

# **Welfare Maximizing Operational Monetary and Fiscal Policy Rules**

Robert Kollmann (\*)

Department of Economics, University of Bonn  
24-42 Adenauerallee, D-53113 Bonn, Germany

Centre for Economic Policy Research, UK

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This paper studies the welfare effects of monetary and fiscal policy rules, in a dynamic general equilibrium model with sticky prices. The model features capital accumulation, exogenous productivity and taste shocks, and government purchases that are valued positively by the private sector. These purchases are financed using a proportional income tax. The government issues nominal one-period bonds. Monetary policy is described by a Taylor-style interest rate rule; fiscal policy is described by rules that set the tax rate and government purchases as functions of government debt and of other macroeconomic variables. Attention focuses on policies that induce stationary fluctuations of real public debt. A quadratic approximation is used to solve the model, and to compute household welfare. The paper determines the policy parameters (consistent with debt being stationary) that maximize household welfare. Optimized monetary policy has a strong anti-inflation stance. Optimized fiscal policy implies procyclical fluctuations of the tax rate and of government purchases.

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(\*) Tel.: 49 228 734073; Fax: 49 228 739100; E-mail: [kollmann@wiwi.uni-bonn.de](mailto:kollmann@wiwi.uni-bonn.de)  
<http://www.wiwi.uni-bonn.de/kollmann>

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## 1. Introduction

Much recent research has focused on the question what *monetary* policy rules are best suited for stabilizing the economy and raising (private sector) welfare (see, e.g., Clarida et al. (1999), Taylor (1999), McCallum (1999), and Woodford (2003) for surveys of the relevant literature). The analysis of *fiscal* policy rules has received less attention.

Existing contributions can be classified into two categories: (i) Dynamic extensions of Ramsey (1927) that determine time paths of fiscal policy instruments that maximize household welfare, subject to "implementability" conditions consisting of the private sector's decision rules (see, e.g., Lucas and Stokey (1983)).<sup>1</sup> (ii) Studies that analyze the macroeconomic consequences of simple, operational feedback rules for fiscal policy instruments (e.g., Taylor (2001)).

The Ramsey approach is appealing as it uses fully micro-based models and focuses on household welfare as the criterion for evaluating the effects of policy. However, solving Ramsey problems raises technical difficulties that are not yet fully resolved.<sup>2</sup> Also, existing studies that use the Ramsey approach focus on models that are highly stylized. Furthermore, Ramsey policy rules are often complicated; this may make it difficult to implement those rules in the real world.

By contrast, previous quantitative studies on the effect of "simple" fiscal policy rules have mostly used models that are not fully micro-based, and these studies use ad hoc criteria to evaluate policy rules (policies are evaluated according to the implied volatilities of output, inflation etc).

The present paper uses a calibrated business cycle model with rigorous micro-foundations. The model has a richer structure than those used in previous analyses of optimal fiscal/monetary policy. In contrast to the Ramsey approach, the paper focuses on operational feedback rules that link the policy instruments to a small set of macro variables. The paper computes the policy response coefficients that maximize household welfare.

The model here assumes a closed economy with capital accumulation and endogenous government purchases that are valued positively by private households; there are two types of exogenous disturbances: shocks to productivity and to the private sector's taste for government purchases. There is monopolistic competition in goods markets. Goods prices are set in a staggered fashion, à la Calvo (1983). Therefore (and because the government issues nominal debt; see below), monetary policy has real effects. Monetary policy is described by a Taylor (1993) style interest rate rule. Previous analyses of optimal policy typically assume that the monopolistic distortion is offset by corrective government subsidies to firms (e.g., Rotemberg and Woodford, 1997); this assumption is made for analytical reasons, but it is unrealistic. This paper assumes that there are no subsidies to firms. Here, the government levies a proportional income tax; it also issues nominal non-state contingent one-period bonds. Fiscal policy is described by rules according to which the income tax rate and government purchases are set as functions of the stock of government debt and of other macroeconomic variables (GDP, inflation rate etc.). The average ratio of government purchases to GDP, the average tax rate and the average debt-to-GDP ratio are calibrated to historical averages observed in OECD economies. Attention focuses on fiscal policies characterized by stationary fluctuations of public debt around the historical average of that

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<sup>1</sup> See also Aiyagari et al. (2001), Benigno and Woodford (2003), Buera and Nicolini (2004), Chamley (1986), Chari et al. (1994), Correia et al. (2001), Schmitt-Grohé and Uribe (2002), Siu (2004) (several of these papers also determine optimal monetary policy).

<sup>2</sup>Ramsey problems are (in general) not concave programming problems. Nevertheless, existing analyses generally cast Ramsey problems as Lagrange problems, and focus on the associated first-order conditions. Existing papers typically do not provide evidence that the relevant second-order conditions hold.

variable. I numerically solve the model, compute household welfare, and determine the response coefficients of the policy rules (consistent with debt being stationary) that maximize welfare.

The simulation results here confirm two widely discussed results from the theoretical literature on Ramsey fiscal and monetary policies: (i) if the government issues nominal debt (as assumed here), then optimal inflation would be extremely volatile under flexible prices (Chari et al, 1991; 1994), while tax rates would show little variation; (ii) under sticky prices, the optimal volatility of optimal is very low (Schmitt-Grohé and Uribe, 2002; Siu, 2004). Intuitively, when prices are flexible, then time-varying inflation does not cause relative price distortions across firms; thus, the policy authority meets its solvency constraint by using unanticipated changes in inflation that make the real debt return state contingent. When prices are sticky, by contrast, time-varying inflation induces inefficient price dispersion across intermediate goods firms (e.g., Rotemberg and Woodford, 1997); this induces a welfare loss--and hence the policy authority relies much less on the "inflation tax", when prices are sticky.

Optimized policy in the sticky prices model here yields a standard deviation of inflation that is close to zero; it also implies procyclical fluctuations of the income tax rate, and of government purchases. The government (almost) fully accommodates shocks to the private sector's taste for government purchases.

I show that very simple optimized rules--under which the interest rate *just* responds to inflation, the tax rate *just* responds to the stock of public debt, and government purchases *just* respond to debt and the private sector's taste for public goods--yield a welfare level that can only very slightly be improved upon by rules that stipulate a direct response to (selected) additional variables. In particular, virtually no welfare gains can be achieved by also tying monetary policy to GDP or the public deficit, or by tying the tax rate to inflation, GDP or government purchases.

As optimized policy in the sticky prices economy entails strict inflation stabilization, the behavior of that economy mimics closely the behavior of an RBC-style economy with flexible prices and real (indexed) debt.

The model is solved using Sims' (2000) quadratic approximation method that is based on a second-order expansion of the equilibrium conditions. In contrast to the linear, certainty-equivalent approximations that are widely used in macroeconomics, this second-order accurate approach allows to capture the effect of risk on agents' decision rules and is thus better suited for welfare analysis. Compared to other non-linear methods (see Judd (1998)), the method allows to easily solve models with a rich structure. The techniques used here can in principle also be applied to much larger macro-econometric simulation models (such as the empirical models with choice theoretic foundations currently developed by, i.a., the IMF and Fed); I hope that the results here will provide a basis for optimal policy analysis based on models of that type.

