appendix C.

4.1 The worst-case model of inflation and output

We begin by analyzing the effects of increased model uncertainty (i.e., an increased preference for robustness) on the worst-case model of inflation and output.

Proposition 8 (Worst-case inflation and output)
In the worst-case model, an increased preference for robustness against misspecification in any equation increases the response of inflation and output to all shocks.

Proof As shown above, the reduced-form coefficients for inflation in the worst-case model are given by

$$a_\pi = \frac{(1+\sigma\gamma)\Sigma_\pi}{E}; \quad a_x = -\frac{\sigma\alpha\Sigma_x}{E}; \quad a_e = \frac{\alpha\Sigma_e}{E}, \tag{80}$$

and the output coefficients are given by $b_j = -Aa_j$ for all j. Thus, all coefficients depend negatively (in absolute value) on E, and the effects of increased robustness (a decrease in any θ_j) on the absolute value of all coefficients have the opposite sign relative to the effects on the coefficient E. These effects are given by

$$-\frac{\partial E}{\partial\theta_\pi} = -\frac{(1+\sigma\gamma)\Sigma_\pi^2}{\theta_\pi^2}; \quad -\frac{\partial E}{\partial\theta_x} = -\frac{\alpha^2\sigma\Sigma_x^2}{(\gamma+\sigma^{-1})\theta_x^2}; \quad -\frac{\partial E}{\partial\theta_e} = -\frac{\alpha^2\sigma^{-1}\Sigma_e^2}{(\gamma+\sigma^{-1})\theta_e^2}, \tag{81}$$

which are all negative. Therefore, the inflation and output coefficients all increase in absolute value when uncertainty about any equation increases (any θ_j falls). □

Thus, with an increased preference for robustness, the central bank fears that inflation and output are more sensitive to shocks, and therefore more volatile. As we do not allow for shock persistence, the central bank fears only that shocks have larger effect on inflation and output.

4.2 The exchange rate and monetary policy

The effects of model uncertainty on the exchange rate in the worst-case model are intimately related to the effects on monetary policy. We therefore discuss these in parallel.

First, as uncertainty about the Phillips curve increases, the central bank will fear that inflation is more responsive to shocks. Therefore, after a positive shock to inflation, the central bank will tighten policy more, leading to a larger exchange rate appreciation in the worst-case model. After a positive exchange rate shock, again the central bank fears that the effects on inflation will be larger, and tightens policy more, leading to a smaller depreciation of the exchange rate. After a positive demand shock, the central bank fears that its policy response will lead to a larger fall in inflation. Therefore, the central bank tightens policy less than if there were no inflation uncertainty, leading to a smaller exchange rate appreciation in the worst-case model.

If the central bank is more uncertain about the determination of output, it fears that shocks have a larger effect on the output gap. A positive inflation shock then leads it to tighten policy less, implying a smaller exchange rate appreciation. After a positive output shock the central bank fears that output will increase further, so the interest rate is increased more, and the exchange rate depreciates by more than when there is no output uncertainty. A positive exchange rate shock leads the central bank to tighten policy to reduce the output gap and inflation and offset the exchange rate depreciation. If output is more uncertain, however, the central bank will tighten policy less, leading to a larger depreciation in the worst-case model.

Finally, if the central bank worries about misspecification in the exchange rate equation, it fears that the exchange rate is very sensitive to shocks. Therefore, after a positive inflation or exchange rate shock, it will tighten policy more. In the worst-case model, this leads to a smaller exchange rate appreciation after an inflation shock and a larger depreciation after an exchange rate shock. After a positive output shock, on the other hand, the central bank will not tighten policy as much to avoid large effects on the exchange rate. In the worst-case model, the net effect is a larger exchange rate appreciation.

These results can be summarized as follows.

Proposition 9 (Worst-case exchange rate under inflation uncertainty)
In the worst-case model, a larger preference for robustness against inflation uncertainty makes the exchange rate more sensitive to inflation shocks, but less sensitive to output and exchange rate shocks.

Proof See Appendix C.1.

Proposition 10 (Worst-case exchange rate under output/exchange rate uncertainty)
In the worst-case model, a larger preference for robustness against output or exchange rate uncertainty makes the exchange rate more sensitive to output and exchange rate shocks, but less sensitive to inflation shocks.

Proof See Appendix C.1.

Proposition 11 (Monetary policy under inflation/exchange rate uncertainty)
A larger preference for robustness against inflation or exchange rate uncertainty makes monetary policy respond more aggressively to inflation and exchange rate shocks, but less aggressively to output shocks.

Proof See Appendix C.2.

Proposition 12 (Monetary policy under output uncertainty)
A larger preference for robustness against output uncertainty makes monetary policy respond more aggressively to output shocks, but less aggressively to inflation and exchange rate shocks.

Proof See Appendix C.2.

In general, we see that there is an ambiguous effect of increased uncertainty (an increased preference for robustness) on the optimal monetary policy rule. Depending on the type of shock or the source of uncertainty, an increased preference for robustness can make policy more or less aggressive in response to shocks.

4.3 The approximating model

Finally, we analyze the effects on the most likely development of the macroeconomy when there is an increase in the central bank's preference for robustness. Note that the conclusions regarding monetary policy in Propositions 11 and 12 hold also in the approximating model.

Proposition 13 (Approximating inflation and output model)
In the approximating model, an increased preference for robustness against inflation or exchange rate uncertainty makes inflation less sensitive and output more sensitive to all shocks, but increased robustness against output uncertainty has the opposite effect.

Proof The inflation coefficients on the inflation and exchange rate shocks are both positive, so increased robustness has opposite effects on these coefficients relative to the coefficients in the policy rule, see equations (63) and (65). The inflation coefficient on the output shock is negative, so the effects on this coefficient are of the same sign as on the coefficient in the policy rule, see equation (64). The effects on the output coefficients will always be of the opposite sign relative to the inflation coefficients, see equations (66)– (68) □

Proposition 14 (Approximating exchange rate model)
In the approximating model, increased robustness against inflation or exchange rate uncertainty makes the exchange rate more sensitive to inflation shocks, but less sensitive to output and exchange rate shocks. Increased robustness against output uncertainty makes the exchange rate more sensitive to output and the exchange rate shocks but less sensitive to inflation shocks.

Proof The effects on the exchange rate coefficients on the inflation and output shocks will be of the same sign as on the inflation and output coefficients in the policy rule, see equations (69) and (70). The effects on the coefficients on the exchange rate shock will be of the opposite sign relative to the exchange rate coefficient in the policy rule, see equation (71). □

4.4 Summary

Table 1 summarizes the effects of increased model uncertainty (an increased preference for robustness) on the reduced-form coefficients. The third column shows the sign of each coefficient, and the next three columns show the effects of an increase in model uncertainty (so a decrease in the θ's) on the absolute values of the reduced-form coefficients. Thus, a positive sign implies that the variable in question is more sensitive to that particular shock when uncertainty increases, and vice versa.

We see that robustness against exchange rate uncertainty has qualitatively very similar effects as robustness against inflation uncertainty. The only exception regards the effects on the exchange rate in the worst-case model. On the other hand, robustness against output uncertainty always has the opposite effects on policy and the approximating model relative to inflation and exchange rate uncertainty.

From Table 1 it is again clear that the effects of model uncertainty on monetary policy are ambiguous: A robust policymaker may respond more or less aggressively to shocks than a non-robust policymaker, depending on both the shock and the source of uncertainty.

5 Concluding remarks

Using a simple model of a small open economy we have analyzed how optimal monetary policy and the behavior of the economy are affected by the central bank's wish to be robust against model misspecification. Our simple model enables us to solve analytically for the optimal robust policy, as well as the central bank's worst-case model and the most likely approximating model. Our framework also allows us to analyze cases when the policymaker is more confident about some equations in the model than others. It thus restricts the evil agent to introduce misspecification where it will hurt the most, but forces it to consider misspecification in equations that are perceived to be particularly uncertain.

