Micro 3 Recap

• Issues in consumer choice
  – Revealed preference: Theory of choice – based on observable choices
  – Aggregation issues: Aggregate demand and aggregate wealth

• General equilibrium in exchange economies
  – Existence of Walrasian equilibrium; Walras’ law
  – The two welfare theorems; Pareto optimality
  – Social welfare functions
• General equilibrium in production economies
  – Walras’ law in a production economy; existence (Debreu, 1959)

• Imperfections
  – Public goods
  – Externalities

• General equilibrium over time
  – Dynamic programming
  – Intro to GE models of finance
1 Revealed preference

- Alternative to theory of choice based on utility functions

- Predictions about consumer behavior derived from assumptions on observable choices, rather than unobservable preferences

- Directly revealed preferred: \( x \) DRP to \( y \) (G)

- Consistency requirements – Weak axiom: If \( y \) is feasible when \( x \) is chosen, then, when \( y \) is chosen, \( x \) must not be feasible (G)

- Why do we like WARP? WARP necessary condition for utility max, i.e \( UM \Rightarrow WARP \); hence, WARP check on whether consumers max utility
• WARP not sufficient; requires Strong Axiom (nec. & suff. for UM)

• More general approach: GARP. Empirically, GARP is satisfied for most income groups over long periods of time; also, concerns for fairness and altruism enter rational utility calculus
2 Aggregation

- Does aggregation preserve properties of individual demand?
  
- Generally, no!
  
- Aggregate demand can be written as function of aggregate wealth, rather than indv. wealth levels, if and only if preferences have Gorman-form

- Preferences of hypothetical representative consumer independent of SWF when indv. prefs of Gorman-form

- Aggregation has regularizing properties – discrete indv. demand can translate into (almost) continuous aggregate demand
3 GE in exchange economies

3.1 Existence

- Definition of WE in exchange economies

- Aggregate excess demand: Satisfies (among other things) Walras’ law – value of AED identically zero. \( (k - 1) \)-market clearing result

- Existence of Walrasian equilibrium for exchange economies
  - Test of logical coherence of GE model
  - Basic (simple) idea: Find price system \( p^* \) such \( \text{AED}(p^*) = 0 \)
Proof: Use Brouwer’s fixed point thm on “price adjustment function” on price simplex and rearrange, using Walras’ law

Simple proof; can show existence for more general preference structure and weaker notions of monotonicity

Numerous objections to WE! Examples: Requires price taking behavior; strong demands on rationality; somewhat far-fetched in dynamic setting

No description of how markets actually operate, but in experimental Walrasian market, WE looks good, with fast convergence (Smith, 1962)
3.2 PE and Welfare theorems

- Equivalence between two common concepts: Weak and strong PE

- 1st thm: Any WE-allocation is PE. Simple proof!

- 2nd thm: Any PE allocation can be supported as WE with lump-sum re-distributions of endowments (req. convex prefs)

- Proof: Requires separating hyperplane theorem (G)
3.3 Welfare maximization

- PE not very specific as normative criterion (G)

- One solution: Postulate existence of Social Welfare Function

- SWF aggregates indv. utilities into social utility (one number)

- Any social welfare maximum is PE . . . and can be sustained as WE with redistribution

- (Calculus characterization of PE ⇒) Every PE allocation max of some welfare function (choose SWF weights appropriately)
4 GE in production economies

- Introduce aggregate net supply

- Generalize AED and Walras’ law to include production

- Main results carry over from exchange economies: Existence, welfare theorems and welfare analysis

- Robinson Crusoe economies and separating hyperplanes (G)
5 Imperfections

- Public goods: Non-excludeability and non-rivalness in consumption (broadcast vs. cable)
  - discrete public goods: reservation prices
  - private provision of public goods not PE; positive externality
  - Lindahl equilibrium; personalized prices
  - Mechanism design
• Externalities
  – Consumption and production externalities – things that affect utility outside of price system
  – Possible solutions: Pigouvian taxes (quotas); compensation mechanisms; create missing markets; rely on bilateral bargaining (Coase Thm)
6 GE over time

- Methods: Stochastic dynamic programming
  
  - Bellman equation and optimality principle: Take best action now and follow optimal plan in the future. Result: Many-period decision problems broken down into series of two-period decision problems
  
  - Envelope theorem and Keynes-Ramsey rule under uncertainty

- Reinterpretation of GE model to include temporal dimension
  
  - same good available in $T$ different periods: $T$ different goods
  
  - results of GE model carry over: existence, optimality etc.
– also: different states of the world; trade in contingent commodities \(\Rightarrow\) efficient allocation of risk (insurance).

– In reality, we use assets
7 Asset markets in GE

- Assets are titles to receive goods or financial payments at future date depending on state

- Two roles of assets
  - move wealth around across time and states of nature (means)
  - diversify risk (variances)

- General conclusion of chapter: Asset prices = risk free rate + risk premium
7.1 CAPM model

- Workhorse model; benchmark for finance theory

- Central assumptions: Perfect capital markets, investors look only at means and variances of asset returns (quadratic u-fn; normally distr. asset returns)

- Solve by min variance s.t. required rate of return – appeal to Mutual fund theorem: everyone holds risky assets in same proportions in eq.

- Expected return function $= \text{risk free rate} + \text{risk premium}$

- Risk premium depends on covariance with market portfolio ($\beta$)
7.2 APT

- Different approach to asset pricing: asset prices depend on individual factors and common (macro) factors

- Idea: Create risk free portfolio by choosing asset structure such that risky factors cancel

- Price this portfolio using arbitrage argument: In eq., risk free portfolio must have return = risk free rate $\Rightarrow$ exp. return = risk free rate + risk premium

- Can show: APT model with market return as only factor $\Rightarrow$ simple CAPM
7.3 C-CAPM

- Asset pricing model based on more general preference structure (improvement over CAPM)

- More of an economic (macro) model – not asset pricing model as such

- Structure similar to dyn.prog. model – yields CAPM-style equation, with consumption instead of wealth

- Assets priced based on covariance with aggregate consumption

- Equity premium example
Complete markets argument can lead to similar pricing formula, but mostly theoretical interest