This study builds on my recent work that computed welfare maximizing "simple" monetary policy rules, for calibrated New Keynesian models (see Kollmann (2002, 2004a)); that work assumed open economies, and it abstracted from fiscal policy. Schmitt-Grohé and Uribe (2004a) use a second-order approximation to compute welfare maximizing simple monetary and fiscal feedback rules for a calibrated New Keynesian model, but that paper focuses on a setting with a lump sum tax, and it assumes exogenous government purchases (the paper here assumes that no lump sum tax is available).<sup>3</sup>

Section 2 of the paper presents the model. Section 3 presents the results and Section 4 concludes.

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<sup>3</sup> Benigno and Woodford (2003) too derive optimal monetary and fiscal policy, based on a second-order approximation; these authors consider a more stylized economy (without capital), for which they are able to obtain analytical results. Kim and Kim (2001) compute welfare maximizing tax policy rules (using a second-order approximation), for a two-country RBC model (without monetary policy).

## 2. The model

A closed economy with a representative household, firms, a monetary authority and a fiscal authority is considered (the structure of preferences, technologies and markets resembles that of Kollmann's (2002, 2004a) open economy models). There is a single final good that is produced by combining a continuum of intermediate goods indexed by  $s \in [0, 1]$ . The final good is produced by perfectly competitive firms, it can be consumed and used for investment. There is monopolistic competition in intermediate goods markets--each intermediate good is produced by a single firm. Intermediate goods producers use capital and labor as inputs. The household owns all producers and the capital stock, which it rents to producers. It also supplies labor. The markets for rental capital and for labor are competitive.

### 2.1. Final good production

The final good is produced using the aggregate technology

$$Z_t = \left\{ \int_0^1 q_t(s)^{(\nu-1)/\nu} ds \right\}^{\nu/(\nu-1)} \quad (1)$$

with  $\nu > 1$ , where  $q_t(s)$  is the quantity of the type  $s$  intermediate good. Let  $p_t(s)$  be the nominal price of that good. Cost minimization in final good production implies:

$$q_t(s) = (p_t(s) / P_t)^{-\nu} Z_t, \quad \text{with } P_t = \left\{ \int_0^1 p_t(s)^{1-\nu} ds \right\}^{1/(1-\nu)}. \quad (2)$$

Perfect competition in the final good market implies that the good's price is  $P_t$  (its marginal cost is  $\left\{ \int_0^1 p_t(s)^{1-\nu} ds \right\}^{1/(1-\nu)}$ ).

### 2.2. Intermediate goods firms

The technology of the firm that produces intermediate good  $s$  is:

$$y_t(s) = \theta_t K_t(s)^\psi L_t(s)^{1-\psi}, \quad 0 < \psi < 1. \quad (3)$$

$y_t(s)$  is the firm's output at date  $t$ .  $K_t(s)$  and  $L_t(s)$  are the amounts of capital and labor used by the firm.  $\theta_t$  is an exogenous random productivity parameter that is identical for all intermediate goods producers.

Let  $R_t$  and  $W_t$  be the rental rate of capital and the wage rate. Cost minimization implies:  $L_t(s)/K_t(s) = \psi^{-1}(1-\psi)R_t/W_t$ . The firm's marginal cost is:  $MC_t = (1/\theta_t)R_t^\psi W_t^{1-\psi} \psi^{-\psi} (1-\psi)^{\psi-1}$ . Demand for the firm's output is given by (3). Its profit is:

$$\pi_t(p_t(s)) = (p_t(s) - MC_t)(p_t(s) / P_t)^{-\nu} Z_t.$$

The representative household receives the profits of intermediate goods firms--at the household level, period  $t$  profits are taxed at the rate  $\tau_t$  (see below).

There is staggered price setting, à la Calvo (1983): intermediate goods firms cannot change prices, in buyer currency, unless they receive a random "price-change signal." The probability of receiving this signal in any particular period is  $1-d$ , a constant. Thus, the mean price-change-interval is  $1/(1-d)$ . Following Yun (1996) and Erceg et al. (2000) it is assumed that when a firm does not receive a "price-change signal," its price is automatically increased at the steady state growth factor of the price level. (Throughout this paper, the term

"steady state" refers to the deterministic steady state.) Firms are assumed to meet all demand at posted prices.

Consider an intermediate good producer that, at time  $t$ , sets a new price,  $p_{t,t}$ . If no "price-change signal" is received between  $t$  and  $t+j$ , the price is  $p_{t,t}\Pi^j$  at  $t+j$ , where  $\Pi$  is the steady state growth factor of the final good price,  $P_t$ . The firm sets  $p_{t,t} = \text{Arg Max}_{\mathbf{p}} \sum_{j=0}^{j=\infty} d^j E_t \{ \rho_{t,t+j} (1-\tau_{t+j}) \pi_{t+j}(\mathbf{p}\Pi^j) / P_{t+j} \}$ , where  $\rho_{t,t+\tau}$  is a pricing kernel (for valuing date  $t+j$  pay-offs) that equals the household's marginal rate of substitution between consumption at  $t$  and at  $t+j$  (see discussion below). Let  $\Xi_{t,t+j} = \rho_{t,t+j} (P_{t+j})^{\nu-1} Z_{t+j}$ . The solution of the maximization problem regarding  $p_{t,t}$  is:

$$p_{t,t} = (\nu/(\nu-1)) \left\{ \sum_{j=0}^{j=\infty} (d\Pi^{-\nu})^j E_t \Xi_{t,t+j} (1-\tau_{t+j}) MC_{t+j} \right\} / \left\{ \sum_{j=0}^{j=\infty} (d\Pi^{1-\nu})^j E_t \Xi_{t,t+j} (1-\tau_{t+j}) \right\}.$$

The final good price  $P_t$  evolves according to:  $(P_t)^{1-\nu} = d(P_{t-1}\Pi)^{1-\nu} + (1-d)(p_{t,t})^{1-\nu}$ .

### 2.3. The representative household

Household preferences are described by:

$$E_0 \sum_{t=0}^{t=\infty} \beta^t U(C_t, L_t, G_t - \gamma_t). \quad (4)$$

$E_t$  denotes the mathematical expectation conditional upon complete information pertaining to period  $t$  and earlier.  $C_t$ ,  $L_t$  and  $G_t$  are, respectively, household consumption, labor effort, and government purchases of the final good;  $\gamma_t$  is an exogenous random shock to the household's taste for government purchases.  $0 < \beta < 1$  is the subjective discount factor.  $U$  is a utility function given by:  $U(C_t, L_t, G_t - \gamma_t) = \ln(C_t) - L_t + \Psi \ln(G_t - \gamma_t)$ , where  $\Psi$  is a parameter.

As indicated earlier, the household owns all domestic producers and accumulates physical capital. The law of motion of the capital stock is:

$$K_{t+1} = K_t(1-\delta) + I_t, \quad (5)$$

where  $I_t$  is gross investment, while  $0 < \delta < 1$  is the depreciation rate of capital. The household also trades in nominal one-period bonds. The household pays taxes,  $T_t$ , to the government. There is a proportional tax on labor income and on "entrepreneurial" income, with tax rate  $\tau_t$ :  $T_t = \tau_t (W_t L_t + \int_0^1 \pi(p_t(s)) ds + R_t K_t - \delta P_t K_t)$ .  $W_t L_t$  is the household's labor income;  $\int_0^1 \pi(p_t(s)) ds$  is the aggregate profit of intermediate goods producers, and  $R_t K_t - \delta P_t K_t$  is income from capital rental (net of a depreciation allowance).