Our analysis of the open-economy model shows that an increase in the central

Table 1: Effects of increased uncertainty on reduced-form coefficients

Equation	Coefficient on	Sign	Source of uncertainty		
			Inflation (θ_π)	Output (θ_x)	Exchange rate (θ_e)
			Worst-case model		
Inflation (π_t)	Inflation (a_π)	+	+	+	+
	Output (a_x)	−	+	+	+
	Exchange rate (a_e)	+	+	+	+
Output (x_t)	Inflation (b_π)	−	+	+	+
	Output (b_x)	+	+	+	+
	Exchange rate (b_e)	−	+	+	+
Exchange rate (e_t)	Inflation (c_π)	−	+	−	−
	Output (c_x)	−	−	+	+
	Exchange rate (c_e)	+	−	+	+
			Policy rule		
Interest rate (i_t)	Inflation (d_π)	+	+	−	+
	Output (d_x)	+	−	+	−
	Exchange rate (d_e)	+	+	−	+
			Approximating model		
Inflation (π_t)	Inflation ($\bar{a}_\pi$)	+	−	+	−
	Output ($\bar{a}_x$)	−	−	+	−
	Exchange rate ($\bar{a}_e$)	+	−	+	−
Output (x_t)	Inflation ($\bar{b}_\pi$)	−	+	−	+
	Output ($\bar{b}_x$)	+	+	−	+
	Exchange rate ($\bar{b}_e$)	−	+	−	+
Exchange rate (e_t)	Inflation ($\bar{c}_\pi$)	−	+	−	+
	Output ($\bar{c}_x$)	−	−	+	−
	Exchange rate ($\bar{c}_e$)	+	−	+	−

For each coefficient in the reduced-form model, Column 3 shows the sign of the coefficient, and Columns 4–6 show the effects of an increased central bank preference for robustness on the absolute value of the coefficient. Thus, +/− implies that model uncertainty makes the variable in question more/less sensitive to that particular shock.

bank's preference for robustness has ambiguous effects on the optimal policy behavior, depending not only on the shock to which the central bank responds, but also on what part of the model the central bank perceives as most uncertain. Although our model is highly stylized, we believe this ambiguity to carry over also to more elaborate models. In numerical applications the effects of increased uncertainty will therefore depend crucially on the calibration of the parameters that determine the central bank's relative faith in the different model equations.

In the closed-economy version of our model, we show that optimal robust policy is always more aggressive than the non-robust policy, confirming the results of previous research. However, although our model is more restrictive than models that include persistent shocks, our analytical approach shows that this result holds in any parameterization of the model, and not only in typical parameterizations.

Key parameters in our approach are the different preferences for robustness relating to the different equations in the model. We envision that future research can use Bayesian techniques in distributing the budgets of misspecification among the model equations based on the probability that each equation is a good representation of human behaviour. This would be a step towards integrating Bayesian and Knightian uncertainty into a single unifying analysis of model uncertainty.

A Model appendix

This Appendix briefly derives our open-economy model from microfoundations. For more details, see Galí and Monacelli (2004), Clarida et al. (2002), or Walsh (2003, Ch. 6.5), who provides a textbook treatment. We deviate from these authors by introducing a time-varying premium on foreign exchange, in order to analyze uncertainty about exchange rate determination.

A.1 Domestic households

Households in the home country consume a CES composite of domestic goods (C_t^d) and imported foreign goods (C_t^m), defined as

$$C_t = \left[(1-\omega)^{1/\delta}(C_t^d)^{(\delta-1)/\delta} + \omega^{1/\delta}(C_t^m)^{(\delta-1)/\delta}\right]^{\delta/(\delta-1)}, \tag{A1}$$

where ω is the share of foreign goods in consumption and δ is the elasticity of substitution across domestic and foreign goods. Households obtain utility from consumption and disutility from supplying labor (N_t) according to

$$U(C_t, N_t) = \frac{C_t^{1-\hat{\sigma}}}{1-\hat{\sigma}} - \frac{N_t^{1+\eta}}{1+\eta}, \tag{A2}$$

where $\hat{\sigma}$ is the elasticity of intertemporal substitution and η is the elasticity of labor supply.

The household chooses paths of consumption, labor supply, and holdings of one-period domestic bonds, which pay the nominal interest rate i_t and foreign bonds, which pay the risk-adjusted interest rate $\exp(\phi_t)i_t^f$, where ϕ_t is a time-varying premium on foreign bond holdings. Intertemporal optimization then gives the log-linearized consumption Euler condition

$$c_t = \mathrm{E}_t c_{t+1} - \frac{1}{\hat{\sigma}}\left[i_t - \mathrm{E}_t \pi_{t+1}^c\right], \tag{A3}$$

where β is the household's discount factor and π_t^c is the consumer price inflation rate, defined as $\pi_t^c \equiv p_t^c - p_{t-1}^c$, where the CPI is given by

$$p_t^c = (1-\omega)p_t + \omega p_t^m, \tag{A4}$$

where p_t and p_t^m are the price levels for domestic and imported goods.

Optimal allocation across domestic and foreign bond holdings gives the uncovered interest parity (UIP) condition

$$i_t = i_t^f + \mathrm{E}_t \Delta s_{t+1} + \phi_t, \tag{A5}$$

where s_t is the nominal exchange rate, and ϕ_t is the premium on foreign exchange. The optimal labor-leisure choice implies that

$$\eta n_t + \hat{\sigma} c_t = w_t - p_t^c. \tag{A6}$$

where w_t is the nominal wage. Finally, relative demand for domestic and imported goods satisfies

$$c_t^d - c_t^m = -\delta \left[p_t - p_t^m \right]. \tag{A7}$$

We define the real exchange rate in terms of the domestic price level as

$$e_t = s_t + p_t^f - p_t, \tag{A8}$$

which, assuming that the law of one price holds, is equal to the terms of trade $p_t^m - p_t$. Then we can express the UIP condition (A5) in real terms as

$$i_t - \mathrm{E}_t \pi_{t+1} = i_t^f - \mathrm{E}_t \pi_{t+1}^f + \mathrm{E}_t \Delta e_{t+1} + \phi_t. \tag{A9}$$

We can then also write the CPI in (A4) as

$$p_t^c = p_t + \omega e_t, \tag{A10}$$

the CPI inflation rate as

$$\pi_t^c = \pi_t + \omega \Delta e_t, \tag{A11}$$

and the labor supply condition in (A6) as

$$\eta n_t + \hat{\sigma} c_t = w_t - p_t - \omega e_t. \tag{A12}$$

Log-linearizing the consumption index (A1), we get

$$c_t = (1 - \omega) c_t^d + \omega c_t^m, \tag{A13}$$

and combining with (A7) and (A8) to eliminate c_t^m gives

$$c_t \;\; = \;\; c_t^d - \omega \delta e_t. \tag{A14}$$

A.2 Domestic firms

Domestic firms act under monopolistic competition and produce a differentiated good using only labor inputs according to the production function

$$Y_t = \exp(a_t) N_t, \tag{A15}$$

where a_t is a productivity disturbance.

Firms face a constant elasticity demand curve for its output, and also face sticky prices, following Calvo (1983), so in each period there is a fixed probability $1-\theta$ that the firm will be able to change its price. When prices can be adjusted, firms maximize the expected discounted value of profits. This implies that inflation in the domestic sector follows the New-Keynesian Phillips curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \hat{\kappa} v_t, \tag{A16}$$

where $\hat{\kappa} \equiv (1-\theta)(1-\beta\theta)/\theta$ and v_t is real marginal cost, given by

$$v_t = w_t - p_t - a_t, \tag{A17}$$

and where $w_t - p_t$ is the real product wage, which is deflated by the domestic price level.