The household's period  $t$  budget constraint is:

$$A_{t+1} + P_t(C_t + I_t) = A_t(1+i_{t-1}) + W_t L_t + \int_0^1 \pi(p_t(s)) ds + R_t K_t - T_t. \quad (6)$$

$A_t$  is the household's net stock of nominal bonds that mature in period  $t$ , while  $i_{t-1}$  is the nominal interest rates on these bonds.

The household chooses a strategy  $\{A_{t+1}, K_{t+1}, C_t, L_t\}_{t=0}^{t=\infty}$  to maximize its expected lifetime utility (4), subject to constraints (5) and (6) and to initial values  $A_0, K_0$ . Ruling out Ponzi schemes, the following equations are first-order conditions of this decision problem:

$$1 = (1+i_t)E_t\{\rho_{t,t+1}(P_t/P_{t+1})\}, \quad (7)$$

$$1 = E_t\{\rho_{t,t+1}([R_{t+1}/P_{t+1} - \delta](1-\tau_{t+1})+1)\}, \quad (8)$$

$$(1-\tau_t)W_t/P_t = C_t, \quad (9)$$

where  $\rho_{t,t+1} = \beta C_t / C_{t+1}$ . (7)-(8) are Euler conditions, and (9) says that the household equates its marginal rate of substitution between consumption and leisure to the after-tax real wage rate.

## 2.4. The government budget constraint

The government purchases  $G_t$  units of the final good, in period  $t$ . Its budget constraint is:

$$P_t G_t + D_t(1+i_{t-1}) = D_{t+1} + T_t,$$

where  $D_t$  is the net stock of nominal government debt that matures in period  $t$ .

## 2.5. Market clearing conditions

Supply equals demand in intermediate goods markets because intermediate goods firms meet all demand at posted prices. Market clearing for the final good, labor, and rental capital requires:  $Z_t = C_t + I_t + G_t$ ,  $L_t = \int_0^1 L_t(s)ds$ ,  $K_t = \int_0^1 K_t(s)ds$ , where  $Z_t$ ,  $L_t$  and  $K_t$  are the supplies of the final good, of labor, and of rental capital, respectively, while  $\int_0^1 L_t(s)ds$  and  $\int_0^1 K_t(s)ds$  represent total demand for labor and capital (by intermediate goods producers). Market clearing for bonds requires:  $A_t = D_t$ .

## 2.6. Policy rules

Much recent research has focused on monetary policy rules that stipulate a response of the interest rate to inflation (e.g., Taylor, 1993a, 1999). This paper considers the following interest rate rule:

$$i_t = i + \Gamma_i^\pi \widehat{\Pi}_t + \Gamma_i^Z Z_t, \quad (10)$$

with  $\widehat{\Pi}_t = (\Pi_t - \Pi)/\Pi$ , where  $\Pi_t = P_t/P_{t-1}$  is the gross final good inflation rate.  $i$  is the steady state nominal interest rate, and (as defined earlier)  $\Pi$  is the steady state gross inflation rate. Throughout the paper, variables without time subscripts denote steady state values, and  $\hat{x}_t = (x_t - x)/x$  is the relative deviation of a variable  $x_t$  from its steady state value,  $x$ .  $Z_t$  in (10) is a vector of other macro variables (GDP, fiscal deficit etc.; see discussion below).  $\Gamma_i^\pi$  and  $\Gamma_i^Z$  are parameters.

The income tax rate and government purchases are set as functions of the (real) stock of government debt, and a vector of other variables,  $Z_t$  (discussed below):

$$\tau_t = \tau + \Gamma^D i (D_t/P_t - \bar{d})/Y + \Gamma_\tau^Z Z_t, \quad (11)$$

$$G_t = G - \Gamma^D i (D_t/P_t - \mathfrak{d}) + \Gamma_G^Z Z_t, \quad (12)$$

where  $\mathfrak{d}$  is the steady state value of real government debt,  $D_t/P_t$ .  $\Gamma^D, \Gamma_\tau^Z, \Gamma_G^Z$  are parameters. I assume that  $\tau_t$  and  $G_t$  are functions of the stock of debt; setting  $\Gamma^D$  at a sufficiently high positive value ensures government solvency.

In the economy considered here, fiscal policy can have strong effects on endogenous variables. Aiyagari et al. (2002) present analytical results about optimal (Ramsey) fiscal policy in an infinitely-lived economy in which the government levies a distorting (income) tax and issues bonds.<sup>4</sup> Aiyagari et al. show that, when the government faces a standard no-Ponzi solvency constraint, then under Ramsey policy, the long run values of debt and tax rates may differ substantially from initial values. For example, under certain assumptions, a government that (initially) has positive debt is predicted to run budget *surpluses*, until it has accumulated a stock of *assets* that generates an interest income that covers all subsequent government purchases; once that stock of assets is reached, the government sets distorting taxes forever to zero (and a Pareto efficient allocation is achieved).

This prediction is at odds with the data: throughout modern history, real world governments have been net debtors. Aiyagari et al. suggest a simple way of ruling out unrealistic levels of government assets: a lower bound on government debt that makes it impossible for the government to finance its stream of purchases without raising a positive tax income:  $\underline{d} \leq D_{t+1}/P_t$ , where  $\underline{d} > -E_t \sum_{j=0}^{\infty} \rho_{t,t+j} G_{t+j} \quad \forall t \geq 0$ . (As less crude approach might be to assume that public debt provides liquidity services to the private sector; see, e.g., Woodford (1990); if the private sector's demand for these liquidity services is sufficiently strong, then a benevolent government would always select a positive level of debt.)

This paper too imposes a lower bound on public debt. In addition to the empirical motivation discussed in the previous paragraph, this assumption is also used for a technical reason: the numerical solution method used in this paper is based on a *local* approximation of the model, around a given steady state (see below). This method is not suited for the analysis of "large" changes in fiscal variables. Imposing a (sufficiently tight) lower bound on debt allows to ensure that the stock of debt shows stationary fluctuations around steady state debt  $\mathfrak{d}$ . The solution method cannot capture inequality constraints on debt that do not bind at all dates. I thus use the following constraint that can easily be implemented using the technique here: the government has to ensure that, with a conditional probability very close to unity, real public debt has to exceed the lower bound  $\underline{d} < \mathfrak{d}$ :

$$\text{Prob}_0(\underline{d} \leq D_{t+1}/P_t) \geq 0.999 \quad \forall t \geq 0, \quad (13)$$

where  $\text{Prob}_t$  is the conditional probability, given date  $t$  information.

Below, I assume  $\underline{d} = \mathfrak{d} - Y$ : government debt may not fall below the steady state debt level by more than the value of steady state GDP,  $Y$ . It appears that, this constraint permits sizable transitory fluctuations of the stock of debt, while ensuring that the mean stock of debt is close to the steady state stock of debt.