A.3 The foreign country

Foreign demand for domestic goods is given by

$$c_t^{df} = y_t^f + \delta e_t, \tag{A18}$$

where y_t^f is foreign income (or output), which satisfies the Euler condition

$$y_t^f = \mathrm{E}_t y_{t+1}^f - \frac{1}{\hat{\sigma}} \left[i_t^f - \mathrm{E}_t \pi_{t+1}^f \right]. \tag{A19}$$

A.4 Equilibrium

Equilibrium requires that production equal consumption, so the production of domestic goods satisfies

$$\begin{aligned} y_t &= (1-\omega) c_t^d + \omega c_t^{df} \\ &= (1-\omega) c_t + (2-\omega)\omega\delta e_t + \omega y_t^f, \end{aligned} \tag{A20}$$

using (A14) and (A18), and combining with the consumption Euler equation (A3) we obtain

$$y_t = \mathrm{E}_t y_{t+1} - \frac{1-\omega}{\hat{\sigma}} \left[i_t - \mathrm{E}_t \pi_{t+1}^c \right] - (2-\omega)\omega\delta \mathrm{E}_t \Delta e_{t+1} - \omega \mathrm{E}_t \Delta y_{t+1}^f. \tag{A21}$$

Denoting by $\bar{z}$ the flexible-price level of the variable z, the flexible-price equilibrium is characterized by the goods market equilibrium condition

$$\bar{y}_t = \bar{c}_t, \tag{A22}$$

the labor market equilibrium condition

$$a_t = (\hat{\sigma} + \eta)\bar{y}_t - \eta a_t + \omega \bar{e}_t, \tag{A23}$$

where we have combined equations (A12), (A17), the log-linearized production function $y_t = a_t + n_t$, and (A22). Assuming that the foreign exchange premium is zero in the flexible-price equilibrium, the real UIP condition (A9), the Euler equation (A21), and the foreign Euler equation (A19) imply that the real interest rate satisfies

$$\begin{aligned} i_t - \mathrm{E}_t \pi_{t+1} &= i_t^f - \mathrm{E}_t \pi_{t+1}^f + \mathrm{E}_t \Delta \bar{e}_{t+1} \\ &= \frac{\hat{\sigma}}{1-\omega} \mathrm{E}_t \Delta \bar{y}_{t+1} - \frac{(2-\omega)\omega\delta\hat{\sigma}}{1-\omega} \mathrm{E}_t \Delta e_{t+1} - \frac{\omega}{1-\omega} \left[i_t^f - \mathrm{E}_t \pi_{t+1}^f \right]. \end{aligned} \tag{A24}$$

Assuming that all disturbances are white noise, all expectations of future variables are zero, so (A24) gives

$$\bar{e}_t = \Psi \left[\bar{y}_t - y_t^f \right], \tag{A25}$$

where

$$\Psi \equiv \frac{\hat{\sigma}}{1 - \omega + (2-\omega)\omega\delta\hat{\sigma}}, \tag{A26}$$

and the labor-market equilibrium condition (A23) then implies that

$$\bar{y}_t = \frac{1}{\hat{\sigma} + \eta + \omega\Psi} \left[(1+\eta) a_t - \omega \Psi y_t^f \right]. \tag{A27}$$

A.5 The final steps

Combining the expression for marginal cost in (A17), the labor supply condition (A12), and using $c_t = y_t = a_t + n_t$ we can express real marginal cost as

$$\begin{aligned} v_t &= \eta n_t + \hat{\sigma} c_t + \omega e_t - a_t \\ &= (\hat{\sigma} + \eta) y_t - \eta a_t + \omega e_t - a_t, \end{aligned} \tag{A28}$$

and in the flexible-price equilibrium, the marginal product of labor satisfies

$$a_t = (\hat{\sigma} + \eta)\bar{y}_t - \eta a_t + \omega \bar{e}_t, \tag{A29}$$

so

$$v_t = (\hat{\sigma} + \eta)x_t + \omega \left[e_t - \bar{e}_t\right], \tag{A30}$$

where x_t is the output gap, defined as

$$x_t \equiv y_t - \bar{y}_t, \tag{A31}$$

and where the flexible-price level of the real exchange rate is, combining (A25) and (A27)

$$\bar{e}_t = \frac{(1+\eta)\Psi}{\hat{\sigma} + \eta + \omega\Psi} a_t - \left[\Psi + \frac{\omega\Psi}{\hat{\sigma} + \eta + \omega\Psi}\right] y_t^f. \tag{A32}$$

This implies that we can write the Phillips curve (A16) as

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \hat{\kappa}(\hat{\sigma} + \eta)x_t + \hat{\kappa}\omega e_t - \hat{\kappa}\omega\bar{e}_t, \tag{A33}$$

and the Euler equation (A21) can be written as

$$\begin{aligned}
x_t &= \mathrm{E}_t x_{t+1} - \frac{1-\omega}{\hat{\sigma}}\left[i_t - \mathrm{E}_t \pi_{t+1}^c\right] - (2-\omega)\omega\delta \mathrm{E}_t \Delta e_{t+1} \\
&+ \mathrm{E}_t \Delta \bar{y}_{t+1} - \omega \mathrm{E}_t \Delta y_{t+1}^f \\
&= \mathrm{E}_t x_{t+1} - \frac{1-\omega}{\hat{\sigma}}\left[i_t - \mathrm{E}_t \pi_{t+1}\right] - \left[(2-\omega)\omega\delta - \frac{\omega(1-\omega)}{\hat{\sigma}}\right] \mathrm{E}_t \Delta e_{t+1} \\
&+ \mathrm{E}_t \Delta \bar{y}_{t+1} - \omega \mathrm{E}_t \Delta y_{t+1}^f.
\end{aligned} \tag{A34}$$

Finally, setting all foreign variables to zero, equations (A33), (A34) and (A9) give a complete description of the small open economy:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + \alpha e_t + \Sigma_\pi \varepsilon_t^\pi, \tag{A35}$$

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left[i_t - \mathrm{E}_t \pi_{t+1}\right] - \gamma\left[\mathrm{E}_t e_{t+1} - e_t\right] + \Sigma_x \varepsilon_t^x, \tag{A36}$$

$$e_t = \mathrm{E}_t e_{t+1} - \left[i_t - \mathrm{E}_t \pi_{t+1}\right] + \Sigma_e \varepsilon_t^e, \tag{A37}$$

where

$$\kappa \equiv \frac{(\hat{\sigma} + \eta)(1-\theta)(1-\beta\theta)}{\theta}, \tag{A38}$$

$$\alpha \equiv \frac{\omega(1-\theta)(1-\beta\theta)}{\theta}, \tag{A39}$$

$$\sigma \equiv \frac{\hat{\sigma}}{1-\omega}, \tag{A40}$$

$$\gamma \equiv (2-\omega)\omega\delta - \frac{\omega(1-\omega)}{\hat{\sigma}}, \tag{A41}$$

$$\varepsilon_t^\pi \equiv -\frac{(1-\theta)(1-\beta\theta)(1+\eta)\omega\Psi}{(\hat{\sigma}+\eta+\omega\Psi)\theta\Sigma_\pi} a_t, \tag{A42}$$

$$\begin{aligned} \varepsilon_t^x &\equiv \frac{1}{\Sigma_x}\mathrm{E}_t\left[\bar{y}_{t+1}-\bar{y}_t\right] \\ &= \frac{1+\eta}{(\hat{\sigma}+\eta+\omega\Psi)\Sigma_x}\left[\mathrm{E}_t a_{t+1}-a_t\right], \end{aligned} \tag{A43}$$

$$\varepsilon_t^e \equiv \frac{1}{\Sigma_e}\phi_t. \tag{A44}$$

B The reduced form

B.1 The worst-case model

To find the reduced from for inflation and the exchange rate in the worst-case model, first write equations (36) and (38) as

$$\pi_t = a_1 \mathrm{E}_t \pi_{t+1} + a_2 e_t + a_3 \varepsilon_t^{\pi}, \tag{B1}$$

$$e_t = \mathrm{E}_t e_{t+1} + c_1 \mathrm{E}_t \pi_{t+1} + c_2 \pi_t + c_3 \varepsilon_t^{x} + c_4 \varepsilon_t^{e}, \tag{B2}$$

where

$$a_1 \equiv \frac{\beta}{C}, \tag{B3}$$

$$a_2 \equiv \frac{\alpha}{C}, \tag{B4}$$

$$a_3 \equiv \frac{\Sigma_\pi}{C}, \tag{B5}$$

$$c_1 \equiv \frac{\sigma A}{1+\sigma\gamma}, \tag{B6}$$

$$c_2 \equiv -\frac{D}{1+\sigma\gamma}, \tag{B7}$$

$$c_3 \equiv -\frac{\sigma \Sigma_x}{1+\sigma\gamma}, \tag{B8}$$

$$c_4 \equiv \frac{\Sigma_e}{1+\sigma\gamma}, \tag{B9}$$

and we seek a solution of the form

$$\pi_t = a_\pi \varepsilon_t^{\pi} + a_x \varepsilon_t^{x} + a_e \varepsilon_t^{e}, \tag{B10}$$

$$e_t = c_\pi \varepsilon_t^{\pi} + c_x \varepsilon_t^{x} + c_e \varepsilon_t^{e}, \tag{B11}$$

where the a_j, c_j coefficients remain to be determined.