The monetary and fiscal authorities make a commitment to set the coefficients  $\Gamma_i^\pi$ ,  $\Gamma_i^Z$ ,  $\Gamma^D, \Gamma_\tau^Z$  and  $\Gamma_G^Z$  at time-invariant values that maximize household welfare (4), subject to the laws of motion of the endogenous implied by household decisions. Note that the welfare criterion is the conditional expected value of household welfare, given information at the initial date  $t = 0$ , and given the initial values of predetermined variables. I assume that at

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<sup>4</sup> The Aiyagari et al. model is more stylized than the structure considered here (their model assumes flexible prices and perfect competition, exogenous government purchases and constant labor productivity, and it abstracts from monetary policy and capital accumulation).

$t = 0$  the predetermined variables equal their (deterministic) steady state values, and that the exogenous variables at  $t = 0$  equal the unconditional means of these variables.

## 2.7. Solution method and welfare computation

The model is solved using Sims' (2000) algorithm/computer code that is based on second-order Taylor expansions of the equilibrium conditions, around a (deterministic) steady state.<sup>5</sup> I numerically maximize the objective function of the policy authorities (attention is restricted to parameter values for which a unique stationary equilibrium exists).

## 2.8. Parameters (non-policy)

The model is calibrated to quarterly data. The steady state *real* interest rates  $r$  is set at  $r = 0.01$ , a value that corresponds roughly to the long-run average (quarterly) return on capital. The subjective discount factor is, hence, set at  $1/(1.01)$ , since  $\beta(1+r) = 1$  holds in steady state. The weight of government consumption in the household's utility function is set at  $\Psi = 0.2$  (this value implies that, in a first-best efficient allocation, government purchases would represent 20% of private consumption).

The steady state price-marginal cost markup factor for intermediate goods is set at  $\nu/(\nu-1) = 1.2$ , consistent with the findings of Martins et al. (1996) for the US and for European countries. The technology parameter  $\psi$  (see (4)) is set at  $\psi = 0.24$ , which entails a 60% steady state labor income/GDP ratio, consistent with US and European data. Aggregate data suggest a quarterly capital depreciation rate of about 2.5%; thus,  $\delta = 0.025$  is used.

Estimates of Calvo-style price setting equations suggest that the average price-change interval is about 4 quarters (e.g., Lopez-Salido (2000)). Hence,  $d$  is set at  $d = 0.75$ . The steady state growth factors of the price levels is set at  $\Pi = 1$  ( $\Pi$  has no effect on real variables, because of indexing); thus the steady state nominal interest rate is  $i = r = 0.01$ .

The exogenous variables follow AR(1) processes:

$$\begin{aligned}\theta_t &= (1 - \rho^\theta) + \rho^\theta \theta_{t-1} + \varepsilon_t^\theta, \quad 0 \leq \rho^\theta < 1, \\ \gamma_t &= \rho^\gamma \theta_{t-1} + \varepsilon_t^\gamma, \quad 0 \leq \rho^\gamma < 1,\end{aligned}\tag{14}$$

where  $\varepsilon_t^\theta$  and  $\varepsilon_t^\gamma$  are independent normally distributed white noises with standard deviation  $\sigma^\theta$  and  $\sigma^\gamma$ , respectively. I set  $\rho^\theta, \sigma^\theta$  at values that are standard in the RBC literature:  $\rho^\theta = 0.95$ ,  $\sigma^\theta = 0.01$ . I assume that the shock to the household's taste for government purchases ( $\gamma_t$ ) has the same autocorrelation and standard deviation as productivity:  $\rho^\gamma = 0.95$ ,  $\sigma^\gamma = 0.01$ .

The steady state fiscal variables are calibrated to values that are in the range of historical averages observed in OECD economies. The steady state tax rate is set at  $\tau = 0.25$ , which implies that the steady state ratio of tax revenue to GDP is 0.22; steady state government purchases are set in such a way that the steady state ratio of government purchases to GDP is 0.20. The implied steady state ratio of government debt to annual GDP is 0.50.

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<sup>5</sup> See Kim et al. (2002) and Kollmann (2002) for a more detailed discussion of the Sims algorithm. Guu and Judd (1993), Gaspar and Judd (1996), Kim and Kim (1999), Collard and Juillard (2001), Schmitt-Grohé and Uribe (2004b) Anderson and Levin (2002) and Kollmann (2004b) also develop solutions of dynamic models based on second-order expansions.

### 3. Results

Simulation results are reported in Tables 1-3. The variables are quarterly.  $Def_t = (D_{t+1} - D_t)/(PY)$  and  $Debt_t = D_{t+1}/(PY)$  are the (secondary) fiscal deficit and government debt, in real terms, normalized by steady state GDP. In the Tables, the statistics for the interest rate ( $i_t$ ), the tax rate ( $\tau_t$ ),  $Def_t$ , and  $Debt_t$  refer to differences of these variables from steady state values ( $i_t$  is a quarterly rate expressed in fractional units), while statistics for the remaining variables refer to relative deviations from steady state values. All statistics are expressed in percentage terms.

Welfare is expressed as the permanent relative change in consumption, compared to the steady state, that yields the conditional expected welfare in the stochastic equilibrium. The welfare measure,  $\zeta^c$ , is defined as:  $(1-\beta)^{-1}U((1+\zeta^c)C, L, G) = E_0\{\sum_{t=0}^{\infty} U(C_t, L_t, G_t)\}$ . I decompose  $\zeta^c$  into components, denoted  $\zeta^{c,m}$  and  $\zeta^{c,v}$ , that reflect the conditional expected values of consumption, hours worked and government purchases  $\{E_0 C_t, E_0 L_t, E_0 G_t\}_{t \geq 0}$ , and their conditional variances  $\{Var_0(C_t), Var_0(L_t), Var_0(G_t)\}_{t \geq 0}$ , respectively. Note that the policy authorities maximize conditional welfare  $\zeta^c$ . (The Tables also report unconditional welfare,  $E\{\sum_{t=0}^{\infty} U(C_t, L_t, G_t)\}$ , expressed as a permanent equivalent change in consumption,  $\zeta^u$ ;  $\zeta^u$  is decomposed in component  $\zeta^{u,m}$  and  $\zeta^{u,v}$ , respectively, that reflect the unconditional means of consumption, hours worked and government purchases, and their unconditional variances, respectively.)

#### 3.1. Exogenous government purchases

The existing literature on fiscal policy rules assumes that government purchases are exogenous. Cols. 1-5 of Table 1 show predicted business cycle statistics for model variants with exogenous government purchases. These variants assume  $\Gamma_i^Z = \Gamma_\tau^Z = 0$  (i.e. the interest rate is solely a function of the interest rate, and the tax rate is solely a function of the stock of debt) and  $G_t = G + \gamma_t$ , where  $\gamma_t$  is the exogenous shock defined in (14). Cols. 1-2 shows results for a version of the baseline model with sticky prices. Col. 3 (labeled "Flex P") assumes flexible prices (in that structure, monetary policy has real effects, because government debt is nominal). Col. 3 (labeled "RBC") considers a version of the (exogenous  $G_t$ ) model with flexible prices and real (indexed) debt (monetary policy has no real effects in that version). Cols.4-5 pertains to the first-best (undistorted) equilibrium (i.e. to the solution of a social planning problem in which household welfare is maximized subject to the economy's resource constraints, taking as given the exogenous path of  $G_t$ ). Panels (a)-(d) of Tables 2 and 3 show impulse responses for these model variants.

##### ***Model version with sticky prices (Cols. 1-2, Table 1)***

Under sticky prices, the optimized parameters of the interest rate rule and of the tax rate rule are  $\Gamma_i^\pi = 68.84$  and  $\Gamma^D = 4.84$ , respectively. Thus, optimized monetary policy has a strict

anti-inflation stance (an increase in inflation triggers a sharp increase in the nominal interest rate); as a result, the standard deviation of the inflation rate is (essentially) zero.

Optimized fiscal policy implies that an increase in the stock of debt by an amount that corresponds to (quarterly) steady state GDP ( $Y$ ), raises the tax rate by 4.8 percentage points (roughly one-fifth of the steady state tax rate). Experiments with alternative values of  $\Gamma^D$  (holding constant  $\Gamma_i^\pi$  at  $\Gamma_i^\pi = 68.84$ ) show that the model has a stationary solution when  $\Gamma^D \geq 1.62$ . Values of  $\Gamma^D$  that are smaller than the optimized coefficients (but larger than 1.62) entail fluctuations in the stock of debt that violate the constraint on debt (13). Despite that constraint, the stock of debt is highly volatile: its standard deviation (normalized by steady state GDP) is 32.8%--which is markedly larger than the standard deviation of GDP, 6.23%, but smaller than the standard deviation of the capital stock (normalized by  $Y$ ), 44.84% (not shown in Table).

The tax rate undergoes non-negligible, highly persistent, countercyclical fluctuations (standard deviation of tax rate: 1.61%; autocorrelation: 0.994; correlation with GDP: -0.69).

Under optimized policy, the standard deviation of consumption (6.12%) is slightly larger than that of output (5.99%). It appears that consumption undergoes wider low-frequency fluctuations than output--at business cycle frequencies, by contrast, consumption is predicted to be less volatile than output (standard deviations of Hodrick-Prescott filtered consumption and output: 1.24% and 2.52%, respectively).

The unconditional mean value of the capital stock and of the tax rate exceed their respective steady state values (by 0.30% and 0.10%, respectively); this helps to understand why mean hours worked and mean consumption are slightly below steady state (by -0.09% and -0.02%, respectively). Conditional welfare is lower than steady state welfare,  $\zeta^c = -0.066\%$  (and the same holds for unconditional welfare:  $\zeta^u = -0.137\%$ ). The "mean component" of the welfare measure is positive,  $\zeta^{c,m} = 0.077\%$ , but that effect is dominated by the negative effect of consumption variance on welfare:  $\zeta^{c,v} = -0.144\%$ .

The predictions that were just discussed are based on the assumption that the economy is simultaneously subjected to shocks to productivity and to government purchases (see Col. 1 in Table 1). Col. 2 shows predictions for the case where there are just government purchases shocks. These shocks explain only about 2% of the variances of consumption and output and only 16% of the variances of the tax rate and of the stock of debt (that are generated when there are simultaneous shocks to productivity and to government purchases).  $G_t$  shocks also have a markedly smaller effect on welfare than productivity shocks ( $\zeta^c = -0.009\%$  when there are just  $G_t$  shocks).

Under staggered price setting, inflation induces inefficient dispersion of prices across intermediate goods producers. When price stickiness is the only form of economic inefficiency (so that the equilibrium under flexible prices is efficient), then optimal monetary policy fully stabilizes inflation--that policy fully eliminates price dispersion across firms (e.g., Rotemberg and Woodford, 1997). The economy here has other distortions (monopolistic competition; distorting taxes)--nevertheless, optimized policy under sticky prices entails (almost) full inflation stabilization.

Full inflation stabilization implies: (i) all firms set identical prices (as is the case under flexible prices); (ii) nominal bonds are riskless, in real terms. This helps to understand why the equilibrium under sticky prices (with strict inflation stabilization) resembles very closely the equilibrium generated by optimized fiscal policy in the "RBC" version of the model (in which prices are fully flexible, and the government issues indexed debt), as can be seen by

comparing Cols. 1 and 4 in Table 1. The optimized fiscal policy parameter  $\Gamma^D$  ( $\approx 5$ ) is very similar in the sticky prices setting and in the "RBC" setting.<sup>6</sup>

A comparison between the distorted (sticky-prices or RBC) economies and the first-best equilibrium (Cols. 5-6) shows: (i) The *level* of economic activity and welfare are noticeably higher in the first-best equilibrium--for example, steady state consumption and hours worked are 75% and 51% higher, respectively; the welfare difference is equivalent to a 21.46% consumption increase (not reported in Table).<sup>7</sup> (ii) In the distorted economy, output, hours worked and investment respond less strongly to productivity shocks, and more strongly to government purchases shocks than in the first-best equilibrium.

### ***Model version with flexible prices (Col. 3, Table 1)***

Optimal policy under flexible prices (with nominal debt) is markedly different from optimal policy under sticky prices: under flex prices, the optimized policy responses are:  $\Gamma_i^\pi = 1.008$ ,  $\Gamma^D = 0.18$ . Note that the optimized  $\Gamma^D$  is smaller than unity. In the "RBC" model variant with real unconditional debt,  $\Gamma^D < 1$  would imply that public debt grows without bounds. In the model variant with nominal debt, however,  $\Gamma^D < 1$  is compatible with stationary public debt, as there the price level can adjust to ensure that real debt is stationary, a mechanism familiar from the "Fiscal Theory of the Price Level". The tax rate fluctuates much less than in the sticky prices variant, but inflation is much more volatile (standard deviation of  $\tau_t$  [ $\Pi_t$ ] : 0.28% [65.47%] under flexible prices, compared to 1.61% [0.00%] under sticky prices).

This finding is consistent with earlier studies on optimal (Ramsey) fiscal/monetary policy in flex-price economies with nominal debt (e.g., Chari et al., 1991; Chari and Kehoe, 1999): in such economies, the government relies heavily on unanticipated changes in inflation to make the real debt return state contingent. Tables 2 and 3 show that--in the setting with flexible prices and nominal debt--the fiscal/monetary authority responds to a positive innovation to government purchases by triggering an unanticipated rise in the price level, which leads to an unanticipated fall in the real value of outstanding government debt; a positive productivity innovation, by contrast, induces an unanticipated fall in the price level, and thus an unanticipated rise in the real value of public debt.

The fact that the tax rate fluctuates very little under flexible prices helps to understand why the *cyclical* behavior of the flex-prices economy mimics very closely the cyclical features of the undistorted (first-best) equilibrium, as can be seen by comparing Col. 3 and Col. 5 in Table 1, and the impulse responses in Panels (b) and (d) in Tables 2 and 3.

Schmitt-Grohé and Uribe (2002) and Siu (2004) recently shown that Ramsey fiscal/monetary policy entails much lower inflation volatility when prices are sticky. My results in Table 1 confirm this result (using simple optimized policy rules in a richer model). As discussed above, when prices are sticky, then time-varying inflation induces inefficient price dispersion across firms--under sticky prices, the fiscal/monetary authority thus selects a time path for inflation that is less volatile, and it relies more heavily on tax changes to meet its budget constraint.