Noting that $\mathrm{E}_t \pi_{t+1} = \mathrm{E}_t e_{t+1} = 0$ and combining (B1)–(B2) with (B10)–(B11) we obtain

$$a_\pi \varepsilon_t^{\pi} + a_x \varepsilon_t^{x} + a_e \varepsilon_t^{e} = a_2 \left[c_\pi \varepsilon_t^{\pi} + c_x \varepsilon_t^{x} + c_e \varepsilon_t^{e} \right] + a_3 \varepsilon_t^{\pi}, \tag{B12}$$

$$c_\pi \varepsilon_t^{\pi} + c_x \varepsilon_t^{x} + c_e \varepsilon_t^{e} = c_2 \left[a_\pi \varepsilon_t^{\pi} + a_x \varepsilon_t^{x} + a_e \varepsilon_t^{e} \right] + c_3 \varepsilon_t^{x} + c_4 \varepsilon_t^{e}. \tag{B13}$$

Thus, the coefficients satisfy

$$a_\pi = a_2 c_\pi + a_3, \tag{B14}$$

$$
\begin{aligned}
a_x &= a_2 c_x, &\text{(B15)}\\
a_e &= a_2 c_e, &\text{(B16)}\\
c_\pi &= c_2 a_\pi, &\text{(B17)}\\
c_x &= c_2 a_x + c_3, &\text{(B18)}\\
c_e &= c_2 a_e + c_4, &\text{(B19)}
\end{aligned}
$$

and the solution of this system is

$$
\begin{aligned}
a_\pi &= a_2 c_2 a_\pi + a_3\\
&= \frac{a_3}{1 - a_2 c_2}, &\text{(B20)}\\
c_\pi &= \frac{a_3 c_2}{1 - a_2 c_2}, &\text{(B21)}\\
c_x &= c_2 a_2 c_x + c_3\\
&= \frac{c_3}{1 - a_2 c_2}, &\text{(B22)}\\
a_x &= \frac{a_2 c_3}{1 - a_2 c_2}, &\text{(B23)}\\
c_e &= c_2 a_2 c_e + c_4\\
&= \frac{c_4}{1 - a_2 c_2}, &\text{(B24)}\\
a_e &= \frac{a_2 c_4}{1 - a_2 c_2}. &\text{(B25)}
\end{aligned}
$$

Thus, the reduced-form coefficients are

$$
\begin{aligned}
a_\pi &= \frac{a_3}{1 - a_2 c_2} = \frac{(1 + \sigma\gamma)\Sigma_\pi}{E} >_\theta 0, &\text{(B26)}\\
a_x &= \frac{a_2 c_3}{1 - a_2 c_2} = -\frac{\sigma\alpha\Sigma_x}{E} <_\theta 0, &\text{(B27)}\\
a_e &= \frac{a_2 c_4}{1 - a_2 c_2} = \frac{\alpha\Sigma_e}{E} >_\theta 0, &\text{(B28)}\\
c_\pi &= \frac{a_3 c_2}{1 - a_2 c_2} = -\frac{D\Sigma_\pi}{E} <_\theta 0, &\text{(B29)}\\
c_x &= \frac{c_3}{1 - a_2 c_2} = -\frac{\sigma C\Sigma_x}{E} <_\theta 0, &\text{(B30)}\\
c_e &= \frac{c_4}{1 - a_2 c_2} = \frac{C\Sigma_e}{E} >_\theta 0, &\text{(B31)}
\end{aligned}
$$

where

$$
\begin{aligned}
E &\equiv (1 - a_2 c_2)(1 + \sigma\gamma)C\\
&= (1 + \sigma\gamma)C + \alpha D >_\theta 0. &\text{(B32)}
\end{aligned}
$$

We also note that

$$
\begin{aligned}
a_x &= -\frac{\sigma\alpha\Sigma_x}{(1+\sigma\gamma)\Sigma_\pi}a_\pi, && \text{(B33)}\\
a_e &= \frac{\alpha\Sigma_e}{(1+\sigma\gamma)\Sigma_\pi}a_\pi, && \text{(B34)}\\
c_\pi &= -\frac{D}{1+\sigma\gamma}a_\pi, && \text{(B35)}\\
c_x &= \frac{C}{\alpha}a_x \\
&= -\frac{\sigma C\Sigma_x}{(1+\sigma\gamma)\Sigma_\pi}a_\pi, && \text{(B36)}\\
c_e &= \frac{C}{\alpha}a_e \\
&= \frac{C\Sigma_e}{(1+\sigma\gamma)\Sigma_\pi}a_\pi. && \text{(B37)}
\end{aligned}
$$

B.2 The policy rule

Using the interest rate equation (33), the reduced form for the interest rate is

$$
\begin{aligned}
i_t &= \sigma B\pi_t + \sigma\gamma e_t + \sigma\Sigma_x\varepsilon_t^x, \\
&= \sigma B\left[a_\pi\varepsilon_t^\pi + a_x\varepsilon_t^x + a_e\varepsilon_t^e\right] + \sigma\gamma\left[c_\pi\varepsilon_t^\pi + c_x\varepsilon_t^x + c_e\varepsilon_t^e\right] + \sigma\Sigma_x\varepsilon_t^x, \\
&= d_\pi\varepsilon_t^\pi + d_x\varepsilon_t^x + d_e\varepsilon_t^e, && \text{(B38)}
\end{aligned}
$$

where

$$
\begin{aligned}
d_\pi &= \sigma\left[Ba_\pi + \gamma c_\pi\right], && \text{(B39)}\\
d_x &= \sigma\left[Ba_x + \gamma c_x + \Sigma_x\right], && \text{(B40)}\\
d_e &= \sigma\left[Ba_e + \gamma c_e\right]. && \text{(B41)}
\end{aligned}
$$

Using (B33)–(B37), we obtain

$$
\begin{aligned}
d_\pi &= \sigma\left[Ba_\pi + \gamma c_\pi\right] \\
&= \sigma\left[B - \frac{\gamma D}{1+\sigma\gamma}\right]a_\pi \\
&= F_\pi a_\pi >_\theta 0, && \text{(B42)}\\
d_x &= \sigma\left[Ba_x + \gamma c_x + \Sigma_x\right] \\
&= \sigma\left[B + \frac{\gamma C}{\alpha}\right]a_x + \sigma\Sigma_x \\
&= F_x a_\pi + \sigma\Sigma_x >_\theta, && \text{(B43)}
\end{aligned}
$$

$$
\begin{aligned}
d_e &= \sigma\left[Ba_e + \gamma c_e\right] \\
&= \sigma\left[B + \frac{\gamma C}{\alpha}\right] a_e \\
&= F_e a_\pi >_\theta, \qquad \text{(B44)}
\end{aligned}
$$

where

$$
\begin{aligned}
F_\pi &\equiv \sigma\left[B - \frac{\gamma D}{1+\sigma\gamma}\right] \\
&= \sigma\left[\left(1 - \frac{\sigma\gamma}{1+\sigma\gamma}\right) B + \frac{\sigma^{-1}\gamma\alpha\Sigma_e\Theta_e^{-1}}{1+\sigma\gamma}\right] \\
&= \frac{\sigma}{1+\sigma\gamma} B + \frac{\gamma\alpha\Sigma_e\Theta_e^{-1}}{1+\sigma\gamma} >_\theta 0, \qquad \text{(B45)} \\
F_x &\equiv -\sigma\left[B + \frac{\gamma C}{\alpha}\right] \frac{\sigma\alpha\Sigma_x}{(1+\sigma\gamma)\Sigma_\pi} \\
&= -\left[\alpha B + \gamma C\right] \frac{\sigma^2\Sigma_x}{(1+\sigma\gamma)\Sigma_\pi} <_\theta 0, \qquad \text{(B46)} \\
F_e &\equiv \sigma\left[B + \frac{\gamma C}{\alpha}\right] \frac{\alpha\Sigma_e}{(1+\sigma\gamma)\Sigma_\pi} \\
&= \left[\alpha B + \gamma C\right] \frac{\sigma\Sigma_e}{(1+\sigma\gamma)\Sigma_\pi} \\
&= -\frac{\Sigma_e}{\sigma\Sigma_x} F_x >_\theta 0. \qquad \text{(B47)}
\end{aligned}
$$

To verify that $d_x >_\theta 0$ even if $F_x <_\theta 0$, note that

$$
\begin{aligned}
d_x &= \sigma\left[\left(B + \frac{\gamma C}{\alpha}\right) a_x + \Sigma_x\right] \\
&= \sigma\left[\Sigma_x - (\alpha B + \gamma C)\frac{\sigma\Sigma_x}{E}\right] \\
&= \sigma\left[E - \sigma\alpha B - \sigma\gamma C\right]\frac{\Sigma_x}{E}, \qquad \text{(B48)}
\end{aligned}
$$

and

$$
\begin{aligned}
&E - \sigma\alpha B - \sigma\gamma C \\
&\quad = (1+\sigma\gamma)C + \alpha D - \sigma\alpha B - \sigma\gamma C \\
&\quad = C + \alpha\left(D - \sigma B\right) \\
&\quad = C - \sigma^{-1}\alpha^2\Sigma_e\Theta_e^{-1} >_\theta 0, \qquad \text{(B49)}
\end{aligned}
$$

using (39) and (52).

B.3 The approximating model

To find the solution for the approximating model, use the policy rule (B38)–(B41) in the equations for inflation, output and the exchange rate, setting all expectations to zero:[8]

$$
\begin{aligned}
i_t &= d_\pi \varepsilon_t^\pi + d_x \varepsilon_t^x + d_e \varepsilon_t^e, && \text{(B50)}\\
\pi_t &= \kappa x_t + \alpha e_t + \Sigma_\pi \varepsilon_t^\pi, && \text{(B51)}\\
x_t &= -\sigma^{-1} i_t + \gamma e_t + \Sigma_x \varepsilon_t^x, && \text{(B52)}\\
e_t &= -i_t + \Sigma_e \varepsilon_t^e. && \text{(B53)}
\end{aligned}
$$

The solution is

$$
\begin{aligned}
\pi_t &= \bar{a}_\pi \varepsilon_t^\pi + \bar{a}_x \varepsilon_t^x + \bar{a}_e \varepsilon_t^e, && \text{(B54)}\\
x_t &= \bar{b}_\pi \varepsilon_t^\pi + \bar{b}_x \varepsilon_t^x + \bar{b}_e \varepsilon_t^e, && \text{(B55)}\\
e_t &= \bar{c}_\pi \varepsilon_t^\pi + \bar{c}_x \varepsilon_t^x + \bar{c}_e \varepsilon_t^e, && \text{(B56)}
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{c}_\pi &= -d_\pi <_\theta 0, && \text{(B57)}\\
\bar{c}_x &= -d_x <_\theta 0, && \text{(B58)}\\
\bar{c}_e &= \Sigma_e - d_e \\
&= \Sigma_e - [\alpha B + \gamma C] \frac{\sigma \Sigma_e}{E} \\
&= [E - \alpha\sigma B - \sigma\gamma C] \frac{\Sigma_e}{E} >_\theta 0, && \text{(B59)}\\
\bar{b}_\pi &= \gamma \bar{c}_\pi - \sigma^{-1} d_\pi \\
&= -(\gamma + \sigma^{-1}) \left[\sigma B + \alpha\gamma \Sigma_e \Theta_e^{-1}\right] \frac{\Sigma_\pi}{E} <_\theta 0, && \text{(B60)}\\
\bar{b}_x &= \Sigma_x + \gamma \bar{c}_x - \sigma^{-1} d_x \\
&= \Sigma_x - \sigma(\gamma + \sigma^{-1}) [E - \alpha\sigma B - \sigma\gamma C] \frac{\Sigma_x}{E} \\
&= \left\{(1 + \sigma\gamma) C + \alpha D - \sigma(\gamma + \sigma^{-1}) \left[C - \sigma^{-1}\alpha^2 \Sigma_e \Theta_e^{-1}\right]\right\} \frac{\Sigma_x}{E} \\
&= \left[\alpha D + (\gamma + \sigma^{-1}) \alpha^2 \Sigma_e \Theta_e^{-1}\right] \frac{\Sigma_x}{E} >_\theta 0, && \text{(B61)}\\
\bar{b}_e &= \gamma \bar{c}_e - \sigma^{-1} d_e
\end{aligned}
$$

[8] Note that in the approximating model, the optimal inflation–output tradeoff $x_t = -A\pi_t$ does not hold, since policy is not optimal for this model.