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<sup>6</sup> As monetary policy has no real effects, in the RBC variant, the behavior of inflation and the nominal interest rate cannot be determined from welfare maximization--and thus no predictions for nominal variables (and the policy parameter  $\Gamma_i^\pi$ ) are reported, for that variant.

<sup>7</sup>The welfare figures ( $\zeta$ ) for the first-best equilibrium shown in Cols, 5-6 of Table 1 are % equivalent variations in consumption, relative to the steady state of the first-best allocation.

### 3.2. Endogenous government purchases

Cols. 7-10 of Table 1 report results for the model version with endogenous  $G_t$ . Cols. 7-8 assume sticky prices, while Cols. 9-10 show results for the first best equilibrium (with endogenous  $G_t$ ).<sup>8</sup> The following policy rule is used in Cols. 7-8:

$$G_t = G - \Gamma^D i(D_t/P_t - \mathbf{d}) + \Gamma_G^\gamma \gamma_t,$$

i.e.  $G_t$  is a function of the stock of debt, and of the preference shock  $\gamma_t$ . (The model variant with exogenous government purchases discussed above is a special case of that rule in which  $\Gamma^D = 0$ ,  $\Gamma_G^\gamma = 1$ .)

The optimized monetary/fiscal policy rules (under sticky prices) again imply very strict inflation stabilization. Optimized fiscal policy fully accommodates changes in the private sector's preference for government consumption ( $\Gamma_G^\gamma \approx 1$ ): on impact,  $G_t$  adjusts roughly one-to-one to  $\gamma_t$  shocks (see Panel (e), Table 3).

The correlation between government purchases and GDP is much higher when  $G_t$  is endogenous (0.90 compared to 0.05 under exogenous  $G_t$ ). This reflects the fact that a positive productivity shock raises household wealth, which increases the household's "demand" for government purchases; a benevolent government thus raises  $G_t$ , in response to the productivity shocks (when  $G_t$  is endogenous).

### 3.3. Other policy rules

Table 3 report predictions for policy rules that permit responses of policy variables to additional variables. Cols. 1-5 assume exogenous government purchases.

Cols. 1-3, consider tax rate rules that allow for a direct response of the tax rate to inflation (rule R1), to GDP (R2) and to government purchases (R3):  $\tau_t = \tau + \Gamma^D i(D_t/P_t - \mathbf{d})/Y + \Gamma^Z Z_t$ , with  $Z_t = \hat{\Pi}_t$ ,  $\hat{Y}_t$  or  $\hat{G}_t$ . (Cols. 1-3 use an interest rate rule that responds just to inflation). The welfare gain from using R1-R3 (compared to the baseline rule considered in Col. 1 of Table 1) is very small: conditional welfare,  $\zeta^c$ , increases by an amount that does not exceed 0.003%. As before, monetary policy has a strict anti-inflation stance.

Cols. 4-5 in Table 3 consider monetary policy rules that exhibit a direct response of the interest rate to GDP (R4) or to the government's budget deficit (R5):  $i_t = i + \Gamma_i^\pi \hat{\Pi}_t + \Gamma^Z Z_t$ , with  $Z_t = \hat{Y}_t$  or  $Def_t$ . The optimized response coefficient for output is very small (0.004). The optimized coefficient attached to the deficit is negative: holding constant the inflation rate, the monetary authority should respond to an increase in the fiscal deficit by lowering the interest rate. However, the welfare gain from the extended Taylor rules R5 and R6 is very small, compared to the baseline rule considered in Table 1 ( $\zeta^c$  increases by less than 0.001%).

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<sup>8</sup> In Cols. 7-8, the steady state value of  $G_t$  is the same as in the model versions with exogenous  $G_t$ . By contrast, in the first-best equilibrium with endogenous  $G_t$  (Cols. 9-10), the steady state value of  $G_t$  is changed; in that equilibrium, the marginal rate of substitution between  $C_t$  and  $G_t$  is set at the corresponding rate of transformation (unity), which implies:  $G_t = \Psi C_t + \gamma_t$ .

Col. 6 in Table 3 assumes endogenous government purchases and postulates that  $G_t$  is set as a function of the stock of public debt and of real GDP (R6). That specification leads to a slight loss in welfare, compared to the government spending rule considered in Col. 7 of Table 1 (where  $G_t$  is set as a function of the preference shock  $\gamma_t$ ).

#### **4. Conclusion**

This paper studies the welfare effects of monetary and fiscal policy rules, in a dynamic general equilibrium model with sticky prices. The model features capital accumulation and endogenous labor effort, and exogenous shocks to productivity and to the household's preference for government purchases. Government purchases are valued positively by the private sector. These purchases are financed using a proportional income tax. The government issues nominal one-period bonds. Monetary policy is described by a Taylor-style interest rate rule; fiscal policy is described by rules that set the tax rate and government purchases as functions of government debt and of macroeconomic variables. Attention focuses on policies that induce stationary fluctuations of public debt. A quadratic approximation is used to solve the model, and to compute household welfare. The paper determines the policy response coefficients rules (consistent with debt being stationary) that maximize household welfare. Optimized monetary policy has a strong anti-inflation stance. Optimized fiscal policy implies procyclical fluctuations of the income tax rate, and procyclical government purchases.

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Table 1. Optimized policy rules and first best allocations