$$
\begin{aligned}
&= \gamma\Sigma_e - (\gamma+\sigma^{-1})\,[\alpha B + \gamma C]\,\frac{\sigma\Sigma_e}{E} \\
&= \Big\{\gamma E - \sigma(\gamma+\sigma^{-1})\,[\alpha B + \gamma C]\Big\}\,\frac{\Sigma_e}{E} \\
&= \Big\{\gamma\Big[C - \sigma^{-1}\alpha^2\Sigma_e\Theta_e^{-1}\Big] - [\alpha B + \gamma C]\Big\}\,\frac{\Sigma_e}{E} \\
&= -\Big[\alpha B + \gamma\sigma^{-1}\alpha^2\Sigma_e\Theta_e^{-1}\Big]\,\frac{\Sigma_e}{E} <_\theta 0, \qquad \text{(B62)} \\
\bar{a}_\pi &= \Sigma_\pi + \kappa\bar{b}_\pi + \alpha\bar{c}_\pi \\
&= \Sigma_\pi - \Big[\alpha + \kappa(\gamma+\sigma^{-1})\Big]\,\Big[\sigma B + \alpha\gamma\Sigma_e\Theta_e^{-1}\Big]\,\frac{\Sigma_\pi}{E} \\
&= \Big\{E - \Big[\alpha + \kappa(\gamma+\sigma^{-1})\Big]\,\Big[\sigma B + \alpha\gamma\Sigma_e\Theta_e^{-1}\Big]\Big\}\,\frac{\Sigma_\pi}{E} \\
&= \Bigg\{(1+\sigma\gamma)C + \alpha\Big[\sigma B - \sigma^{-1}\alpha\Sigma_e\Theta_e^{-1}\Big] \\
&\quad - \Big[\alpha + \kappa(\gamma+\sigma^{-1})\Big]\,\Big[\sigma B + \alpha\gamma\Sigma_e\Theta_e^{-1}\Big]\Bigg\}\frac{\Sigma_\pi}{E} \\
&\quad - \Big[\alpha + \kappa(\gamma+\sigma^{-1})\Big]\,\alpha\gamma\Sigma_e\Theta_e^{-1}\Bigg\}\frac{\Sigma_\pi}{E} \\
&= \Bigg\{(1+\sigma\gamma)\left[1 + \kappa A - \frac{\Sigma_\pi^2}{\theta_\pi}\right] - \kappa(\gamma+\sigma^{-1})\sigma\Big[A - \alpha\Sigma_x\Theta_x^{-1}\Big] \\
&\quad -\sigma^{-1}\alpha^2\Sigma_e\Theta_e^{-1} - \Big[\alpha + \kappa(\gamma+\sigma^{-1})\Big]\,\alpha\gamma\Sigma_e\Theta_e^{-1}\Bigg\}\frac{\Sigma_\pi}{E} \\
&= \Bigg\{(1+\sigma\gamma)\left[1 - \frac{\Sigma_\pi^2}{\theta_\pi}\right] + \kappa(\gamma+\sigma^{-1})\alpha\sigma\Sigma_x\Theta_x^{-1} - \sigma^{-1}\alpha^2\Sigma_e\Theta_e^{-1} \\
&\quad - \Big[\alpha + \kappa(\gamma+\sigma^{-1})\Big]\,\alpha\gamma\Sigma_e\Theta_e^{-1}\Bigg\}\frac{\Sigma_\pi}{E} >_\theta 0, \qquad \text{(B63)} \\
\bar{a}_x &= \kappa\bar{b}_x + \alpha\bar{c}_x \\
&= \Big\{\kappa E - \Big[\alpha + \kappa(\gamma+\sigma^{-1})\Big]\,\sigma\,[E - \alpha\sigma B - \sigma\gamma C]\Big\}\,\frac{\Sigma_x}{E} \\
&= \Big\{\kappa\,[(1+\sigma\gamma)C + \alpha D] - \Big[\alpha + \kappa(\gamma+\sigma^{-1})\Big]\,\sigma\,\Big[C - \sigma^{-1}\alpha^2\Sigma_e\Theta_e^{-1}\Big]\Big\}\,\frac{\Sigma_x}{E} \\
&= \Big\{\alpha\,[\kappa D - \sigma C] + \Big[\alpha + \kappa(\gamma+\sigma^{-1})\Big]\,\alpha^2\Sigma_e\Theta_e^{-1}\Big\}\,\frac{\Sigma_x}{E} \\
&= \Bigg\{\alpha\left[\kappa\left(\sigma A - \alpha\sigma\Sigma_x\Theta_x^{-1} - \sigma^{-1}\alpha\Sigma_e\Theta_e^{-1}\right) - \sigma\left(1 + \kappa A - \frac{\Sigma_\pi^2}{\theta_\pi}\right)\right] \\
&\quad + \Big[\alpha + \kappa(\gamma+\sigma^{-1})\Big]\,\alpha^2\Sigma_e\Theta_e^{-1}\Bigg\}\frac{\Sigma_x}{E} \\
&= \Bigg\{\alpha\left[-\alpha\sigma\kappa\Sigma_x\Theta_x^{-1} - \sigma^{-1}\alpha\kappa\Sigma_e\Theta_e^{-1} - \sigma + \sigma\frac{\Sigma_\pi^2}{\theta_\pi}\right] \\
&\quad + \Big[\alpha + \kappa(\gamma+\sigma^{-1})\Big]\,\alpha^2\Sigma_e\Theta_e^{-1}\Bigg\}\frac{\Sigma_x}{E} <_\theta 0, \qquad \text{(B64)}
\end{aligned}
$$

$$
\begin{aligned}
\bar{a}_e &= \kappa \bar{b}_e + \alpha \bar{c}_e \\
&= (\alpha + \kappa\gamma)\Sigma_e - \left[\alpha + \kappa(\gamma + \sigma^{-1})\right] [\alpha B + \gamma C] \frac{\sigma \Sigma_e}{E} \\
&= \{(\alpha + \kappa\gamma) [E - \alpha\sigma B - \sigma\gamma C] - \kappa [\alpha B + \gamma C]\} \frac{\Sigma_e}{E} \\
&= \left\{(\alpha + \kappa\gamma) \left[C - \sigma^{-1}\alpha^2 \Sigma_e \Theta_e^{-1}\right] - \kappa [\alpha B + \gamma C]\right\} \frac{\Sigma_e}{E} \\
&= \left\{\alpha [C - \kappa B] - (\alpha + \kappa\gamma)\sigma^{-1}\alpha^2 \Sigma_e \Theta_e^{-1}\right\} \frac{\Sigma_e}{E} \\
&= \left\{\alpha \left[1 + \kappa A - \frac{\Sigma_\pi^2}{\theta_\pi} - \kappa A - \alpha\kappa\Sigma_x \Theta_x^{-1}\right] - (\alpha + \kappa\gamma)\sigma^{-1}\alpha^2 \Sigma_e \Theta_e^{-1}\right\} \frac{\Sigma_e}{E} \\
&= \left\{\alpha \left[1 - \frac{\Sigma_\pi^2}{\theta_\pi} - \alpha\kappa\Sigma_x \Theta_x^{-1}\right] - (\alpha + \kappa\gamma)\sigma^{-1}\alpha^2 \Sigma_e \Theta_e^{-1}\right\} \frac{\Sigma_e}{E} >_\theta 0. \qquad \text{(B65)}
\end{aligned}
$$

C The effects of uncertainty

This Appendix provides some proofs of the propositions in Section 4.

First, we define

$$\Theta_j^{-1} \equiv \frac{\Sigma_j}{(\gamma + \sigma^{-1})\theta_j} > 0, \tag{C1}$$

for $j = \pi, x, e$, and we note that $\lim_{\theta_j \to \infty} \Theta_j^{-1} = 0$.

C.1 Proofs of Propositions 9 and 10

Preliminaries

Note first that the exchange rate coefficients are given by

$$c_\pi = -\frac{D}{1+\sigma\gamma} a_\pi < 0, \tag{C2}$$

$$c_x = \frac{C}{\alpha} a_x < 0, \tag{C3}$$

$$c_e = \frac{C}{\alpha} a_e > 0. \tag{C4}$$

Thus their derivatives depend on the derivatives of the inflation coefficients and those of C and D. These latter are given by

$$\frac{\partial C}{\partial \theta_\pi} = \frac{\Sigma_\pi^2}{\theta_\pi^2} > 0, \tag{C5}$$

$$\frac{\partial C}{\partial \theta_x} = \frac{\partial C}{\partial \theta_e} = 0, \tag{C6}$$

$$\frac{\partial D}{\partial \theta_\pi} = 0, \tag{C7}$$

$$\begin{aligned} \frac{\partial D}{\partial \theta_x} &= \sigma \frac{\partial B}{\partial \theta_x} \\ &= \frac{\sigma \alpha \Sigma_x \Theta_x^{-1}}{\theta_x} > 0, \end{aligned} \tag{C8}$$

$$\frac{\partial D}{\partial \theta_e} = \frac{\sigma^{-1} \alpha \Sigma_e \Theta_e^{-1}}{\theta_e} > 0. \tag{C9}$$

Proof of Proposition 9

The effects of increased robustness against inflation uncertainty on the inflation and output coefficients in the exchange rate equation are given by

$$-\frac{\partial |c_\pi|}{\partial \theta_\pi} = -\frac{1}{1+\sigma\gamma}\left[a_\pi \frac{\partial D}{\partial \theta_\pi} + D \frac{\partial a_\pi}{\partial \theta_\pi}\right]$$

$$
\begin{aligned}
&= \frac{(1+\sigma\gamma)D\Sigma_\pi^3}{E^2\theta_\pi^2} > 0, && \text{(C10)}\\
-\frac{\partial|c_x|}{\partial\theta_\pi} &= \frac{1}{\alpha}\left[a_x\frac{\partial C}{\partial\theta_\pi} + C\frac{\partial a_x}{\partial\theta_\pi}\right] \\
&= -\frac{1}{\alpha}\left[\frac{\sigma\alpha\Sigma_x}{E}\frac{\Sigma_\pi^2}{\theta_\pi^2} - C\frac{\sigma\alpha(1+\sigma\gamma)\Sigma_\pi^2\Sigma_x}{E^2\theta_\pi^2}\right] \\
&= -\left[E-(1+\sigma\gamma)C\right]\frac{\sigma\Sigma_\pi^2\Sigma_x}{E^2\theta_\pi^2} \\
&= -\frac{\sigma\alpha D\Sigma_\pi^2\Sigma_x}{E^2\theta_\pi^2} < 0. && \text{(C11)}
\end{aligned}
$$

The coefficient on the exchange rate is given by

$$
c_e = -\frac{\Sigma_e}{\sigma\Sigma_x}c_x, \qquad \text{(C12)}
$$

so all derivatives have the opposite sign to those of c_x. □

Proof of Proposition 10

The effects of increased robustness against output and exchange rate uncertainty on the inflation and output coefficients in the exchange rate equation are given by

$$
\begin{aligned}
-\frac{\partial|c_\pi|}{\partial\theta_x} &= -\frac{1}{1+\sigma\gamma}\left[a_\pi\frac{\partial D}{\partial\theta_x} + D\frac{\partial a_\pi}{\partial\theta_x}\right] \\
&= -\frac{1}{1+\sigma\gamma}\left[\frac{(1+\sigma\gamma)\Sigma_\pi}{E}\frac{\sigma\alpha\Sigma_x\Theta_x^{-1}}{\theta_x} - D\frac{\sigma^2\alpha^2\Sigma_\pi\Sigma_x^2}{E^2\theta_x^2}\right] \\
&= -\frac{1}{1+\sigma\gamma}\left[\frac{\sigma^2\alpha\Sigma_\pi\Sigma_x^2}{E\theta_x^2} - D\frac{\sigma^2\alpha^2\Sigma_\pi\Sigma_x^2}{E^2\theta_x^2}\right] \\
&= -\left[E-\alpha D\right]\frac{\sigma^2\alpha\Sigma_\pi\Sigma_x^2}{(1+\sigma\gamma)E^2\theta_x^2} \\
&= -(1+\sigma\gamma)C\frac{\sigma^2\alpha\Sigma_\pi\Sigma_x^2}{(1+\sigma\gamma)E^2\theta_x^2} < 0 && \text{(C13)}\\
-\frac{\partial|c_\pi|}{\partial\theta_e} &= -\frac{1}{1+\sigma\gamma}\left[a_\pi\frac{\partial D}{\partial\theta_e} + D\frac{\partial a_\pi}{\partial\theta_e}\right] \\
&= -\frac{1}{1+\sigma\gamma}\left[\frac{(1+\sigma\gamma)\Sigma_\pi}{E}\frac{\sigma^{-1}\alpha\Sigma_e\Theta_e^{-1}}{\theta_e} - D\frac{\alpha^2\Sigma_\pi\Sigma_e^2}{E^2\theta_e^2}\right] \\
&= -\frac{1}{1+\sigma\gamma}\left[\frac{\alpha\Sigma_\pi\Sigma_e^2}{E\theta_e^2} - D\frac{\alpha^2\Sigma_\pi\Sigma_e^2}{E^2\theta_e^2}\right] \\
&= -\left[E-\alpha D\right]\frac{\alpha\Sigma_\pi\Sigma_e^2}{(1+\sigma\gamma)E^2\theta_e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -(1+\sigma\gamma)C\frac{\alpha\Sigma_\pi\Sigma_e^2}{(1+\sigma\gamma)E^2\theta_e^2} < 0, & \text{(C14)}\\
-\frac{\partial|c_x|}{\partial\theta_x} &= \frac{1}{\alpha}\left[a_x\frac{\partial C}{\partial\theta_x} + C\frac{\partial a_x}{\partial\theta_x}\right]\\
&= \frac{\sigma^3\alpha^2 C\Sigma_x^3}{(1+\sigma\gamma)E^2\theta_x^2} > 0, & \text{(C15)}\\
-\frac{\partial|c_x|}{\partial\theta_e} &= \frac{1}{\alpha}\left[a_x\frac{\partial C}{\partial\theta_e} + C\frac{\partial a_x}{\partial\theta_e}\right]\\
&= \frac{\sigma\alpha^2 C\Sigma_x\Sigma_e^2}{(1+\sigma\gamma)E^2\theta_e^2} > 0, & \text{(C16)}
\end{aligned}
$$

and again all derivatives of c_e have the opposite sign to those of c_x. □

C.2 Proofs of Propositions 11 and 12

Preliminaries

Note that we can write the reduced-form coefficients for the policy rule as

$$
\begin{aligned}
d_\pi &= F_\pi a_\pi >_\theta 0, & \text{(C17)}\\
d_x &= F_x a_\pi + \sigma\Sigma_x >_\theta 0, & \text{(C18)}\\
d_e &= F_e a_\pi >_\theta 0, & \text{(C19)}
\end{aligned}
$$

where

$$
\begin{aligned}
F_\pi &\equiv \sigma\left[B - \frac{\gamma D}{1+\sigma\gamma}\right]\\
&= \sigma\left[\left(1 - \frac{\sigma\gamma}{1+\sigma\gamma}\right)B + \frac{\sigma^{-1}\gamma\alpha\Sigma_e\Theta_e^{-1}}{1+\sigma\gamma}\right]\\
&= \frac{\sigma}{1+\sigma\gamma}B + \frac{\gamma\alpha\Sigma_e\Theta_e^{-1}}{1+\sigma\gamma} >_\theta 0, & \text{(C20)}\\
F_x &\equiv -\left[\alpha B + \gamma C\right]\frac{\sigma^2\Sigma_x}{(1+\sigma\gamma)\Sigma_\pi} <_\theta 0, & \text{(C21)}\\
F_e &\equiv \left[\alpha B + \gamma C\right]\frac{\sigma\Sigma_e}{(1+\sigma\gamma)\Sigma_\pi}\\
&= -\frac{\Sigma_e}{\sigma\Sigma_x}F_x >_\theta 0, & \text{(C22)}
\end{aligned}
$$

and it is useful to recall that

$$E - \sigma\alpha B - \sigma\gamma C$$

$$= C + \alpha (D - \sigma B)$$
$$= C - \frac{\sigma^{-1}\alpha^2\Sigma_e^2}{(\gamma + \sigma^{-1})\theta_e} >_\theta 0. \tag{C23}$$

Proof of Proposition 11

The effects on the policy rule coefficients of increased robustness against inflation and exchange rate uncertainty are given by

$$-\frac{\partial |d_\pi|}{\partial \theta_\pi} = -\left[a_\pi \frac{\partial F_\pi}{\partial \theta_\pi} + F_\pi \frac{\partial a_\pi}{\partial \theta_\pi} \right]$$
$$= \left[\frac{\sigma}{1+\sigma\gamma} B + \frac{\gamma\alpha\Sigma_e\Theta_e^{-1}}{1+\sigma\gamma} \right] \frac{(1+\sigma\gamma)^2\Sigma_\pi^3}{E^2\theta_\pi^2} > 0 \tag{C24}$$

$$-\frac{\partial |d_\pi|}{\partial \theta_e} = -\left[a_\pi \frac{\partial F_\pi}{\partial \theta_e} + F_\pi \frac{\partial a_\pi}{\partial \theta_e} \right] \tag{C25}$$
$$= \frac{(1+\sigma\gamma)\Sigma_\pi}{E} \frac{\gamma\alpha\Sigma_e\Theta_e^{-1}}{(1+\sigma\gamma)\theta_e} + \left[\frac{\sigma}{1+\sigma\gamma} B + \frac{\gamma\alpha\Sigma_e\Theta_e^{-1}}{1+\sigma\gamma} \right] \frac{\alpha^2\Sigma_\pi\Sigma_e^2}{E^2\theta_e^2} > 0,$$