	Exogenous government purchases						Endogenous gov't purchases			
	Sticky Prices		Flex P	RBC	First best		Sticky Prices		First best	
	$\theta, \gamma$	$\gamma$	$\theta, \gamma$	$\theta, \gamma$	$\theta, \gamma$	$\gamma$	$\theta, \gamma$	$\gamma$	$\theta, \gamma$	$\gamma$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Standard deviations (in %)										
$Y$	5.99	0.88	5.00	5.98	5.58	0.38	7.28	0.57	6.12	0.42
$C$	6.12	0.99	4.42	6.12	4.30	0.09	5.74	0.63	4.22	0.10
$I$	18.47	2.42	19.50	18.45	19.02	0.41	25.51	2.92	20.06	0.46
$\Pi$	0.00	0.00	65.47	---	---	---	0.00	0.00	---	---
$i$	0.11	0.01	66.06	---	---	---	0.17	0.01	---	---
$G$	3.20	3.20	3.20	3.20	3.20	3.20	4.58	1.55	5.25	3.10
$\tau$	1.61	0.65	0.28	1.62	---	---	0.94	0.39	---	---
Debt	33.04	13.34	28.75	30.04	---	---	1.69	0.70	---	---
Def	0.99	0.40	128.75	0.99	---	---	0.41	0.14	---	---
Correlations with GDP										
$i$	-0.22	0.04	-0.22	---	---	---	-0.11	-0.13	---	---
$G$	0.05	0.37	0.13	0.55	0.06	0.99	0.90	-0.21	0.75	0.99
$\tau$	-0.69	-0.61	0.31	-0.68	---	---	-0.90	-0.83	---	---
Autocorrelations										
$Y$	0.96	0.95	0.93	0.96	0.92	0.94	0.97	0.72	0.93	0.94
$i$	0.93	0.93	0.99	---	---	---	0.93	0.97	---	---
$G$	0.95	0.95	0.95	0.95	0.95	0.95	0.94	0.74	0.97	0.95
$\tau$	0.99	0.99	0.99	0.99	---	---	0.97	0.97	---	---
Debt	0.99	0.99	0.96	0.99	---	---	0.97	0.99	---	---
Means (in %)										
$Y$	0.02	-0.01	0.29	0.03	0.10	0.00	0.18	0.00	0.10	0.00
$C$	-0.02	-0.01	0.31	-0.02	0.07	0.00	0.16	0.00	0.07	0.00
$L$	-0.09	-0.01	0.17	-0.09	0.01	0.00	-0.03	-0.00	-0.01	0.00
$K$	0.30	-0.01	0.68	0.29	0.30	0.00	0.66	0.01	0.29	0.00
$G$	0.00	0.00	0.00	0.00	0.00	0.00	-0.06	-0.00	0.07	0.00
$\tau$	0.10	0.01	-0.14	0.10	---	---	0.01	0.00	---	---
Debt	2.27	0.12	-10.80	2.22	---	---	0.02	0.00	---	---
Welfare (% equivalent permanent variation in consumption)										
$\zeta^u$	-0.137	-0.009	0.090	-0.138	-0.023	-0.000	-0.010	-0.005	-0.010	-0.000
$\zeta^{u,m}$	0.049	-0.004	0.188	0.049	0.068	-0.000	0.176	-0.000	0.097	-0.000
$\zeta^{u,v}$	-0.187	-0.005	-0.097	-0.187	-0.092	-0.000	-0.186	-0.005	-0.107	-0.000
$\zeta^c$	-0.066	-0.003	-0.046	-0.066	-0.031	-0.000	-0.022	-0.005	-0.019	-0.000
$\zeta^{c,m}$	0.077	-0.000	0.035	0.077	0.046	-0.000	0.134	-0.000	0.071	-0.000
$\zeta^{c,v}$	-0.144	-0.003	-0.083	-0.143	-0.078	-0.000	-0.157	-0.005	-0.097	-0.000
Policy parameters										
$\Gamma_i^\pi$	68.84	68.84	1.01	---	---	---	6165	6165	---	---
$\Gamma^D$	4.84	4.84	0.18	4.84	---	---	55.84	55.84	---	---
$\Gamma_G^\gamma$	---	---	---	---	---	---	1.01	1.01	---	---

Table 2. % responses to 1 standard deviation productivity innovations

	<i>Y</i>	<i>C</i>	<i>I</i>	<i>L</i>	<i>K</i>	<i>P</i>	<i>i</i>	<i>G</i>	$\tau$	<i>D</i>	$\theta$
(a) Exogenous government purchases—Baseline model											
$\tau = 0$	1.55	0.82	7.67	0.72	0.19	0.00	0.03	0.00	0.00	-0.39	1.00
$\tau = 4$	1.31	0.94	5.25	0.45	0.76	0.00	0.02	0.00	-0.06	-1.42	0.81
$\tau = 24$	0.63	0.81	0.69	0.05	1.24	0.00	-0.01	0.00	-0.17	-3.63	0.29
$\tau = 100$	0.12	0.17	0.06	0.06	0.27	-0.01	-0.00	0.00	-0.08	-1.71	0.01
(b) Exogenous government purchases—Flexible prices, nominal debt											
$\tau = 0$	1.71	0.75	9.33	0.94	0.23	-2.30	-3.40	0.00	0.01	6.48	1.00
$\tau = 4$	1.30	0.85	5.60	0.40	0.85	-11.51	-3.48	0.00	0.02	5.46	0.81
$\tau = 24$	0.39	0.59	-0.04	-0.22	1.09	-45.17	-3.15	0.00	0.01	2.41	0.29
$\tau = 100$	-0.00	0.01	-0.06	-0.02	0.02	-86.50	-2.35	0.00	0.01	0.57	0.01
(c) Exogenous government purchases—Flexible prices, indexed debt (RBC model)											
$\tau = 0$	1.53	0.83	7.54	0.70	0.19	---	---	0.00	-0.00	-0.38	1.00
$\tau = 4$	1.31	0.94	5.22	0.45	0.75	---	---	0.00	-0.06	-1.40	0.81
$\tau = 24$	0.65	0.82	0.74	0.07	1.26	---	---	0.00	-0.18	-3.48	0.29
$\tau = 100$	0.11	0.16	0.04	0.06	0.27	---	---	0.00	-0.08	-1.44	0.01
(d) Exogenous government purchases—First Best											
$\tau = 0$	2.07	0.66	9.40	1.41	0.23	---	---	0.00	---	---	1.00
$\tau = 4$	1.51	0.82	5.45	0.69	0.86	---	---	0.00	---	---	0.81
$\tau = 24$	0.37	0.58	-0.23	-0.21	0.97	---	---	0.00	---	---	0.29
$\tau = 100$	0.00	0.01	-0.03	-0.01	0.03	---	---	0.00	---	---	0.01
(e) Endogenous government purchases—Baseline model											
$\tau = 0$	1.50	0.84	7.21	0.65	0.18	0.00	0.06	0.00	-0.00	-0.38	1.00
$\tau = 4$	1.96	1.03	8.11	1.23	1.06	0.00	0.03	1.12	-0.21	-0.38	0.81
$\tau = 24$	0.64	0.80	0.10	-0.04	1.55	0.00	-0.03	0.45	-0.09	-0.15	0.29
$\tau = 100$	0.01	0.03	-0.06	-0.01	0.06	0.00	-0.00	0.01	-0.00	-0.00	0.01
(f) Endogenous government purchases—First best											
$\tau = 0$	2.20	0.61	9.92	1.59	0.24	---	---	0.61	---	---	1.00
$\tau = 4$	1.64	0.80	5.74	0.84	0.91	---	---	0.80	---	---	0.81
$\tau = 24$	0.43	0.58	-0.25	-0.14	1.02	---	---	0.58	---	---	0.29
$\tau = 100$	0.01	0.01	-0.03	-0.01	0.03	---	---	0.01	---	---	0.01

Notes:  $\tau$ : periods after shock. Columns labeled *Y*, *C*, etc. show responses of the corresponding variables. Period  $\tau$  responses for capital and bonds (*K*,*B*) pertain to stocks at the *end* of  $\tau$  --  $K_{\tau+1}, B_{\tau+1}/P_t$ . The impulse responses are generated as follows. At a given date, say *T*, all state variables are set at steady state values. A "baseline" path for the endogenous variables is computed by setting all exogenous innovations to zero in periods  $t \geq T$ . Then responses to one-time 1 standard deviation exogenous innovation at *T* are computed; the Table reports differences/relative deviations (that have been multiplied by 100, i.e. expressed in percentage terms) of these responses from the "baseline" path; responses of interest rates (*i*) and *Debt*: differences from baseline path (*Debt*: in real terms and normalized by steady state GDP); responses of remaining variables: relative deviations from baseline path.