$$-\frac{\partial |d_x|}{\partial \theta_\pi} = -\left[a_\pi \frac{\partial F_x}{\partial \theta_\pi} + F_x \frac{\partial a_\pi}{\partial \theta_\pi} \right]$$
$$= \frac{(1+\sigma\gamma)\Sigma_\pi}{E} \frac{\gamma\sigma^2\Sigma_\pi\Sigma_x}{(1+\sigma\gamma)\theta_\pi^2} - [\alpha B + \gamma C] \frac{\sigma^2\Sigma_x}{(1+\sigma\gamma)\Sigma_\pi} \frac{(1+\sigma\gamma)^2\Sigma_\pi^3}{E^2\theta_\pi^2}$$
$$= \frac{\gamma\sigma^2\Sigma_\pi^2\Sigma_x}{E\theta_\pi^2} - [\alpha B + \gamma C] \frac{(1+\sigma\gamma)\sigma^2\Sigma_\pi^2\Sigma_x}{E^2\theta_\pi^2}$$
$$= -[(1+\sigma\gamma)(\alpha B + \gamma C) - \gamma E] \frac{\sigma^2\Sigma_\pi^2\Sigma_x}{E^2\theta_\pi^2}$$
$$= -[\alpha B + \gamma C - \gamma (E - \sigma\alpha B - \sigma\gamma C)] \frac{\sigma^2\Sigma_\pi^2\Sigma_x}{E^2\theta_\pi^2} < 0, \tag{C26}$$

$$-\frac{\partial |d_x|}{\partial \theta_e} = -\left[a_\pi \frac{\partial F_x}{\partial \theta_e} + F_x \frac{\partial a_\pi}{\partial \theta_e} \right]$$
$$= -[\alpha B + \gamma C] \frac{\sigma^2\Sigma_x}{(1+\sigma\gamma)\Sigma_\pi} \frac{\alpha^2\Sigma_\pi\Sigma_e^2}{E^2\theta_e^2} < 0, \tag{C27}$$

$$-\frac{\partial |d_e|}{\partial \theta_\pi} = -\left[a_\pi \frac{\partial F_e}{\partial \theta_\pi} + F_e \frac{\partial a_\pi}{\partial \theta_\pi} \right]$$
$$= -\frac{(1+\sigma\gamma)\Sigma_\pi}{E} \frac{\gamma\sigma\Sigma_\pi\Sigma_e}{(1+\sigma\gamma)\theta_\pi^2} + [\alpha B + \gamma C] \frac{\sigma\Sigma_e}{(1+\sigma\gamma)\Sigma_\pi} \frac{(1+\sigma\gamma)^2\Sigma_\pi^3}{E^2\theta_\pi^2}$$
$$= -\frac{\gamma\sigma\Sigma_\pi^2\Sigma_e}{E\theta_\pi^2} + [\alpha B + \gamma C] \frac{\sigma(1+\sigma\gamma)\Sigma_\pi^2\Sigma_e}{E^2\theta_\pi^2}$$
$$= -[\gamma E - (1+\sigma\gamma)(\alpha B + \gamma C)] \frac{\sigma\Sigma_\pi^2\Sigma_e}{E^2\theta_\pi^2}$$

$$= -\left[\gamma\left(E - \sigma\alpha B - \sigma\gamma C\right) - \left(\alpha B + \gamma C\right)\right] \frac{\sigma \Sigma_\pi^2 \Sigma_e}{E^2 \theta_\pi^2} > 0, \tag{C28}$$

$$-\frac{\partial |d_e|}{\partial \theta_e} = -\left[a_\pi \frac{\partial F_e}{\partial \theta_e} + F_e \frac{\partial a_\pi}{\partial \theta_e}\right]$$

$$= \left[\alpha B + \gamma C\right] \frac{\sigma \Sigma_e}{(1+\sigma\gamma)\Sigma_\pi} \frac{\alpha^2 \Sigma_\pi \Sigma_e^2}{E^2 \theta_e^2} > 0. \quad \square \tag{C29}$$

Proof of Proposition 12

The effects on the policy rule coefficients of increased robustness against output uncertainty are given by

$$-\frac{\partial |d_\pi|}{\partial \theta_x} = -\left[a_\pi \frac{\partial F_\pi}{\partial \theta_x} + F_\pi \frac{\partial a_\pi}{\partial \theta_x}\right]$$

$$= -\frac{(1+\sigma\gamma)\Sigma_\pi}{E} \frac{\sigma\alpha\Sigma_x \Theta_x^{-1}}{(1+\sigma\gamma)\theta_x} + \left[\frac{\sigma}{1+\sigma\gamma} B + \frac{\gamma\alpha\Sigma_e\Theta_e^{-1}}{1+\sigma\gamma}\right] \frac{\sigma^2\alpha^2\Sigma_\pi\Sigma_x^2}{E^2\theta_x^2}$$

$$= -\frac{\sigma^2\alpha\Sigma_\pi\Sigma_x^2}{(1+\sigma\gamma)E\theta_x^2} + \left[\sigma B + \gamma\alpha\Sigma_e\Theta_e^{-1}\right] \frac{\sigma^2\alpha^2\Sigma_\pi\Sigma_x^2}{(1+\sigma\gamma)E^2\theta_x^2}$$

$$= -\left[E - \sigma\alpha B - \gamma\alpha^2\Sigma_e\Theta_e^{-1}\right] \frac{\sigma^2\alpha\Sigma_\pi\Sigma_x^2}{(1+\sigma\gamma)E^2\theta_x^2} < 0, \tag{C30}$$

$$-\frac{\partial |d_x|}{\partial \theta_x} = -\left[a_\pi \frac{\partial F_x}{\partial \theta_x} + F_x \frac{\partial a_\pi}{\partial \theta_x}\right]$$

$$= \frac{(1+\sigma\gamma)\Sigma_\pi}{E} \frac{\sigma^2\alpha^2\Sigma_x^2\Theta_x^{-1}}{(1+\sigma\gamma)\Sigma_\pi\theta_x} - \left[\alpha B + \gamma C\right] \frac{\sigma^2\Sigma_x}{(1+\sigma\gamma)\Sigma_\pi} \frac{\sigma^2\alpha^2\Sigma_\pi\Sigma_x^2}{E^2\theta_x^2}$$

$$= \frac{\sigma^3\alpha^2\Sigma_x^3}{(1+\sigma\gamma)E\theta_x^2} - \left[\alpha B + \gamma C\right] \frac{\sigma^4\alpha^2\Sigma_x^3}{(1+\sigma\gamma)E^2\theta_x^2}$$

$$= \left[E - \sigma\alpha B - \sigma\gamma C\right] \frac{\sigma^3\alpha^2\Sigma_x^3}{(1+\sigma\gamma)E^2\theta_x^2} > 0, \tag{C31}$$

$$-\frac{\partial |d_e|}{\partial \theta_x} = -\left[a_\pi \frac{\partial F_e}{\partial \theta_x} + F_e \frac{\partial a_\pi}{\partial \theta_x}\right]$$

$$= -\frac{(1+\sigma\gamma)\Sigma_\pi}{E} \frac{\sigma\alpha^2\Sigma_x\Sigma_e\Theta_x^{-1}}{(1+\sigma\gamma)\Sigma_\pi\theta_x} + \left[\alpha B + \gamma C\right] \frac{\sigma\Sigma_e}{(1+\sigma\gamma)\Sigma_\pi} \frac{\sigma^2\alpha^2\Sigma_\pi\Sigma_x^2}{E^2\theta_x^2}$$

$$= -\frac{\sigma^2\alpha^2\Sigma_x^2\Sigma_e}{(1+\sigma\gamma)E\theta_x^2} + \left[\alpha B + \gamma C\right] \frac{\sigma^3\alpha^2\Sigma_x^2\Sigma_e}{(1+\sigma\gamma)E^2\theta_x^2}$$

$$= -\left[E - \sigma\alpha B - \sigma\gamma C\right] \frac{\sigma^2\alpha^2\Sigma_x^2\Sigma_e}{(1+\sigma\gamma)E^2\theta_x^2} < 0. \quad \square \tag{C32}$$

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