Table 3. % responses to 1 standard deviation government purchases shocks

	<i>Y</i>	<i>C</i>	<i>I</i>	<i>L</i>	<i>K</i>	<i>P</i>	<i>i</i>	<i>G</i>	$\tau$	<i>D</i>	$\gamma$
(a) Exogenous government purchases—Baseline model											
$\tau = 0$	0.26	-0.08	0.92	0.34	0.02	0.00	0.01	1.00	0.00	0.13	1.00
$\tau = 4$	0.17	-0.06	0.40	0.20	0.07	0.00	0.00	0.81	0.02	0.62	0.81
$\tau = 24$	-0.05	-0.09	-0.32	-0.06	-0.02	0.00	-0.00	0.29	0.08	1.62	0.29
$\tau = 100$	-0.05	-0.06	-0.05	-0.03	-0.10	0.00	0.00	0.01	0.04	0.74	0.01
(b) Exogenous government purchases—Flexible prices, nominal debt											
$\tau = 0$	0.18	-0.05	0.18	0.24	0.00	1.12	1.60	1.00	-0.01	-2.94	1.00
$\tau = 4$	0.17	-0.03	0.21	0.21	0.03	5.68	1.57	0.81	-0.01	-2.45	0.81
$\tau = 24$	0.06	0.00	0.01	0.07	0.04	30.93	1.50	0.29	-0.01	-1.08	0.29
$\tau = 100$	0.01	0.01	0.01	0.01	0.01	179.35	1.37	0.01	-0.01	-0.31	0.01
(c) Exogenous government purchases—Flexible prices, indexed debt (RBC model)											
$\tau = 0$	0.26	-0.08	0.97	0.35	0.02	---	---	1.00	-0.00	0.13	1.00
$\tau = 4$	0.17	-0.07	0.41	0.20	0.08	---	---	0.81	0.03	0.62	0.81
$\tau = 24$	-0.05	-0.10	-0.34	-0.06	-0.03	---	---	0.29	0.08	1.56	0.29
$\tau = 100$	-0.05	-0.06	-0.04	-0.03	-0.10	---	---	0.02	0.03	0.62	0.01
(d) Exogenous government purchases—First Best											
$\tau = 0$	0.12	-0.04	0.20	0.16	0.00	---	---	1.00	---	---	1.00
$\tau = 4$	0.10	-0.03	0.11	0.12	0.02	---	---	0.81	---	---	0.81
$\tau = 24$	0.03	-0.00	-0.00	0.03	0.02	---	---	0.29	---	---	0.29
$\tau = 100$	0.00	-0.00	-0.00	0.00	0.00	---	---	0.01	---	---	0.01
(e) Endogenous government purchases—Baseline model											
$\tau = 0$	0.29	-0.09	1.13	0.38	0.03	0.00	-0.00	1.01	0.00	0.13	1.00
$\tau = 4$	-0.14	-0.10	-1.10	-0.18	-0.06	0.00	-0.00	0.29	0.10	0.18	0.81
$\tau = 24$	-0.05	-0.09	-0.04	-0.01	-0.16	0.00	0.00	0.09	0.04	0.07	0.29
$\tau = 100$	-0.00	-0.00	0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.01
(f) Endogenous government purchases—First best											
$\tau = 0$	0.14	-0.04	0.23	0.18	0.01	---	---	0.95	---	---	1.00
$\tau = 4$	0.11	-0.03	0.13	0.14	0.02	---	---	0.78	---	---	0.81
$\tau = 24$	0.04	-0.00	-0.01	0.04	0.02	---	---	0.28	---	---	0.29
$\tau = 100$	0.00	-0.00	-0.00	0.00	0.00	---	---	0.01	---	---	0.01

Notes:  $\tau$ : periods after shock. Columns labeled *Y*, *C*, etc. show responses of the corresponding variables. See Table 3 for further information.

Table 4. Other policy rules

	R1	R2	R3	R4	R5	R6
	(1)	(2)	(3)	(4)	(5)	(6)
Standard deviations (in %)						
$Y$	6.23	6.80	6.02	5.98	5.99	7.12
$C$	6.18	6.58	6.16	6.12	6.14	5.49
$I$	21.41	23.75	18.45	18.46	18.32	24.62
$\Pi$	0.03	0.00	0.00	0.00	0.01	0.00
$i$	0.09	0.14	0.11	0.11	0.10	0.16
$G$	3.20	3.20	3.20	3.20	3.20	4.70
$\tau$	1.62	1.63	1.62	1.61	1.61	0.73
Debt	32.85	30.10	30.32	33.04	30.32	1.38
Def	0.98	0.81	0.96	0.99	0.98	0.36
Correlations with GDP						
$i$	-0.16	-0.15	-0.22	-0.27	-0.27	-0.00
$G$	0.05	0.07	0.05	0.05	0.05	0.98
$\tau$	-0.69	-0.80	-0.69	-0.67	-0.69	-0.96
Autocorrelations						
$Y$	0.94	0.95	0.96	0.96	0.96	0.97
$i$	0.90	0.93	0.94	0.93	0.94	0.92
$G$	0.95	0.95	0.95	0.95	0.95	0.97
$\tau$	0.99	0.99	0.99	0.99	0.99	0.96
Debt	0.99	0.99	0.99	0.99	0.99	0.96
Means (in %)						
$Y$	0.04	0.03	0.03	0.03	0.03	0.17
$C$	0.00	-0.03	-0.01	-0.02	-0.01	0.15
$L$	-0.08	-0.13	-0.09	-0.09	-0.09	-0.03
$K$	0.36	0.43	0.31	0.30	0.31	0.61
$G$	0.00	0.00	0.00	0.00	0.00	-0.02
$\tau$	0.10	0.15	0.10	0.10	0.10	0.01
Debt	1.60	4.20	1.92	2.22	1.94	0.02
Welfare (% equivalent permanent variation in consumption)						
$\zeta^u$	-0.125	-0.149	-0.133	-0.137	-0.133	-0.005
$\zeta^{u,m}$	0.065	0.067	0.056	0.049	0.055	0.168
$\zeta^{u,v}$	-0.191	-0.216	-0.190	-0.187	-0.188	-0.173
$\zeta^c$	-0.063	-0.063	-0.066	-0.066	-0.066	-0.017
$\zeta^{c,m}$	0.084			0.077		
$\zeta^{c,v}$	-0.147			-0.143		
Policy parameters						
$\Gamma_i^\pi$	-2.78	193.79	158.10	132.72	85.24	1000
$\Gamma^D$	5.68	3.78	5.30	4.84	5.27	52.34
$\Gamma^Z$	12.88	-0.10	0.004	0.0004	-1.08	0.03

$$\text{R1: } \tau_t = \tau + \Gamma^D i (D_t/P_t - \mathbf{d})/Y + \Gamma^Z \widehat{\Pi}_t; \quad \text{R2: } \tau_t = \tau + \Gamma^D i (D_t/P_t - \mathbf{d})/Y + \Gamma^Z \widehat{Y}_t;$$

$$\text{R3: } \tau_t = \tau + \Gamma^D i (D_t/P_t - \mathbf{d})/Y + \Gamma^Z \widehat{G}_t; \quad \text{R4: } i_t = i + \Gamma_i^\pi \widehat{\Pi}_t + \Gamma^Z \widehat{Y}_t;$$

$$\text{R5: } i_t = i + \Gamma_i^\pi \widehat{\Pi}_t + \Gamma^Z \text{Def}_t;$$

$$\text{R6: } G_t/Y = G/Y - \Gamma^D i (D_t/P_t - \mathbf{d})/Y + \Gamma^Z \widehat{Y}_t.$$